MCDM by Normalized Euclidean Distance in Intuitionistic Multi-Fuzzy Sets

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Abstract *- In this paper we propose an efficient approach for multi-criteria decision making (MCDM) based on intuitionistic multi-fuzzy sets (IMFS). Intuitionistic multi-fuzzy set of an universe set is very useful in providing a flexible model to elaborate uncertainty and vagueness involved in decision making. In this paper we used the concept of IMFS and proposed its application in the field of determining the best mechanism under the satisfaction of rider's aptitude by using Normalized Euclidean Distance method to measure the distance between the each mechanism of heavy motor vehicle (HMV) and each rider's aptitude with respect to multi-criteria respectively. Finally, optimal determination is obtained by looking for the smallest distance between each rider's aptitude and each HMV's mechanism.*

Keywords - *Fuzzy set, Intuitionistic fuzzy set (IFS), Intuitionistic multi-fuzzy set (IMFS), Normalized Euclidean Distance*

I. INTRODUCTION

Lofti A. Zadeh [36] proposed Fuzzy set in 1965 and it allows the uncertainty of a set with a membership degree (μ) between 0 and 1. That is, the membership function $\mu \in [0,1]$ and the non-membership function (ν) equals to ($1-\mu$) which is also in [0,1]. The generalisation of the fuzzy set, that is, Intuitionistic Fuzzy set (IFS) was introduced by Krasssimir T. Atanassov [1, 2, 3, 4] in 1983. The IFS represents the uncertainty with respect to both membership ($\mu \in [0,1]$) and non-membership ($\nu \in [0,1]$) such that $0 \le \mu + \nu \le 1$. The number $\pi = (1 - \mu - \nu)$ is called the hesitation degree or index of IFS. The study of distance measures of IFS's gave by several authors like Y. H. Li, D. L. Olson, Q. Zheng, Li and Cheng, Liang and Shi [10, 11, 12, 13, 14, 35]. They gives lot of measures, each representing specific properties and behaviour in real-life decision making and pattern recognition works. Based on Hamming distance, Szmidt and Kacprzyk [30, 31, 32] was introduced the distance and similarity measure between IFS's and its application is widely used in various fields like medical diagnosis, logic programming, decision making. Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc., since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object (Szmidt and Kacprzyk, 1997, 2001). Atanassov (1999, 2012) carried out many applications of IFS using distance measures approach. Distance measure between IFS's is an important concept in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction.

The Multi-Fuzzy set (MFS) was introduced by Sabu Sebastian and T.V.Ramakrishnan [29] in 2010 can occur more than once with the possibly of the same or the different membership values, which is based on the Multiset [5] repeats the occurrences of any element. And recently, the new concept Intuitionistic Fuzzy Multisets was proposed by T.K. Shinoj and Sunil Jacob John [28].

As various distance methods of IFS are extended to IMFS distance measures [11, 12, 26, 27, 28], this paper is an extension of the new measure on IMFS using the method of Normalized Euclidean Distance.

II. CONCEPT OF INTUITIONISTIC MULTI-FUZZY SETS

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition [36]

Let X be a non-empty set. A **fuzzy set** A drawn from X is defined as $A = \{\langle x, \mu(x) \rangle / x \in X\}$, where $\mu(x) : X \to [0, 1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership. The generalization of fuzzy sets are the Intuitionistic fuzzy set (IFS) which was proposed by **Atanassov [1, 2]** with independent memberships and non-memberships.

2.2 Definition [15 to 25]

Let X be a non-empty set. A multi-fuzzy set (MFS) A of X is defined as $A = \{ < x, \mu_A(x) > : x \in X \}$ where $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$ and $\mu_i : X \to [0, 1]$, $\forall$ i=1, 2, …, k. Here 'k' is the finite dimension of A. Also note that, for all i, $\mu_i(x)$ is a decreasingly ordered sequence of elements. That is, $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_k(x)$, $\forall x \in X$.

2.3 Definition [1, 2, 3]

Let X be a non-empty set. An **Intuitionistic Fuzzy Set (IFS)** A of X is an object of the form $A = \{ < x, \mu(x), \nu(x) > : x \in X \}$, where $\mu : X \to [0,1]$ and $\nu : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively with $0 \leq \mu(x) + \nu(x) \leq 1$, $\forall x \in X$.

Furthermore, we have $\pi(x) = (1 - \mu(x) - \nu(x))$ is called the **index or hesitation margin** of x in IFS A. $\pi(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi(x) \in [0,1]$. That is, $\pi : X \to [0,1]$ and $0 \leq \pi(x) \leq 1$, $\forall x \in X$. $\pi(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

For example, let A be an IFS with $\mu(x) = 0.4$ and $\nu(x) = 0.5$ which implies that $\pi(x) = (1 - 0.4 - 0.5) = 0.1$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.4, the degree that the object x does not belongs to IFS A is 0.5 and the degree of hesitancy is 0.1".

2.4 Remark [23]

(i) Every fuzzy set A on a non-empty set X is obviously an intuitionistic fuzzy set having the form $A = \{ < x, \mu(x), 1-\mu(x) > : x \in X \}$.

(ii) In the definition 2.3, when $\mu(x) + \nu(x) = 1$, that is, when $\nu(x) = 1- \mu(x) = \mu^c(x)$, A is called fuzzy set.

2.5 Definition [23, 24]

Let $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$ where $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$ and $\nu_A(x) = (\nu_1(x), \nu_2(x), \dots, \nu_k(x))$ such that $0 \leq \mu_i(x) + \nu_i(x) \leq 1$, for all i, $\forall x \in X$. Also for each i = 1, 2, …, k, $\mu_i : X \to [0,1]$, $\nu_i : X \to [0,1]$. Here, $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_k(x)$, $\forall x \in X$. That is, μ_i's are decreasingly ordered sequence. That is, $0 \leq \mu_i(x) + \nu_i(x) \leq 1$, $\forall x \in X$, for all i=1, 2, …, k. Then the set A is said to be an **Intuitionistic Multi-Fuzzy Set (IMFS)** with dimension k of X.

Furthermore, we have $\pi_A(x) = (1_k - \mu_A(x) - \nu_A(x))$ is called the **index of intuitionistic multi-fuzzy set or hesitation margin** of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IMFS A and $\pi_A(x) \in [0,1]^k$. That is, $\pi_i : X \to [0,1]$ and $0 \leq \pi_i(x) \leq 1$, $\forall$ i = 1, 2, … , k and $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IMFS A or not. Here $1_k = (1, 1, \dots, 1)$ (k times).

For example, let A be an IMFS of dimension two with $\mu_A(x) = (0.4, 0.6)$ and $\nu_A(x) = (0.3, 0.2)$ which implies that $\pi_A(x) = (1-0.4-0.3, 1-0.6-0.2) = (0.3, 0.2)$. It can be interpreted as "the degree that the object x belongs to IMFS A is (0.4, 0.6), the degree that the object x does not belongs to IMFS A is (0.3, 0.2) and the degree of hesitancy is (0.3, 0.2)".

2.6 Remark [28]

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.7 Definition [26, 27]

The **Cardinality** of the membership function $\mu_A(x)$ and the non-membership function $\nu_A(x)$ is the length of an element x in an IMFS A denoted as $\eta(A)$ and it is defined as $\eta(A) = |\mu_A(x)| = |\nu_A(x)|$. If A, B, C are the IMFS's defined on X, then their cardinality $\eta = \max\{\eta(A), \eta(B), \eta(C)\}$.

2.8 Definition

Let X be non-empty set such that IMFS's A, B, C $\in$ X. Then **distance measure** d is a mapping $d : X \times X \to [0,1]$ if d(A, B) satisfies the following axioms :

(i) $0 \leq d(A, B) \leq 1$;

(ii) $d(A, B) = 0 \Leftrightarrow A = B$ (That is, faithful condition);

(iii) $d(A, B) = d(B, A)$ (That is, symmetric);

(iv) $d(A, B) + d(B, C) \geq d(A, C)$ (That is, triangle inequality);

(v) if $A \subseteq B \subseteq C$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.

Then d(A, B) is a distance measure between IMFS's A and B. Distance measure is a term that describes the difference between intuitionistic multi-fuzzy sets and can be considered as a dual concept of similarity measure. They have some good geometric properties and satisfied the conditions of metric distance.

2.9 Definition [30, 31, 32]

The **Normalized Euclidean Distance** $\boldsymbol{\ell^E(A, B)}$ between two IMFS's A and B is defined as : For $X = \{ x_1, x_2, \dots, x_k \}$ and $\eta = \max\{ \eta(A), \eta(B) \}$,

$$\ell^E(A, B) = \sqrt{\frac{1}{2\eta k} \sum_{j=1}^{\eta} \sum_{i=1}^{k} \begin{bmatrix} \left(\mu_A^j(x_i) - \mu_B^j(x_i)\right)^2 \\ + \left(\nu_A^j(x_i) - \nu_B^j(x_i)\right)^2 \\ + \left(\pi_A^j(x_i) - \pi_B^j(x_i)\right)^2 \end{bmatrix}}$$

2.10 Remark

Here X denotes the set of all multi-criterias and η represents the cardinality of the IMFS.

III. Application of IMFS's in MCDM problem of determining the mechanism of HMV under the satisfaction of rider's aptitude

Most of them expects the mechanism of HMV is more attractive, efficiently and sensitively. During the time of travelling, the rider's aptitude and the mechanism are both taking a more important role for safety than other reasons. We choose the vehicle in such a way that suitable in financially, safely, ridingly, comfortably, etc. So in our study we take the important five such reasons like as mileage, maintenance cost, pulling power, resale value and safety facts.

The term mileage in HMV depends on vehicle cc, driver skill, location of driving, load on the vehicle, vehicle condition, speed, single usage/multi usage, etc. These are all combinely determines the mileage of HMV. From these we take three main factors such as vehicle cc, vehicle condition and speed to determine the mileage.

The term maintenance cost in HMV depends on vehicle manufacturing (brand of vehicle: due to the cost of spare parts), vehicle's type of usage, service frequently, etc. From these we take three main factors such as vehicle manufacturing, vehicle's usage and service frequently to determine the maintenance cost.

The term pulling power in HMV mainly depends on the torque. When we achieve high speed, gets lower torque and when we achieve lower speed, gets higher torque. To the purpose of climbing on hills, we will need to achieve the higher torque. It also depends on vehicle cc, type of fuel (petrol or diesel), load on the vehicle, speed, gear position, etc. From these we take three main factors such as vehicle cc, load on the vehicle and speed to determine the pulling power.

The term resale value in HMV depends on the vehicle manufacturing, year of model, vehicle condition, vehicle age, running kilometre, etc. From these we take three main factors such as vehicle's manufacturing, vehicle's condition and vehicle's age to determine the resale value.

The term safety facts in HMV depends on the vehicle facility(cost), vehicle handling(driving), vehicle maintenance, condition of the vehicle, vehicle's brand, etc. From these we take three main factors such as vehicle's brand, vehicle maintenance and vehicle facility to determine the safety facts.

Using these three factors in each criteria, we form a three dimensional IMFS in each criteria. Then these all five criterias are combinely form a multi-criteria to determine the best HMV mechanism under the satisfaction of rider's aptitude. Then finally optimal decision was done by calculating the distance of each rider's aptitude from each HMV's mechanism with respect to the specified multi-criteria.

Let $A = \{ A_1, A_2, A_3 \}$ be the set of riders of heavy motor vehicles, $B = \{ B_1, B_2, B_3 \}$ be the set of mechanisms of heavy motor vehicles (HMV's) and C = { mileage, maintenance cost, pulling power, resale value, safety facts } be the set of criterias which are common to both rider's aptitude and HMV's mechanism that decides both suitable for which the efficiency and eminency of each other.

The table 1 below shows rider's aptitude and their eminency proved under the specified multi-criteria. Performances on each criteria is described by three dimensional IMFS's. That is, memberships $(\mu_i)_{i=1,2,3}$ due to various factors, non-memberships $(\nu_i)_{i=1,2,3}$ due to that factors and the correspondin hesitation margin $(\pi_i)_{i=1,2,3}$. After the various observations and tests, the rider's aptitude was obtained the following membership degrees on the various factors as shown in the table below:

Table 1: Rider's Aptitude vs Criterias

		Criterias				
		Mileage	Maintenance cost	Pulling power	Resale value	Safety facts
Rider's Aptitude	A_1	(0.5, 0.4, 0.4) (0.3, 0.3, 0.4) (0.2, 0.3, 0.2)	(0.4, 0.3 0.3) (0.3, 0.6, 0.3) (0.3, 0.1, 0.4)	(0.6, 0.4, 0.5) (0.3, 0.3, 0.3) (0.1, 0.3, 0.2)	(0.4, 0.3, 0.2) (0.3, 0.3, 0.3) (0.3, 0.4, 0.5)	(0.5, 0.4, 0.2) (0.4, 0.3, 0.3) (0.1, 0.3, 0.5)
	A_2	(0.3, 0.2, 0.2) (0.5, 0.4, 0.3) (0.2, 0.4, 0.5)	(0.3, 0.4, 0.6) (0.3, 0.3, 0.2) (0.4, 0.3, 0.2)	(0.5, 0.6, 0.4) (0.3, 0.2, 0.4) (0.2, 0.2, 0.2)	(0.5, 0.3, 0.2) (0.2, 0.2, 0.4) (0.3, 0.5, 0.4)	(0.3, 0.2, 0.1) (0.4, 0.3, 0.4) (0.3, 0.5, 0.5)
	A_3	(0.7, 0.6, 0.5) (0.3, 0.3, 0.3) (0.0, 0.1, 0.2)	(0.2, 0.3, 0.4) (0.4, 0.3, 0.4) (0.4, 0.4, 0.2)	(0.4, 0.5, 0.6) (0.3, 0.5, 0.0) (0.3, 0.0, 0.4)	(0.3, 0.4, 0.3) (0.4, 0.3, 0.2) (0.3, 0.3, 0.5)	(0.2, 0.2, 0.4) (0.5, 0.4, 0.0) (0.3, 0.4, 0.6)

The table 2 below shows mechanisms of HMV and its efficiency tested under the specified multi-criteria.

Table 2: Mechanism of HMV vs Criterias

		Criterias				
		Mileage	Maintenance cost	Pulling power	Resale value	Safety facts
HMV's Mechanism	B_1	(0.6, 0.4, 0.5) (0.4, 0.3, 0.4) (0.0, 0.3, 0.1)	(0.5, 0.7 0.4) (0.3, 0.0, 0.3) (0.2, 0.3, 0.3)	(0.5, 0.6, 0.4) (0.3, 0.2, 0.3) (0.2, 0.2, 0.3)	(0.2, 0.3, 0.1) (0.4, 0.4, 0.1) (0.4, 0.3, 0.8)	(0.4, 0.5, 0.6) (0.3, 0.3, 0.3) (0.3, 0.2, 0.1)
	B_2	(0.5, 0.5, 0.3) (0.3, 0.2, 0.2) (0.2, 0.3, 0.5)	(0.2, 0.3, 0.1) (0.4, 0.4, 0.0) (0.4, 0.3, 0.9)	(0.5, 0.4, 0.6) (0.3, 0.4, 0.2) (0.2, 0.2, 0.2)	(0.3, 0.2, 0.3) (0.3, 0.3, 0.2) (0.4, 0.5, 0.5)	(0.5, 0.6, 0.7) (0.3, 0.4, 0.2) (0.2, 0.0, 0.1)
	B_3	(0.5, 0.4, 0.2) (0.4, 0.3, 0.0) (0.1, 0.3, 0.8)	(0.3, 0.4, 0.5) (0.4, 0.6, 0.3) (0.3, 0.0, 0.2)	(0.2, 0.2, 0.1) (0.3, 0.3, 0.3) (0.5, 0.5, 0.6)	(0.3, 0.2, 0.1) (0.4, 0.3, 0.2) (0.3, 0.5, 0.7)	(0.7, 0.6, 0.5) (0.2, 0.3, 0.4) (0.1, 0.1, 0.1)

Table 3 below shows the final calculation of the Normalized Euclidean Distance between each rider's aptitude and each HMV's mechanism with reference to the common set of criterias C, by using the definition 2.9, between two IMFS's.

Table 3: Rider's Aptitude vs HMV's Mechanism

$l^E(A_i, B_i)$		HMV's Mechanism		
		B_1	B_2	B_3
Rider's Aptitude	A_1	0.2160	**0.2144**	0.2542
	A_2	**0.2516**	0.2828	0.2792
	A_3	**0.2323**	0.2670	0.3183

From the above table, the shortest distance gives the optimal decision making on determining the best mechanism under the satisfaction of rider's aptitude. The mechanism B_2 of HMV is suitable to the Rider's aptitude A_1 and the mechanism B_1 of HMV is suitable to both the Riders A_2 and A_3. Also we conclude that the mechanism B_3 of HMV is not suitable to the acquired eminency of Riders or the riders are not getting eminency in the HMV mechanism B_3.

IV. CONCLUSION

This novel application of intuitionistic multi-fuzzy sets in a MCDM on determining the best mechanism under the satisfaction of rider's aptitude is of great significance because it provides the optimal, accurate and proper choice based on the multi-criteria which is common to both riders and mechanisms. It is a very tedious decision making problem since it has a reverberatory effect on group of efficiency and eminency if not properly handled. In the proposed application, we used Normalized Euclidean Distance measure to calculate the distance of each rider's aptitude from each HMV's mechanism in respect to the specified multi-criteria, to obtained results.

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