Integrals involving Multivariable Aleph-function, Aleph-function of one variable and general class of polynomials of several variables

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ABSTRACT
In this paper, we evaluate two general class of Eulerian integrals involving a general class of multivariable polynomials, Aleph-function of one variable and generalized multivariable Aleph-function.The mains results of our document are quite general in nature and capable of yielding a very large number of integrals involving polynomials and various special functions occuring in the problem of mathematical analysis and mathematical . physics and mechanics.

Keywords :generalized multivariable Aleph-function, Aleph-function, class of multivariable polynomials, generalized hypergeometric function, finite integral,.

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1. Introduction and preliminaries.

In this paper we establish two general class of Eulerian integral concerning the multivariable Aleph-function, the Aleph-function and general class of multivariable polynomials. The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}}\left(\begin{array}{c|} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right.$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]$ $,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$

$\cdots\cdots\cdots\cdots\cdots\cdots$ $,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i(k)}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i(k)}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i(k)}}\delta_{ji^{(k)}}^{(k)}>0,\ \text{with}\ k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \tag{1.2}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,)\,,max(\,|z_1|\ldots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\ldots|z_r|\,)\rightarrow\infty$$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\;;\;V=m_1,n_1;\cdots;m_r,n_r \tag{1.3}$$

$$\mathrm{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.4}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.5}$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.6}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.7}$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.8}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}:\mathrm{C} \\ \cdot & \cdots \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B}:\mathrm{D}\end{array}\right) \tag{1.9}$$

The generalized polynomials of multivariable defined by Srivastava [6], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.10}$$

The Aleph- function , introduced by Südland [7] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,\mathfrak{c}_i;r}^{M,N}\left(\mathrm{z}\;\middle|\;\begin{array}{c}(\,\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[\mathfrak{c}_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\(\,\mathrm{b}_j,B_j)_{1,m},[\mathfrak{c}_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right)=\frac{1}{2\pi\omega}\int_L\;\Omega_{P_i,Q_i,\mathfrak{c}_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \tag{1.11}$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,\mathfrak{c}_i;r}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}\mathfrak{c}_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.12}$$

With $|argz|<\frac{1}{2}\pi\Omega$ where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-\mathfrak{c}_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0\,,i=1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [7].The serie representation of Aleph-function is given by Chaurasia et al [2].

$$\aleph^{M,N}_{P_i,Q_i,\mathfrak{c}_i;r}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,\mathfrak{c}_i,r}(s)}{B_G g!}z^{-s} \tag{1.13}$$

With $s = \eta_{G,g} = \dfrac{b_G + g}{B_G}$, $P_i < Q_i$, $|z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,\mathfrak{c}_i;r}(s)$ is given in (1.2) (1.14)

2. Required formulas

We have the following results , see Rathie et al [4].

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}[1+ax+b(1-x)]^{-2\rho-1}{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]\mathrm{d}x$$

$$= 2^{\alpha+\beta-2\rho}\ \frac{\Gamma(\rho-\frac{\alpha}{2}-\frac{\beta}{2})\Gamma(\frac{\alpha+\beta+2}{2})\Gamma(\rho)}{(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho}\Gamma(\alpha)\Gamma(\beta)}$$

$$\times\left[\frac{(2\rho-\alpha+\beta)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2}-1)\Gamma(\rho-\frac{\beta}{2}+\frac{1}{2})}-\frac{(2\rho-\alpha+\beta)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2}+1)\Gamma(\rho-\frac{\beta}{2}+\frac{1}{2})}\right] \tag{2.1}$$

where $Re(\rho) > 0, Re(2\rho-\alpha-\beta) > 0$, a and b are constants , such the expression and $1+ax+b(1-x)$ is not zero.

$$\int_0^1 x^{\rho-1}(1-x)^{\rho}\left[1+ax+b(1-x)\right]^{-2\rho+1}{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta}{2};\frac{x(1+a)}{1+ax+b(1-x)}\right]\mathrm{d}x$$

$$= 2^{\alpha+\beta-2\rho-1}\frac{\Gamma(\rho-\frac{\alpha}{2}-\frac{\beta}{2}-1)\Gamma(\frac{\alpha+\beta}{2})\Gamma(\rho-1)}{(1+a)^{\rho}(1+b)^{\rho}\Gamma(\alpha)\Gamma(\beta)}$$

$$\times\left[\frac{(2\rho-\alpha+\beta-2)\Gamma(\frac{\alpha}{2}+\frac{1}{2})\Gamma(\frac{\beta}{2})}{\Gamma(\rho-\frac{\alpha}{2})\Gamma(\rho-\frac{\beta}{2}-\frac{1}{2})}+\frac{(2\rho+\alpha-\beta)\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta+1}{2})}{\Gamma(\rho-\frac{\beta}{2})\Gamma(\rho-\frac{\alpha}{2}-\frac{1}{2})}\right] \tag{2.2}$$

where $Re(\rho) > 0, Re(2\rho-\alpha-\beta) > 0$, a and b are constants , such the expression ; In this document the quantity $1+ax+b(1-x)$ is not zero.

3 Finite integrals

We evaluate the following two finite integrals involving hypergeometric functions and multivariable Aleph-functions.

Let $g(t) = \dfrac{4(t+at)(1+b)(1-t)}{[1+at+b(1-t)]^2}$, $U_{33} = p_i+3, q_i+3, \tau_i; R$

and $a' = \dfrac{(-N_1)_{M_1K_1}}{K_1!}\cdots\dfrac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]$ (3.1)

Formula 1

$$\int_0^1 t^{\rho-1}(1-t)^{\rho}[1+at+(1-b)]^{-2\rho-1}{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{t(1+a)}{1+at+b(1-t)}\right]$$

$$\aleph_{P_i,Q_i,\mathfrak{c}_i;r'}^{M,N}(x\,(g(t))^{c})\; S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1\,(g(t))^{c_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{y}_s\,(g(t))^{c_s}\end{array}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1\,(g(t))^{u_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r\,(g(t))^{u_r}\end{array}\right)\mathrm{d}t$$

$$= 2^{\alpha+\beta-2\rho-1}\,\frac{\Gamma(\frac{\alpha+\beta+2}{2})}{(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho+1}\Gamma(\alpha)\Gamma(\beta)}\left[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$

$$a'\,\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\,y_1^{K_1}\cdots y_s^{K_s}x^{\eta_{G,g}}\left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r\end{array}\right|\right.$$

$$\begin{array}{c}(1\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\alpha+\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\\ \cdot\ \cdot\ \cdot\\ (1\text{-}\rho+\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\alpha}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\end{array}$$

$$\left.\begin{array}{c}(\frac{\alpha-\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\frac{\beta+1}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),B:D\end{array}\right)-\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_r\end{array}\right|$$

$$\begin{array}{c}(1\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\alpha+\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\\ \cdot\ \cdot\ \cdot\\ (1\text{-}\rho-\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\end{array}$$

$$\left.\left.\left.\begin{array}{c}(\frac{\beta-\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),A:C\\ \cdot\ \cdot\ \cdot\\ (\frac{1+\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),B:D\end{array}\right)\right\}\right] \tag{2.2}$$

Provided that

a) $min\{c,d,c_i,d_i,u_j,v_j\}>0, i=1,\cdots,s; j=1,\cdots,r$

b) $Re(\rho-\alpha-\beta)>0\,,Re(\rho)>0\,,1+at+b(1-t)$ is not zero.

c) $Re[\rho+c\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{r}u_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

d) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

e) $|argx|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-\mathfrak{c}_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$

Formula 2

$$\int_0^1 t^{\rho-1}(1-t)^{\rho-2}[1+at+(1-b)]^{-2\rho+1}\,{}_2F_1\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{t(1+a)}{1+at+b(1-t)}\Big]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(x\,(g(t))^c)\,S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{y}_1\,(g(t))^{c_1}\\ \cdot\,\cdot\,\cdot\\ \cdot\,\cdot\,\cdot\\ \mathrm{y}_s\,(g(t))^{c_s}\end{pmatrix}\aleph_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix} \mathrm{z}_1\,(g(t))^{u_1}\\ \cdot\,\cdot\,\cdot\\ \cdot\,\cdot\,\cdot\\ \mathrm{z}_r\,(g(t))^{u_r}\end{pmatrix}\mathrm{d}t$$

$$= 2^{\alpha+\beta-2\rho}\,\frac{\Gamma(\frac{\alpha+\beta}{2})}{(1+a)^{\rho}(1+b)^{\rho-1}\Gamma(\alpha)\Gamma(\beta)}\Bigg[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}$$

$$a'\,\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\,y_1^{K_1}\cdots y_s^{K_s}x^{\eta_{G,g}}\left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c|} \mathrm{z}_1\\ \cdot\,\cdot\,\cdot\\ \cdot\,\cdot\,\cdot\\ \mathrm{z}_r\end{array}\right.\right.$$

$$\begin{array}{c}(2\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(2-\rho+\frac{\alpha+\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\\ \cdot\,\cdot\,\cdot\\ (2\text{-}\rho+\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\alpha}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\end{array}$$

$$\left.\begin{array}{c}(1+\frac{\alpha-\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),A:C\\ \cdot\,\cdot\,\cdot\\ (\frac{3}{2}+\frac{\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),B:D\end{array}\right)+\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c|} \mathrm{z}_1\\ \cdot\,\cdot\,\cdot\\ \cdot\,\cdot\,\cdot\\ \mathrm{z}_r\end{array}\right.$$

$$\begin{array}{c}(2\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\beta-\alpha}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\\ \cdot\,\cdot\,\cdot\\ (2\text{-}\rho-\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),(1-\rho+\frac{\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),\end{array}$$

$$\left.\left.\begin{array}{c}(1+\frac{\beta-\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),A:C\\ \cdot\,\cdot\,\cdot\\ (\frac{3}{2}+\frac{\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,\cdots,u_r),B:D\end{array}\right)\right\}\Bigg] \tag{3.3}$$

Provided that

a) $min\{c,d,c_i,d_i,u_j,v_j\}>0, i=1,\cdots,s; j=1,\cdots,r$

b) $Re(\rho-\alpha-\beta)>2$, $Re(\rho)>1$, $1+at+b(1-t)$ is not zero.

c) $Re\big[\rho+c\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+\sum_{i=1}^{r}u_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}\big]>1$

d) $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

e) $|argx| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - \mathfrak{c}_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0$

Proof of (3.2). Let $M = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$

We first replace the multivariable Aleph-function on the L.H.S of (3.2) by its Mellin-barnes contour integral (1.1), the Aleph-function and general class of polynomials of several variables in series using respectively (1.11) and (1.10),Now we interchange the order of summation and integrations (which is permissible under the conditions stated) . We get :

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a'\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}$$

$$\int_0^1 t^{\rho-1}(1-t)^{\rho}[1+at+(1-b)]^{-2\rho-1}\,{}_2F_1\Big[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{t(1+a)}{1+at+b(1-t)}\Big]$$

$$\Big\{M\Big\{(g(t))^{c\eta_{G,g}+\sum_{i=1}^{s}c_iK_i+\sum_{i=1}^{r}u_is_i}\Big\}\mathrm{d}s_1\cdots\mathrm{d}s_r\Big\}\mathrm{d}t \tag{3.4}$$

Now intechanging the order of integrations , we get

$$\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]} a'\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}x^{\eta_{G,g}}y_1^{K_1}\cdots y_s^{K_s}\,M\Big\{\int_0^1 t^{\rho+c\eta_{G,g}+\sum_{i=1}^{s}K_ic_i+\sum_{i=1}^{r}u_is_i-1}$$

$$(1-t)^{\rho+c\eta_{G,g}+\sum_{i=1}^{s}K_ic_i+\sum_{i=1}^{r}s_ju_j}[1+at+(1-b)]^{-2(\rho-c\eta_{G,g}-\sum_{i=1}^{s}K_ic_i+\sum_{j=1}^{r}s_ju_j)-1}\mathrm{d}t\Big\}$$
$$\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.5}$$

Finally, we evaluate the inner integrals with the help of (2.1) and interpreting the resulting with the Mellin-Barnes contour integral as a multivariable Aleph-function, we obtain the desired result (3.2).

To prove (3.3), we use the similar method with the help of (2.2).

4. Multivariable I-function

If $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}} \rightarrow 1$, the Aleph-function of several variables degenere to the I-function of several variables. The following finite integrals have been derived in this section for multivariable I-functions defined by Sharma et al [3].

a) $$\int_0^1 t^{\rho-1}(1-t)^{\rho}[1+at+(1-b)]^{-2\rho-1}\,{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{t(1+a)}{1+at+b(1-t)}\right]$$

$$\aleph_{P_i,Q_i,\mathfrak{c}_i;r'}^{M,N}(x\,(g(t))^{c})\,S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\mathrm{y}_1\,(g(t))^{c_1}\\ \cdots\\ \cdots\\ \mathrm{y}_s\,(g(t))^{c_s}\end{pmatrix} I_{U:W}^{0,\mathfrak{n}:V}\begin{pmatrix}\mathrm{z}_1\,(g(t))^{u_1}\\ \cdots\\ \cdots\\ \mathrm{z}_r\,(g(t))^{u_r}\end{pmatrix}\mathrm{d}t$$

$$= 2^{\alpha+\beta-2\rho-1} \frac{\Gamma(\frac{\alpha+\beta+2}{2})}{(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho+1}\Gamma(\alpha)\Gamma(\beta)} \left[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$

$$a' \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!} y_1^{K_1}\cdots y_s^{K_s} x^{\eta_{G,g}} \left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right) I_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\right|\right.$$

$$\begin{array}{c}(1\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),(1-\rho+\frac{\alpha+\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), \\ \cdot\cdot\cdot \\ (1\text{-}\rho+\frac{\alpha-\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),(1-\rho+\frac{\alpha}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),\end{array}$$

$$\left.\begin{array}{c}(\frac{\alpha-\beta}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), A:C \\ \cdot\cdot\cdot \\ (\frac{\beta+1}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), B:D\end{array}\right) - \Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right) I_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\right|$$

$$\begin{array}{c}(1\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),(1-\rho+\frac{\alpha+\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), \\ \cdot\cdot\cdot \\ (1\text{-}\rho-\frac{\alpha-\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),(1-\rho+\frac{\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r),\end{array}$$

$$\left.\left.\left.\begin{array}{c}(\frac{\beta-\alpha}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), A:C \\ \cdot\cdot\cdot \\ (\frac{1+\alpha}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_iK_i : u_1,\cdots,u_r), B:D\end{array}\right)\right\}\right] \tag{4.1}$$

with the same notations and validity conditions that (3.2).

b) $$\int_0^1 t^{\rho-1}(1-t)^{\rho-2}[1+at+(1-b)]^{-2\rho+1}\,{}_2F_1\left[\alpha,\beta;\frac{\alpha+\beta+2}{2};\frac{t(1+a)}{1+at+b(1-t)}\right]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(x\,(g(t))^c)\, S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{y}_1\,(g(t))^{c_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{y}_s\,(g(t))^{c_s} \end{array}\right) I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1\,(g(t))^{u_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r\,(g(t))^{u_r} \end{array}\right)\mathrm{d}t$$

$$= 2^{\alpha+\beta-2\rho} \frac{\Gamma(\frac{\alpha+\beta}{2})}{(1+a)^{\rho}(1+b)^{\rho-1}\Gamma(\alpha)\Gamma(\beta)} \left[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$

$$a' \frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!} y_1^{K_1}\cdots y_s^{K_s} x^{\eta_{G,g}} \left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right) I_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\right|\right.$$

$$(2\text{-}\rho - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), (2-\rho+\frac{\alpha+\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r),$$
$$\cdots$$
$$(2\text{-}\rho+\tfrac{\alpha-\beta}{2} - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), (1-\rho+\frac{\alpha}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r),$$

$$\left.\begin{matrix}(1+\frac{\alpha-\beta}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), A : C\\ \cdots \\ (\frac{3}{2}+\frac{\beta}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), B : D\end{matrix}\right) + \Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right) I_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{matrix}z_1\\ \cdots\\ \cdots\\ z_r\end{matrix}\ \middle|\right.$$

$$(2\text{-}\rho - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), (1-\rho+\frac{\beta-\alpha}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r),$$
$$\cdots$$
$$(2\text{-}\rho-\tfrac{\alpha-\beta}{2} - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), (1-\rho+\frac{\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r),$$

$$\left.\begin{matrix}(1+\frac{\beta-\alpha}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), A : C\\ \cdots \\ (\frac{3}{2}+\frac{\alpha}{2}\text{-}\rho - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, \cdots, u_r), B : D\end{matrix}\right)\Bigg\}\Bigg] \tag{4.2}$$

with the same notations and validity conditions that (3.3).

5. Aleph-function of two variables

If $r = 2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following integrals.

a) $$\int_0^1 t^{\rho-1}(1-t)^{\rho}[1+at+(1-b)]^{-2\rho-1}\, {}_2F_1\left[\alpha, \beta; \frac{\alpha+\beta+2}{2}; \frac{t(1+a)}{1+at+b(1-t)}\right]$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(x\,(g(t))^{c})\, S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{matrix}y_1\,(g(t))^{c_1}\\ \cdots\\ \cdots\\ y_s\,(g(t))^{c_s}\end{matrix}\right) \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{matrix}z_1\,(g(t))^{u_1}\\ \cdots\\ \cdots\\ z_2\,(g(t))^{u_2}\end{matrix}\right)\mathrm{d}t$$

$$= 2^{\alpha+\beta-2\rho-1}\frac{\Gamma(\frac{\alpha+\beta+2}{2})}{(\alpha-\beta)(1+a)^{\rho}(1+b)^{\rho+1}\Gamma(\alpha)\Gamma(\beta)}\left[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$

$$a'\,\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G\, g!}\, y_1^{K_1}\cdots y_s^{K_s} x^{\eta_{G,g}}\left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{matrix}z_1\\ \cdots\\ \cdots\\ z_2\end{matrix}\ \middle|\right.\right.$$

$$(1\text{-}\rho - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, u_2), (1-\rho+\frac{\alpha+\beta}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, u_2),$$
$$\cdots$$
$$(1\text{-}\rho+\tfrac{\alpha-\beta}{2} - c\eta_{G,g} - \textstyle\sum_{i=1}^{s} c_i K_i : u_1, u_2), (1-\rho+\frac{\alpha}{2} - c\eta_{G,g} - \sum_{i=1}^{s} c_i K_i : u_1, u_2),$$

$$\left.\begin{matrix}(\frac{\alpha-\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),A:C\\ \cdots\\ (\frac{\beta+1}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),B:D\end{matrix}\right)-\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_2\end{matrix}\right|$$

$$\begin{matrix}(1\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(1-\rho+\frac{\alpha+\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\\ \cdots\\ (1\text{-}\rho-\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(1-\rho+\frac{\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\end{matrix}$$

$$\left.\left.\left.\begin{matrix}(\frac{\beta-\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),A:C\\ \cdots\\ (\frac{1+\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),B:D\end{matrix}\right)\right\}\right] \tag{5.1}$$

with the same notations and validity conditions that (3.2) with $r=2$

b) $\int_0^1 t^{\rho-1}(1-t)^{\rho-2}[1+at+(1-b)]^{-2\rho+1}\,{}_2F_1\Big[\alpha,\beta;\dfrac{\alpha+\beta+2}{2};\dfrac{t(1+a)}{1+at+b(1-t)}\Big]$

$$\aleph_{P_i,Q_i,\mathfrak{c}_i;r'}^{M,N}(x\,(g(t))^{c})\,S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{matrix}\mathrm{y}_1\,(g(t))^{c_1}\\ \cdots\\ \cdots\\ \mathrm{y}_s\,(g(t))^{c_s}\end{matrix}\right)\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{matrix}\mathrm{z}_1\,(g(t))^{u_1}\\ \cdots\\ \cdots\\ \mathrm{z}_2\,(g(t))^{u_2}\end{matrix}\right)\mathrm{d}t$$

$$=2^{\alpha+\beta-2\rho}\,\frac{\Gamma(\frac{\alpha+\beta}{2})}{(1+a)^{\rho}(1+b)^{\rho-1}\Gamma(\alpha)\Gamma(\beta)}\left[\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\right.$$

$$a'\,\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}\,y_1^{K_1}\cdots y_s^{K_s}x^{\eta_{G,g}}\left\{\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(\frac{\beta}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_2\end{matrix}\right|\right.$$

$$\begin{matrix}(2\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(2-\rho+\frac{\alpha+\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\\ \cdots\\ (2\text{-}\rho+\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(1-\rho+\frac{\alpha}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\end{matrix}$$

$$\left.\begin{matrix}(1+\frac{\alpha-\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),A:C\\ \cdots\\ (\frac{3}{2}+\frac{\beta}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),B:D\end{matrix}\right)+\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\aleph_{U_{33}:W}^{0,\mathfrak{n}+3:V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_2\end{matrix}\right|$$

$$\begin{matrix}(2\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(1-\rho+\frac{\beta-\alpha}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\\ \cdots\\ (2\text{-}\rho-\frac{\alpha-\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),(1-\rho+\frac{\beta}{2}-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),\end{matrix}$$

$$\left.\left.\left.\begin{array}{c}(1+\frac{\beta-\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),A:C\\ \cdots\\ (\frac{3}{2}+\frac{\alpha}{2}\text{-}\rho-c\eta_{G,g}-\sum_{i=1}^{s}c_iK_i:u_1,u_2),B:D\end{array}\right)\right\}\right] \tag{5.2}$$

with the same notations and validity conditions that (3.3) with $r=2$

6. Conclusion

Due to general nature of the multivariable aleph-function and the Eulerian integrals involving here, our formulas are capable to be reduced into many known and news integrals involving the special functions of one and several variables and polynomials of one and several variables.

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