

Certain finite double integrals involving the hypergeometric function and Aleph-function

Frédéric Ayant
*Teacher in High School , France
E-mail : fredericayant@gmail.com

Dinesh Kumar
Department of Mathematics and Statistics
Jai Narain Vyas university
JODHPUR-342005, INDIA
E-mail address:dinesh_dino03@yahoo.com

Abstract : The aim of this document is to evaluate four finite double integrals involving the product of two hypergeometric functions and the Aleph-function. At the end of this paper , we evaluate few particular cases.

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1 Introduction and notations

The Aleph- function , introduced by Südländ [8] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph_{p_i, q_i, c_i; r}^{m, n} \left(z \mid \begin{matrix} (a_j, A_j)_{1, n}, [c_i(a_{ji}, A_{ji})]_{n+1, p_i; r} \\ (b_j, B_j)_{1, m}, [c_i(b_{ji}, B_{ji})]_{m+1, q_i; r} \end{matrix} \right)$$

$$= \frac{1}{2\pi\omega} \int_L \Omega_{p_i, q_i, c_i; r}^{m, n}(s) z^{-s} ds \tag{1.1}$$

for all z different to 0 and

$$\Omega_{p_i, q_i, c_i; r}^{m, n}(s) = \frac{\prod_{j=1}^m \Gamma(b_j + \beta_j s) \prod_{j=1}^n \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^r c_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji} s)} \tag{1.2}$$

With

$$|\arg z| < \frac{1}{2}\pi\Omega \quad \text{Where } \Omega = \sum_{j=1}^m \beta_j + \sum_{j=1}^n \alpha_j - c_i \left(\sum_{j=m+1}^{q_i} \beta_{ji} + \sum_{j=n+1}^{p_i} \alpha_{ji} \right) > 0 \quad \text{with } i = 1, \dots, r$$

For convergence conditions and other details of Aleph-function , see Südländ et al [8].

We shall use notation the following :

$$A = (a_j, A_j)_{1, n}, [c_i(a_{ji}, A_{ji})]_{n+1, p_i; r} \text{ and } B = (b_j, B_j)_{1, m}, [c_i(b_{ji}, B_{ji})]_{m+1, q_i; r}$$

For more details ,see D.Kumar et all [3].

2 Hypergeometric function

We have the following results , see Rathie et al [7]

$$\begin{aligned}
& \int_0^1 x^{\rho-1} (1-x)^\rho [1+ax+(1-b)]^{-2\rho-1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] dx \\
&= 2^{\alpha+\beta-2\rho} \frac{\Gamma(\rho - \frac{\alpha}{2} - \frac{\beta}{2}) \Gamma(\frac{\alpha+\beta+2}{2}) \Gamma(\rho)}{(\alpha-\beta)(1+a)^\rho (1+b)^\rho \Gamma(\alpha) \Gamma(\beta)} \\
&\times \left[\frac{2\rho - \alpha + \beta \Gamma(\frac{\alpha}{2} + \frac{1}{2}) \Gamma(\frac{\beta}{2})}{\Gamma(\rho - \frac{\alpha}{2} - 1) \Gamma(\rho - \frac{\beta}{2} + \frac{1}{2})} - \frac{(2\rho - \alpha + \beta) \Gamma(\frac{\alpha}{2} + \frac{1}{2}) \Gamma(\frac{\beta}{2})}{\Gamma(\rho - \frac{\alpha}{2} + 1) \Gamma(\rho - \frac{\beta}{2} + \frac{1}{2})} \right] \tag{2.1}
\end{aligned}$$

Where $Re(\rho) > 0$, $Re(2\rho - \alpha - \beta) > 0$, a and b are constants , such the expression

$1 + ax + b(1 - x)$ is not zero.

$$\begin{aligned}
& \int_0^1 x^{\rho-1} (1-x)^\rho [1+ax+b(1-x)]^{-2\rho+1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] dx \\
&= 2^{\alpha+\beta-2\rho-1} \frac{\Gamma(\rho - \frac{\alpha}{2} - \frac{\beta}{2} - 1) \Gamma(\frac{\alpha+\beta}{2}) \Gamma(\rho - 1)}{(1+a)^\rho (1+b)^\rho \Gamma(\alpha) \Gamma(\beta)} \\
&\times \left[\frac{(2\rho - \alpha + \beta - 2) \Gamma(\frac{\alpha}{2} + \frac{1}{2}) \Gamma(\frac{\beta}{2})}{\Gamma(\rho - \frac{\alpha}{2}) \Gamma(\rho - \frac{\beta}{2} - \frac{1}{2})} + \frac{(2\rho + \alpha - \beta) \Gamma(\frac{\alpha}{2}) \Gamma(\frac{\beta+1}{2})}{\Gamma(\rho - \frac{\beta}{2}) \Gamma(\rho - \frac{\alpha}{2} - \frac{1}{2})} \right] \tag{2.2}
\end{aligned}$$

Where $Re(\rho) > 0$, $Re(2\rho - \alpha - \beta) > 0$, a and b are constants , such the expression

$1 + ax + b(1 - x)$ is not zero.

$$\begin{aligned}
& \int_0^{\pi/2} e^{i(2w+1)\pi\theta} (\sin\theta)^{w-1} (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta} \cos\theta \right] d\theta \\
&= \frac{e^{i\pi(w+1)/2} \Gamma(w) \Gamma(w - \frac{\alpha'-\beta'}{2}) \Gamma(\frac{\alpha'-\beta'}{2} + 1)}{2^{2w-\alpha'-\beta'+2} \Gamma(\alpha' - \beta') \Gamma(\alpha') \Gamma(\beta')} \\
&\times \left[\frac{(2w - \alpha' - \beta') \Gamma(\frac{\alpha'+1}{2}) \Gamma(\frac{\beta'}{2})}{\Gamma(w - \frac{\alpha'}{2} + 1) \Gamma(w - \frac{\beta'-1}{2})} - \frac{(2w + \alpha' - \beta') \Gamma(\frac{\alpha'}{2}) \Gamma(\frac{\beta'+1}{2})}{\Gamma(w - \frac{\beta'}{2} + 1) \Gamma(w - \frac{\alpha'-1}{2})} \right] \tag{2.3}
\end{aligned}$$

where $Re(w) > 0$ and $Re(2w - \alpha' - \beta') > 0$

$$\begin{aligned}
& \int_0^{\pi/2} e^{i(2w+1)\pi\theta} (\sin\theta)^{w-1} (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'}{2}; e^{i\theta} \cos\theta \right] d\theta \\
&= \frac{e^{i\pi(w+1)/2} \Gamma(w - 1) \Gamma(w - \frac{\alpha'-\beta'}{2} - 1) \Gamma(\frac{\alpha'+\beta'}{2})}{2^{2w-\alpha'-\beta'} \Gamma(\alpha') \Gamma(\beta')} \\
&\times \left[\frac{(2w - \alpha' - \beta' - 2) \Gamma(\frac{\alpha'+1}{2}) \Gamma(\frac{\beta'}{2})}{\Gamma(w - \frac{\alpha'}{2}) \Gamma(w - \frac{\beta'+1}{2})} - \frac{(2w + \alpha' - \beta' - 2) \Gamma(\frac{\alpha'}{2}) \Gamma(\frac{\beta'+1}{2})}{\Gamma(w - \frac{\beta'}{2}) \Gamma(w - \frac{\alpha'+1}{2})} \right] \tag{2.4}
\end{aligned}$$

with the validity conditions : $Re(\rho) > 0, Re(w) > 0, |argz| < \frac{1}{2}\pi\Omega,$

$Re(2\rho - \alpha - \beta + 2\rho_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 0$ and $Re(2w - \alpha' - \beta' + 2w_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 0$

$$\begin{aligned}
& \text{b) } \int_0^1 \int_0^{\pi/2} x^{\rho-1} (1-x)^\rho [1+ax+b(1-x)]^{-2\rho+1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] \\
& e^{i\pi(2w-1)\theta} (\sin\theta)^{w-2} (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'}{2}; e^{i\theta} \cos\theta \right] \\
& \times \aleph_{p_i, q_i, c_i; r}^{m, n} \left(zx^{\rho_1} (1-x)^{\rho_1} [1+ax+b(1-x)]^{-2\rho_1} e^{2i\theta w_1} (\sin\theta)^{w_1} (\cos\theta)^{w_1} \left| \begin{matrix} A \\ B \end{matrix} \right. \right) d\theta dx \\
& = \frac{2^{\alpha+\beta-2\rho-1} \Gamma(\frac{\alpha+\beta+2}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+a)^\rho(1+b)^\rho} \left[\Gamma(\frac{\alpha+\beta}{2}) \Gamma(\frac{\beta}{2}) \aleph_5(z) - \Gamma(\frac{\alpha}{2}) \Gamma(\frac{\alpha+\beta}{2}) \aleph_6(z) \right] \\
& \times \frac{e^{i\pi(w+1)/2} \Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w-\alpha'+1} \Gamma(\alpha'-\beta') \Gamma(\alpha') \Gamma(\beta')} \left[\Gamma(\frac{\alpha'+1}{2}) \Gamma(\frac{\beta'}{2}) \aleph_7(z) - \Gamma(\frac{\alpha'+1}{2}) \Gamma(\frac{\beta'+2}{2}) \aleph_8(z) \right] \quad (3.6)
\end{aligned}$$

where

$$\begin{aligned}
\aleph_5(z) &= \aleph_{p_i+3, q_i+3, c_i; r}^{m, n+3} \left(\frac{2^{-2\rho_1} z}{(1+a)^{\rho_1} (1+b)^{\rho_1}} \left| \begin{matrix} A, (2+\alpha-\beta-2\rho, \rho_1), (2-\rho, \rho_1) \\ B, (3-2\rho+\alpha+\beta, 2\rho_1), (1+\alpha/2-\rho, \rho_1) \end{matrix} \right. \right) \\
& , \left(\begin{matrix} (2+(\alpha+\beta)/2-\rho, \rho_1) \\ ((\beta+3)/2-\rho, \rho_1) \end{matrix} \right) \quad (3.7)
\end{aligned}$$

$$\begin{aligned}
\aleph_6(z) &= \aleph_{p_i+3, q_i+3, c_i; r}^{m, n+3} \left(\frac{2^{-2\rho_1} z}{(1+a)^{\rho_1} (1+b)^{\rho_1}} \left| \begin{matrix} A, (2-\alpha+\beta-2\rho, \rho_1), (2-\rho, \rho_1) \\ B, (3-2\rho-\alpha+\beta, 2\rho_1), (1+\beta/2-\rho, \rho_1) \end{matrix} \right. \right) \\
& , \left(\begin{matrix} (2+(\alpha+\beta)/2-\rho, \rho_1) \\ ((\alpha+3)/2-\rho, \rho_1) \end{matrix} \right) \quad (3.8)
\end{aligned}$$

$$\begin{aligned}
\aleph_7(z) &= \aleph_{p_i+3, q_i+3, c_i; r}^{m, n+3} \left(\frac{ze^{i\pi w_1/2}}{4w_1} \left| \begin{matrix} A, (2+\alpha'-\beta'-2w, w_1), (2-w, w_1) \\ B, (3-2w-\alpha'+\beta', 2w_1), (1+\alpha/2-w, w_1) \end{matrix} \right. \right) \\
& , \left(\begin{matrix} (2+(\alpha'+\beta')/2-w, w_1) \\ ((\beta+3)/2-w, w_1) \end{matrix} \right) \quad (3.9)
\end{aligned}$$

$$\begin{aligned}
\aleph_8(z) &= \aleph_{p_i+3, q_i+3, c_i; r}^{m, n+3} \left(\frac{ze^{i\pi w_1/2}}{4w_1} \left| \begin{matrix} A, (2-\alpha'+\beta'-2w, w_1), (2-w, w_1) \\ B, (3-2w-\alpha'+\beta', 2w_1), (1+\beta/2-w, w_1) \end{matrix} \right. \right) \\
& , \left(\begin{matrix} (2+(\alpha'+\beta')/2-w, w_1) \\ ((\alpha+3)/2-w, w_1) \end{matrix} \right) \quad (3.10)
\end{aligned}$$

with the validity conditions : $Re(\rho) > 1, Re(w) > 1, |argz| < \frac{1}{2}\pi\Omega,$ and

$$Re(2\rho - \alpha - \beta + 2\rho_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 2 \text{ and } Re(2w - \alpha' - \beta' + 2w_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 2$$

$$\begin{aligned} \text{c) } & \int_0^1 \int_0^{\pi/2} x^{\rho-1} (1-x)^\rho [1+ax+b(1-x)]^{-2\rho+1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] \\ & \times e^{i\pi(2w-1)\theta} (\sin\theta)^{w-1} (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'}{2}; e^{i\theta} \cos\theta \right] \\ & \times \aleph_{p_i, q_i, c_i; r}^{m, n} \left(zx^{\rho_1} (1-x)^{\rho_1} [1+ax+b(1-x)]^{-2\rho_1} e^{2i\theta w_1} (\sin\theta)^{w_1} (\cos\theta)^{w_1} \left| \begin{matrix} A \\ B \end{matrix} \right. \right) d\theta dx \\ & = \frac{2^{\alpha+\beta-2\rho-1}}{\Gamma(\alpha)\Gamma(\beta)(\alpha-\beta)(1+a)^\rho(1+b)^\rho} \left[\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_1(z) - \Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_2(z) \right] \\ & \times \frac{e^{i\pi(w-1)/2}\Gamma\left(\frac{\alpha'+\beta'}{2}\right)}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')} \left[\Gamma\left(\frac{\alpha'+1}{2}\right)\Gamma\left(\frac{\beta'}{2}\right)\aleph_7(z) - \Gamma\left(\frac{\alpha'}{2}\right)\Gamma\left(\frac{\beta'+1}{2}\right)\aleph_8(z) \right] \end{aligned} \quad (3.11)$$

Where $\aleph_1(z)$, $\aleph_2(z)$, $\aleph_7(z)$ and $\aleph_8(z)$ are mentioned in (3.2), (3.3), (3.9) and (3.10) respectively and the validity conditions are the following :

$$Re(\rho) > 0, Re(w) > 1, |argz| < \frac{1}{2}\pi\Omega, \text{ and } Re(2\rho - \alpha - \beta + 2\rho_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 0$$

$$, Re(2w - \alpha' - \beta' + 2w_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 0$$

$$\begin{aligned} \text{d) } & \int_0^1 \int_0^{\pi/2} x^\rho (1-x)^{\rho-2} [1+ax+b(1-x)]^{-2\rho+1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] \\ & \times e^{i\pi(2w+1)\theta} (\sin\theta)^w (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta} \cos\theta \right] \\ & \times \aleph_{p_i, q_i, c_i; r}^{m, n} \left(zx^{\rho_1} (1-x)^{\rho_1} [1+ax+b(1-x)]^{-2\rho_1} e^{2i\theta w_1} (\sin\theta)^{w_1} (\cos\theta)^{w_1} \left| \begin{matrix} A \\ B \end{matrix} \right. \right) d\theta dx \\ & = \frac{2^{\alpha+\beta-2\rho-1}\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\alpha+\beta+2}{2}\right)}{\Gamma(\alpha)\Gamma(\beta)(1+a)^\rho(1+b)^\rho} \left[\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_5(z) - \Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_6(z) \right] \\ & \times \frac{e^{i\pi(w+1)/2}\Gamma\left(\frac{\alpha'+\beta'+2}{2}\right)}{2^{2w+\alpha'-\beta'+1}\Gamma(\alpha')\Gamma(\beta')\Gamma(\alpha'-\beta')} \left[\Gamma\left(\frac{\alpha'+1}{2}\right)\Gamma\left(\frac{\beta'}{2}\right)\aleph_3(z) - \Gamma\left(\frac{\alpha'}{2}\right)\Gamma\left(\frac{\beta'+1}{2}\right)\aleph_4(z) \right] \end{aligned} \quad (3.12)$$

Where $\aleph_5(z)$, $\aleph_6(z)$, $\aleph_3(z)$ and $\aleph_4(z)$ are mentioned by (3.4), (3.5), (3.7) and (3.8) respectively and the validity conditions are :

$$Re(\rho) > 0, Re(w) > 1, |argz| < \frac{1}{2}\pi\Omega, \text{ and } Re(2\rho - \alpha - \beta + 2\rho_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 0$$

$$, Re(2w - \alpha' - \beta' + 2w_1 \min_{1 \leq j \leq n} \frac{b_j}{\beta_j}) > 2$$

Proof : To establish (3.1), we express the Aleph_function on the left hande side using (1.1) in Mellin-Barnes contour integral and interchanging the order of integration which is justifiable due to absolute convergence of the integrals , we

have :

$$\begin{aligned} & \frac{1}{2\pi\omega} \int_L \Omega_{p_i, q_i, c_i; r}^{m, n}(s) \left(\left(\int_0^1 x^{\rho+\rho_1 s-1} (1-x)^{\rho+\rho_1 s} [1+ax+b(1-x)]^{-2\rho-2\rho_1 s-1} \right. \right. \\ & \times {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{x(1+a)}{1+ax+b(1-x)} \right] dx \left. \right) \left(\int_0^{\pi/2} e^{i(2w+2w_1 s+1)\theta} (\sin\theta)^{w+w_1 s} \right. \\ & \left. \left. \times (\cos\theta)^{w+w_1 s} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta} \cos\theta \right] d\theta \right) z^{-s} ds \end{aligned}$$

We evaluate the inner integrals with the help of (2.1) and (2.3) and applying (1.1) , we get the R.H.S of (3.1) in terms of product of Aleph-functions. The other integrals calculate in the similar method

4 Particular cases

If $a = b$ in (3.6) , we obtain :

$$\begin{aligned} & \int_0^1 \int_0^{\pi/2} x^{\rho-1} (1-x)^\rho (1+b)^{-2\rho+1} {}_2F_1 \left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; x \right] e^{i\pi(2w+1)\theta} (\sin\theta)^{w-2} \\ & \times (\cos\theta)^{w-1} {}_2F_1 \left[\alpha', \beta'; \frac{\alpha'+\beta'+2}{2}; e^{i\theta} \cos\theta \right] \\ & \times \aleph_{p_i, q_i, c_i; r}^{m, n} \left(z x^{\rho_1} (1-x)^{\rho_1} (1+b)^{-2\rho_1} e^{2iw_1\theta} (\sin\theta)^{w_1} (\cos\theta)^{w_1} \left| \begin{matrix} A \\ B \end{matrix} \right. \right) d\theta dx \\ & = \frac{2^{\alpha+\beta-2\rho_1-1} \Gamma(\frac{\alpha+\beta}{2})}{\Gamma(\alpha)\Gamma(\beta)(1+b)^{2\rho}} \left[\Gamma(\frac{\alpha+1}{2})\Gamma(\frac{\beta}{2})\aleph_5(z) - \Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})\aleph_6(z) \right] \\ & \times \frac{e^{i\pi(w_1-1)/2} \Gamma(\frac{\alpha'+\beta'+2}{2})}{2^{2w+\alpha'-\beta'+1} \Gamma(\alpha')\Gamma(\beta')} \left[\Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'}{2})\aleph_7(z) - \Gamma(\frac{\alpha'+1}{2})\Gamma(\frac{\beta'+2}{2})\aleph_8(z) \right] \end{aligned} \quad (4.1)$$

Remarks : if $c_i = 1$ for $i = 1, \dots, r$, the Aleph-function degenerate into the L-function defined by V.P. Saxena [6], for more details see D.kumar et al [1,5]. If $r = 1$, the L-function degenerate into the fox's H-function , see Ronghe [4].

5 Conclusion

The aleph-function, presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as I-function ,Fox's H-function see [2] , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modified Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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*Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE