On some summation formulae for the Aleph-function of several variables

F.Y. $AYANT^1$

1 Teacher in High School , France

ABSTRACT
In this paper we aim to establish three interesting summation formulas for the multivariable Aleph-function . The results are derived with the help of classical summation theorems due to Watson, Dixon and Whipple. A few results are also obtained as special cases of our main findings. Since the multivariable Aleph-function is the most generalized function of several variables and its includes as specials cases many formulas involving special functions of one and several variables.

Keywords:Multivariable Aleph-function,Mellin-Barnes contour, Summation theorems, I-function of several variables, Aleph-function of two variables.

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1. Introduction and preliminaries.

In this paper we establish three summation formulas concerning the multivariable Aleph-function, the Aleph-function and general class of multivariable polynomials. The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \right|$

$[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}] \ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}] :$
$\ldots\ldots\ldots \ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}] :$

$[(c_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(c_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$[(d_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(d_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}] \Bigg)$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i(k)}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i(k)}}\delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1\cdots,r, i=1,\cdots,R \ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.2}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,)\,,max(\,|z_1|\ldots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\ldots|z_r|\,)\rightarrow \infty$$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \tag{1.3}$$

$$W=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.4}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.5}$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.6}$$

$$C_1=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,$$

$$C_r=\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.7}$$

$$D_1=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,$$

$$D_r=\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.8}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{0,n:V}\left(\begin{array}{c|c} z_1 & \mathrm{A}\ ;\ \mathrm{C}_1;\cdots;C_r \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \mathrm{B}\ ;\ \mathrm{D}_1;\cdots;D_r \end{array}\right) \tag{1.9}$$

Let $W=W^{(1)};\cdots;W^{(r)}$ where $W^{(k)}=p_{i^{(k)}},q_{i^{(k)}},\tau_{i^{(k)}};R^{(k)}, C=C_1;\cdots;C_r$ and $D=D_1;\cdots;D_r$

2. Required results

In our present investigation, we shall require the following classical summations theorems.

Watson's Theorem (Bailey [2])

$${}_3F_2\left(\begin{array}{c} \mathrm{a,\ b,\ c} \\ \frac{1}{2}(a+b+1),\ 2\mathrm{c} \end{array};1\right)=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(c+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}(a+b+1)\right)\Gamma\left(c+\frac{1}{2}(1-a-b)\right)}{\Gamma\left(\frac{1}{2}(a+1)\right)\Gamma\left(\frac{1}{2}(b+1)\right)\Gamma\left(c+\frac{1}{2}(1-a)\right)\Gamma\left(c+\frac{1}{2}(1-b)\right)} \tag{2.1}$$

provided $Re(2c-a-b)>-1$ and the parameters are such that series on the left is defined.

Dixon's Theorem (Bailey [2])

$${}_3F_2\left(\begin{array}{c} \mathrm{a,\ b,\ c} \\ \mathrm{1+a\text{-}b,\ 1+a\text{-}c} \end{array};1\right)=\frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma\left(1-b-c+\frac{a}{2}\right)}{\Gamma(a+1)\Gamma\left(1-b+\frac{a}{2}\right)\Gamma\left(1-c+\frac{a}{2}\right)\Gamma(1+a-b-c)} \tag{2.2}$$

provided $Re(a-2b-2c) > -2$

Whipple's Theorem (Bailey [2])

$$ {}_3F_2\left(\begin{matrix} \text{a, b, c} \\ \text{f, 2c+1-f ; 1}\end{matrix}\right) = \frac{2^{1-2c}\pi\Gamma(f)\Gamma(2c+1-f)}{\Gamma\left(c+\frac{1+a-f}{2}\right)\Gamma\left(\frac{a+f}{2}\right)\Gamma\left(c+\frac{1+b-f}{2}\right)\Gamma\left(\frac{b+f}{2}\right)} \tag{2.3} $$

provided $b = 1-a$ and $Re(c) > 0$

3. Main summation formulae

In this section, the following three general summation formulae will be established.

Let $U_{11} = p_i+1, q_i+1, \tau_i; R$ and $U_{12} = p_i+1, q_i+2, \tau_i; R$, we have

Theorem 1

$$ \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{\left(\frac{a+b+1}{2}\right)_k k!} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix} \text{z}_1 \\ \cdots \\ \cdots \\ \text{z}_r \end{matrix}\left|\begin{matrix} (\text{1-k-c:u}_1,\cdots,u_r), A : C \\ \cdots \\ (\text{1-k-2c:2u}_1,\cdots,2u_r), B : D \end{matrix}\right.\right) = \frac{2^{1-2c}\pi\Gamma\left(c+\frac{1+a+b}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)} $$

$$ \aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix} 4^{-u_1}z_1 \\ \cdots \\ \cdots \\ 4^{-u_r}z_r \end{matrix}\left|\begin{matrix} \left(\frac{a+b+1}{2}-c:u_1,\cdots,u_r\right), A : C \\ \cdots \\ \left(\frac{a+1}{2}-c:u_1,\cdots,u_r\right),\left(\frac{b+1}{2}-c:u_1,\cdots,u_r\right) B : D \end{matrix}\right.\right) \tag{3.1} $$

Provided that

a) $u_1,\cdots,u_r \geqslant 0$ (both this numbers are not simulteneously zero)

b) $Re[c+\sum_{i=1}^{r} u_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$; $Re[c+\frac{1-a-b}{2}+\sum_{i=1}^{r} u_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > 0$

c) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

Let $V+1 = m_1, n_1+1; m_2, n_2; \cdots; m_r, n_r : V_{11} = m_1+1, n_1+1; m_2, n_2; \cdots; m_r, n_r$

$W+2 = W^{(1)}+2; W^{(2)}; \cdots; W^{(r)} : W_{31} = W_{31}^{(1)}; W^{(2)}; \cdots; W^{(r)}$ where

$W_{31}^{(1)} = p_{i^{(1)}}+3, q_{i^{(1)}}+1, \tau_{i^{(1)}}; R^{(1)}$, we obtain

Theorem 2

$$ \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(1+a-b)_k k!} \aleph_{U:W+2}^{0,\mathfrak{n}:V+1}\left(\begin{matrix} \text{z}_1 \\ \cdots \\ \cdots \\ \text{z}_r \end{matrix}\left|\begin{matrix} \text{A : (1-k-c;u), (1+a+k-c;u), C}_1; C_2;\cdots;C_r \\ \cdots \\ \text{B : D}_1;\cdots;D_r \end{matrix}\right.\right) $$

$$= \frac{\Gamma\left(1+\frac{a}{2}\right)\Gamma(1+a-b)}{\Gamma(a+1)\Gamma\left(1+\frac{a}{2}-b\right)} \aleph_{U:W_{31}}^{0,\mathfrak{n}:V_{11}} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array} \middle| \begin{array}{c} \mathrm{A};\ (\text{1-c;u}),\ \left(1+\frac{a}{2}-c:u\right);(1+a+k-c:u,\ \mathrm{C}_1; \\ \cdot\cdot\cdot \\ \mathrm{B}:\left(1+\frac{a}{2}-b-c:u\right),D_1; \end{array} \right.$$

$$\left. \begin{array}{c} \mathrm{C}_2;\cdots;C_r \\ \cdot\cdot\cdot \\ \mathrm{D}_2;\cdots;D_r \end{array} \right) \tag{3.2}$$

Provided that

a) $u \geqslant 0$

b) $Re\left[c+u \min_{1\leqslant j\leqslant m_1} \frac{d_j^{(1)}}{\delta_j^{(1)}}\right] > 0\,;Re\left[c-\frac{a}{2}+b+u_1 \min_{1\leqslant j\leqslant n_1} \frac{(c_j^{(1)}-1)}{\gamma_j^{(1)}}\right] < 1$

c) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

Let $U_{11} = p_i+1, q_i+1, \tau_i; R$ and $U_{12} = p_i+1, q_i+2, \tau_i; R$, we have

Theorem 3

$$\sum_{k=0}^{\infty} \frac{(a)_k(1-a)_k}{(b)_k k!} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array} \middle| \begin{array}{c} (\text{1-k-c:u}_1,\cdots,u_r), A:C \\ \cdot\cdot\cdot \\ (\text{b-k-2c:2u}_1,\cdots,2u_r), B:D \end{array} \right) = \frac{2^{1-2c}\pi\Gamma(b)}{\Gamma\left(\frac{a+b}{2}\right)\Gamma\left(\frac{b-a+1}{2}\right)}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V} \left(\begin{array}{c} 4^{-u_1}z_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ 4^{-u_r}z_r \end{array} \middle| \begin{array}{c} (\text{1-c:u}_1,\cdots,u_r), A:C \\ \cdot\cdot\cdot \\ \left(\frac{b-a+1}{2}-c:u_1,\cdots,u_r\right), \left(\frac{b+a}{2}-c:u_1,\cdots,u_r\right) B:D \end{array} \right) \tag{3.3}$$

Provided that

a) $u_1,\cdots,u_r \geqslant 0$ (both this numbers are not simulteneously zero)

b) $Re\left[c+\sum_{i=1}^{r} u_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}\right] > 0$

c) $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.2)

Proof : Let $M = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$

In order to establish the general summation (3.1), we proceed as follows. Denoting the left-hand side of (3.1) by L, using the definition of Aleph-function of several variables with the help of (1.1), we get

$$L = \sum_{k=0}^{\infty} \frac{(a)_k(b)_k}{\left(\frac{a+b+1}{2}\right)_k k!} M \left\{ \frac{\Gamma(k+c+\sum_{i=1}^{r} u_i s_i)}{\Gamma(k+2c+2\sum_{i=1}^{r} u_i s_i)} \right\} \mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.4}$$

Now, changing the order of summation and integration (which is permissible under the conditions stated), we have after some simplification

$$L = M\left\{\frac{\Gamma(k+c+\sum_{i=1}^{r}u_is_i)}{\Gamma(k+2c+2\sum_{i=1}^{r}u_is_i)}\sum_{k=0}^{\infty}\frac{(a)_k(b)_k(c+\sum_{i=1}^{r}u_is_i)_k}{\left(\frac{a+b+1}{2}\right)_k(2c+2\sum_{i=1}^{r}u_is_i)_k k!}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.5}$$

Summing the inner-serie, we have

$$L = M\left\{\frac{\Gamma(k+c+\sum_{i=1}^{r}u_is_i)}{\Gamma(k+2c+2\sum_{i=1}^{r}u_is_i)}\,{}_3F_2\left(\begin{matrix}\text{a, b, c }+\sum_{i=1}^{r}u_is_i\\ \frac{1}{2}(a+b+1),\ 2(\text{c}+\sum_{i=1}^{r}u_is_i)\end{matrix};1\right)\right\}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.6}$$

Finally we observe that the ${}_3F_2$ appearing in the inner side can be evaluated with the help of result (2.1) and after simplification, interpreting the resulting Mellin-Barnes contour integral as a multivariable Aleph-function, we obtain the desired result (3.1).
To prove (3.2) and (3.3), we use the similar method that (3.1) using respectively the help of results (2.2) and (2.3).

4. Multivariable I-function

If $\tau_i, \tau_{i^{(1)}}, \cdots, \tau_{i^{(r)}} \rightarrow 1$, the Aleph-function of several variables degenere to the I-function of several variables. We obtain the following three general summation formulas in this section for multivariable I-functions defined by Sharma et al [3].

Corollary 1

$$\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{\left(\frac{a+b+1}{2}\right)_k k!}I_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}\text{z}_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \text{z}_r\end{matrix}\ \middle|\ \begin{matrix}(\text{1-k-c:u}_1,\cdots,u_r),A:C\\ \cdot\cdot\cdot\\ (\text{1-k-2c:2u}_1,\cdots,2u_r),B:D\end{matrix}\right) = \frac{2^{1-2c}\pi\Gamma\left(c+\frac{1+a+b}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{matrix}4^{-u_1}z_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ 4^{-u_r}z_r\end{matrix}\ \middle|\ \begin{matrix}\left(\frac{a+b+1}{2}-c:u_1,\cdots,u_r\right),A:C\\ \cdot\cdot\cdot\\ \left(\frac{a+1}{2}-c:u_1,\cdots,u_r\right),\left(\frac{b+1}{2}-c:u_1,\cdots,u_r\right)B:D\end{matrix}\right) \tag{4.1}$$

where the same notations and validity conditions that (3.1).

Corollary 2

$$\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(1+a-b)_k k!}I_{U:W+2}^{0,\mathfrak{n}:V+1}\left(\begin{matrix}\text{z}_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \text{z}_r\end{matrix}\ \middle|\ \begin{matrix}\text{A : (1-k-c;u), (1+a+k-c;u), C}_1;C_2;\cdots;C_r\\ \cdot\cdot\cdot\\ \text{B : D}_1;\cdots;D_r\end{matrix}\right)$$

$$= \frac{\Gamma\left(1+\frac{a}{2}\right)\Gamma(1+a-b)}{\Gamma(a+1)\Gamma\left(1+\frac{a}{2}-b\right)}I_{U:W_{31}}^{0,\mathfrak{n}:V_{11}}\left(\begin{matrix}\text{z}_1\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\\ \text{z}_r\end{matrix}\ \middle|\ \begin{matrix}\text{A; (1-c;u), }\left(1+\frac{a}{2}-c:u\right);(1+a+k-c:u,\text{ C}_1;\\ \cdot\cdot\cdot\\ \text{B : }\left(1+\frac{a}{2}-b-c:u\right),D_1;\end{matrix}\right.$$

$$\left.\begin{matrix}\text{C}_2;\cdots;C_r\\ \cdot\cdot\cdot\\ \text{D}_2;\cdots;D_r\end{matrix}\right) \tag{4.2}$$

where the same notations and validity conditions that (3.2).

Corollary 3

$$\sum_{k=0}^{\infty}\frac{(a)_k(1-a)_k}{(b)_k k!} I_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \cdots \\ \mathrm{z}_r \end{array}\middle| \begin{array}{c} (\text{1-k-c:u}_1,\cdots,u_r), A:C \\ \cdots \\ (\text{b-k-2c:2u}_1,\cdots,2u_r), B:D \end{array}\right) = \frac{2^{1-2c}\pi\Gamma(b)}{\Gamma\left(\frac{a+b}{2}\right)\Gamma\left(\frac{b-a+1}{2}\right)}$$

$$I_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} 4^{-u_1}z_1 \\ \cdots \\ \cdots \\ 4^{-u_r}z_r \end{array}\middle| \begin{array}{c} (\text{1-c:u}_1,\cdots,u_r), A:C \\ \cdots \\ \left(\frac{b-a+1}{2}-c:u_1,\cdots,u_r\right),\left(\frac{b+a}{2}-c:u_1,\cdots,u_r\right)B:D \end{array}\right) \tag{4.3}$$

where the same notations and validity conditions that (3.3).

5. Aleph-function of two variables

If $r=2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following general summation formulas.

Corollary 4

$$\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{\left(\frac{a+b+1}{2}\right)_k k!} \aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \cdots \\ \mathrm{z}_2 \end{array}\middle| \begin{array}{c} (\text{1-k-c:u}_1,u_2), A:C \\ \cdots \\ (\text{1-k-2c:2u}_1,2u_2), B:D \end{array}\right) = \frac{2^{1-2c}\pi\Gamma\left(c+\frac{1+a+b}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(\frac{b+1}{2}\right)}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} 4^{-u_1}z_1 \\ \cdots \\ \cdots \\ 4^{-u_2}z_2 \end{array}\middle| \begin{array}{c} \left(\frac{a+b+1}{2}-c:u_1,u_2\right), A:C \\ \cdots \\ \left(\frac{a+1}{2}-c:u_1,u_2\right),\left(\frac{b+1}{2}-c:u_1,u_2\right)B:D \end{array}\right) \tag{5.1}$$

where the same notations and validity conditions that (3.1).

Corollary 5

$$\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(1+a-b)_k k!} \aleph_{U:W+2}^{0,\mathfrak{n}:V+1}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \cdots \\ \mathrm{z}_2 \end{array}\middle| \begin{array}{c} \text{A : (1-k-c;u), (1+a+k-c;u), C}_1;C_2 \\ \cdots \\ \text{B : D}_1;D_2 \end{array}\right) = \frac{\Gamma\left(1+\frac{a}{2}\right)\Gamma(1+a-b)}{\Gamma(a+1)\Gamma\left(1+\frac{a}{2}-b\right)}$$

$$\aleph_{U:W_{31}}^{0,\mathfrak{n}:V_{11}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \cdots \\ \mathrm{z}_2 \end{array}\middle| \begin{array}{cc} \text{A; (1-c;u), } \left(1+\frac{a}{2}-c:u\right);(1+a+k-c:u, & \text{C}_1;\text{ C}_2 \\ \cdots & \cdots \\ \text{B}:\left(1+\frac{a}{2}-b-c:u\right),D_1; & \text{D}_2 \end{array}\right) \tag{5.2}$$

where the same notations and validity conditions that (3.2).

Corollary 6

$$\sum_{k=0}^{\infty}\frac{(a)_k(1-a)_k}{(b)_k k!}\aleph_{U_{11}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\middle| \begin{array}{c} (\text{1-k-c:u}_1,u_2),A:C \\ \cdot\cdot\cdot \\ (\text{b-k-2c:2u}_1,2u_2),B:D \end{array}\right)=\frac{2^{1-2c}\pi\Gamma(b)}{\Gamma\left(\frac{a+b}{2}\right)\Gamma\left(\frac{b-a+1}{2}\right)}$$

$$\aleph_{U_{12}:W}^{0,\mathfrak{n}+1:V}\left(\begin{array}{c} 4^{-u_1}z_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ 4^{-u_2}z_2 \end{array}\middle| \begin{array}{c} (\text{1-c:u}_1,u_2),A:C \\ \cdot\cdot\cdot \\ \left(\frac{b-a+1}{2}-c:u_1,u_2\right),\left(\frac{b+a}{2}-c:u_1,u_2\right)B:D \end{array}\right) \tag{5.3}$$

where the same notations and validity conditions that (3.3). If $\tau_i=\tau_i'=\tau_i''=1$, we obtain the same general summation formulas with the I-function of two variables defined by Sharma et al [4].

6. Conclusion

In this paper, we have established three general summation formulas involving the multivariable Aleph-function by using some classical summation theorems. Due to general nature of the multivariable aleph-function involving here, our formulas are capable to be reduced into many known and news summations involving the special functions of one and several variables.

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Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE