Triple finite series relations involving the multivariable Aleph-function

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Abstract
The aim of this document is to establish finite triple series relations for the multivariable Aleph-function. These relations are quite general in nature, from which a large number of new results can be obtained simply by specializing the parameters.

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1. Introduction and preliminaries.

The object of this document is to evaluate four finite triple summations involving the multivariable aleph-function. These function generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;p_{i(r)},q_{i(r)};\tau_{i(r)};R^{(r)}} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\right|$

$[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]$, $[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]$:
.... , $[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]$:

$[(c_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(c_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$[(d_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(d_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\Bigg)$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.2}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. The numbers. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\dots|z_r|^{\alpha_r}\,)\,,max(\,|z_1|\dots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\dots|z_r|^{\beta_r}\,)\,,min(\,|z_1|\dots|z_r|\,)\rightarrow \infty$$

where, with $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \tag{1.3}$$

$$\text{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.4}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.5}$$

$$B=\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.6}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.7}$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.8}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \text{A : C} \\ \vdots & \cdots \\ z_r & \text{B : D}\end{array}\right) \tag{1.9}$$

2. Series relations

We have the following four finites sums ;

Formula 1

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t}{u!v!t!}\aleph_{p_i+5,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+4:V}\left(\begin{array}{c|c} z_1 & \text{(1-a-v-t };\theta_1,\cdots,\theta_r), \\ \vdots & \cdots \\ z_r & \text{(1-c-u-v-t };\delta_1,\cdots,\delta_r),\end{array}\right.$$

$$\begin{array}{cc}\text{(1-c+a-u };\delta_1-\theta_1,\cdots,\delta_r-\theta_r), & \text{(1-c+b-u -t};\delta_1-\phi_1,\cdots,\delta_r-\phi_r), \\ \cdots & \cdots \\ \text{(d+m-u };\delta_1-\phi_1-\theta_1,\cdots,\delta_r-\phi_r-\theta_r), & \text{(b+p-a-t };\theta_1-\phi_1,\cdots,\theta_r-\phi_r),\end{array}$$

$$\left.\begin{array}{c}\text{(1+d-n+v};\delta_1-\theta_1-\phi_1,\cdots,\delta_r-\theta_r-\phi_r),(1-b-v;\phi_1,\cdots,\phi_r),A:C \\ \cdots \\ \text{B : D}\end{array}\right)$$

$$=(-)^{m+n+p}\aleph_{p_i+5,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+4:V}\left(\begin{array}{c|cc} z_1 & \text{(1-a-m };\theta_1,\cdots,\theta_r), & \text{(1-c+a-n-p };\delta_1-\theta_1,\cdots,\delta_r-\theta_r), \\ \vdots & \cdots & \cdots \\ z_r & \text{(1-c-m-n };\delta_1,\cdots,\delta_r), & \text{(b-a };\theta_1-\phi_1,\cdots,\theta_r-\phi_r),\end{array}\right.$$

$$(1\text{+d } ;\delta_1-\theta_1-\phi_1,\cdots,\delta_r-\theta_r-\phi_r),(1\text{+b-c-n } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r),$$
$$\cdots \qquad\qquad \cdots$$
$$(\text{d}; \ \delta_1-\theta_1-\phi_1,\cdots,\delta_r-\theta_r-\phi_r), \qquad \cdots,$$

$$\left.\begin{matrix}(1\text{-b-m-p } ;\phi_1,\cdots,\phi_r),A:C\\ \cdots \\ \text{B : D}\end{matrix}\right) \tag{2.1}$$

with the validity conditions : $\delta_i,\theta_i,\phi^{i)}>0,\delta_i>\theta_i+\phi_i;i=1,\cdots,r$ and

$|argz_k| < \dfrac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2) and $d=a+b-c$

Formula 2

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(a)_{v+t}}{u!v!t!}\aleph_{p_i+5,q_i+2,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{matrix}\text{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \text{z}_r\end{matrix}\middle|\begin{matrix}(1\text{-c+a-u } ;\delta_1,\cdots,\delta_r),\\ \cdots\\ (1\text{-c-u-v-t } ;\delta_1,\cdots,\delta_r),\end{matrix}\right.$$

$$\begin{matrix}(1\text{-b-v } ;\phi_1,\cdots,\phi_r), & (1\text{-c+b-u-t } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r),(1\text{+a-b-p+t } ;\phi_1,\cdots,\phi_r),\\ \cdots & \cdots \qquad\qquad \cdots\\ (\text{d+m-u } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r), & \text{B : D}, \qquad\qquad \cdots\end{matrix}$$

$$\left.\begin{matrix}(1\text{+d-n+s } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r),A:C\\ \cdots\\ \cdots\end{matrix}\right)$$

$$=(-)^{m+n+p}(a)_m\aleph_{p_i+5,q_i+2,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{matrix}\text{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \text{z}_r\end{matrix}\middle|\begin{matrix}(1\text{-c+a-n-p } ;\delta_1,\cdots,\delta_r), & (1\text{-b-m-p } ;\phi_1,\cdots,\phi_r),\\ \cdots & \cdots\\ (1\text{-c-m-n-p } ;\delta_1,\cdots,\delta_r), & (1\text{+d-n } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r),\end{matrix}\right.$$

$$\left.\begin{matrix}(1\text{+d } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r), & (1\text{+a-b } ;\phi_1,\cdots,\phi_r), & (1\text{+b-n-c } ;\delta_1-\phi_1,\cdots,\delta_r-\phi_r),A:C\\ \cdots & \cdots & \cdots\\ \text{B : D}, & \cdots & \cdots\end{matrix}\right) \tag{2.2}$$

with the validity conditions : $\delta_i>\phi_i>0;i=1,\cdots,r$ and $|argz_k| < \dfrac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2) and $d=a+b-c$

Formula 3

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(b)_v}{u!v!t!}\aleph_{p_i+4,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{matrix}\text{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \text{z}_r\end{matrix}\middle|\begin{matrix}(1\text{-a-v -t};\theta_1,\cdots,\theta_r),\\ \cdots\\ (1\text{-c-u-v-t } ;\delta_1,\cdots,\delta_r),\end{matrix}\right.$$

$$\left.\begin{matrix} \text{(1-c+a-u;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\text{(1-c+b-u-t };\delta_1,\cdots,\delta_r),\text{(1+d-n+v;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),A:C \\ \cdots \qquad \cdots \qquad \cdots \\ \text{(d+m-u; }\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\ \text{(b+p-a-t };\theta_1,\cdots,\theta_r), \qquad \text{B : D,} \end{matrix}\right)$$

$$=(-)^{m+n+p}(b)_{m+p}\aleph^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3,\tau_i;R:W}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix}\ \middle|\ \begin{matrix} \text{(1-a-m;}\theta_1,\cdots,\theta_r),\quad \text{(1+b-c-n };\delta_1,\cdots,\delta_r), \\ \cdots \qquad \cdots \\ \text{(1-c-m-n-p };\delta_1,\cdots,\delta_r),\quad \text{(b-a };\theta_1,\cdots,\theta_r), \end{matrix}\right.$$

$$\left.\begin{matrix} \text{(1+d };\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\text{(1-c+a-n-p;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),A:C \\ \cdots \qquad \cdots \\ \text{(d; }\delta_1-\theta_1,\cdots,\delta_r-\theta_r), \qquad \text{B : D,} \end{matrix}\right) \tag{2.3}$$

with the validity conditions : $\delta_i > \theta_i > 0; i = 1,\cdots,r$ and $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

Formula 4

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(c-b)_{v+t}(c-a)_u}{(1+a-b-p)_t u!v!t!}\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2,\tau_i;R:W}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix}\ \middle|\ \begin{matrix} \text{(1-b-v };\delta_1,\cdots,\delta_r), \\ \cdots \\ \text{(n-d-v };\delta_1,\cdots,\delta_r), \end{matrix}\right.$$

$$\left.\begin{matrix} \text{(1-d-m+u };\delta_1,\cdots,\delta_r),\text{(1-a-v-t;}\delta_1,\cdots,\delta_r),A:C \\ \cdots \qquad \cdots \\ \text{(1-c-u-v-t };\delta_1,\cdots,\delta_r), \qquad \text{B : D,} \end{matrix}\right)$$

$$=(-)^{m+n}\frac{(c-a)_{n+p}(c-b)_n}{(b-a)_p}\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2,\tau_i;R:W}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix}\ \middle|\ \begin{matrix} \text{(1-a-m };\delta_1,\cdots,\delta_r),\text{(1+d };\delta_1,\cdots,\delta_r), \\ \cdots \qquad \cdots \\ \text{(-d };\delta_1,\cdots,\delta_r), \qquad \cdots, \end{matrix}\right.$$

$$\left.\begin{matrix} \text{(1-b-m-p };\delta_1,\cdots,\delta_r),A:C \\ \cdots \\ \text{(1-c-m-n-p };\delta_1,\cdots,\delta_r),B:D \end{matrix}\right) \tag{2.4}$$

with the validity conditions : $\delta_i > 0; i = 1,\cdots,r$ and $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

Proof of (2.1)

Substituting contour integral (1.1) for the Aleph-function of several variables in left-hand side of (2.1), changing the order of summation and integration which is justified as the series involved are finite and using the following result of Paradhan [2,p.33]

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(a)_{v+t}(c-a)_u(c-b)_{u+t}(b)_v}{(c)_{u+v+t}(1-d-m)_u(1+d-n)_v(1+a-b-p)_v u!v!t!}$$

$$=\frac{(a)_m(c-a)_{n+p}(b)_{m+p}(c-a)_n}{(-d)_n(b-a)_p(d)_m(c)_{m+n+p}} \text{ with } d=a+b-c \text{ (not an integer)} \tag{2.5}$$

Interpreting the result, thus obtained with the help pf (1.1), we get the desired result. To prove (2.2) to (2.4), we use the similar methods to (3.1).

3. Multivariable I-function

If $\tau_i, \tau_{i^{(1)}}, \cdots, \tau_{i^{(r)}} \rightarrow 1$, the Aleph-function of several variables degenere to the I-function of several variables. The finite triple sums have been derived in this section for multivariable I-functions defined by Sharma et al [3]. In these section, we note

$$B_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \tag{3.1}$$

Formula 1

$$\sum_{u=0}^{m} \sum_{v=0}^{n} \sum_{t=0}^{p} \frac{(-m)_u(-n)_v(-p)_t}{u!v!t!} I_{p_i+5,q_i+3;R:W}^{m,\mathfrak{n}+4:V} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \middle| \begin{array}{c} (\text{1-a-v-t };\theta_1, \cdots, \theta_r), \\ \cdots \\ (\text{1-c-u-v-t };\delta_1, \cdots, \delta_r), \end{array} \right.$$

$$\begin{array}{cc} (\text{1-c+a-u };\delta_1 - \theta_1, \cdots, \delta_r - \theta_r), & (\text{1-c+b-u -t};\delta_1 - \phi_1, \cdots, \delta_r - \phi_r), \\ \cdots & \cdots \\ (\text{d+m-u };\delta_1 - \phi_1 - \theta_1, \cdots, \delta_r - \phi_r - \theta_r), & (\text{b+p-a-t };\theta_1 - \phi_1, \cdots, \theta_r - \phi_r), \end{array}$$

$$\left. \begin{array}{c} (\text{1+d-n+v};\delta_1 - \theta_1 - \phi_1, \cdots, \delta_r - \theta_r - \phi_r), (1 - b - v; \phi_1, \cdots, \phi_r), A : C \\ \cdots \\ \text{B : D} \end{array} \right)$$

$$= (-)^{m+n+p} I_{p_i+5,q_i+3;R:W}^{m,\mathfrak{n}+4:V} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array} \middle| \begin{array}{cc} (\text{1-a-m };\theta_1, \cdots, \theta_r), & (\text{1-c+a-n-p };\delta_1 - \theta_1, \cdots, \delta_r - \theta_r), \\ \cdots & \cdots \\ (\text{1-c-m-n };\delta_1, \cdots, \delta_r), & (\text{b-a };\theta_1 - \phi_1, \cdots, \theta_r - \phi_r), \end{array} \right.$$

$$\begin{array}{cc} (\text{1+d };\delta_1 - \theta_1 - \phi_1, \cdots, \delta_r - \theta_r - \phi_r), & (\text{1+b-c-n };\delta_1 - \phi_1, \cdots, \delta_r - \phi_r), \\ \cdots & \cdots \\ (\text{d}; \delta_1 - \theta_1 - \phi_1, \cdots, \delta_r - \theta_r - \phi_r), & \cdots, \end{array}$$

$$\left. \begin{array}{c} (\text{1-b-m-p };\phi_1, \cdots, \phi_r), A : C \\ \cdots \\ \text{B : D} \end{array} \right) \tag{3.2}$$

with the validity conditions : $\delta_i, \theta_i, \phi_i > 0, \delta_i > \theta_i + \phi_i; i = 1, \cdots, r$ and

$|argz_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is defined by (3.1) and $d = a + b - c$

Formula 2

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(a)_{v+t}}{u!v!t!}I^{m,\mathfrak{n}+3:V}_{p_i+5,q_i+2;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\middle|\begin{array}{l}\text{(1-c+a-u };\delta_1,\cdots,\delta_r),\\ \cdots\\ \text{(1-c-u-v-t };\delta_1,\cdots,\delta_r),\end{array}\right.$$

$$\begin{array}{lll}\text{(1-b-v };\phi_1,\cdots,\phi_r), & \text{(1-c+b-u-t };\delta_1-\phi_1,\cdots,\delta_r-\phi_r),\text{(1+a-b-p+t };\phi_1,\cdots,\phi_r),\\ \cdots & \cdots & \cdots\\ \text{(d+m-u };\delta_1-\phi_1,\cdots,\delta_r-\phi_r), & \text{B : D}, & \cdots\end{array}$$

$$\left.\begin{array}{l}\text{(1+d-n+s };\delta_1-\phi_1,\cdots,\delta_r-\phi_r),A:C\\ \cdots\\ \cdots\end{array}\right)$$

$$=(-)^{m+n+p}I^{m,\mathfrak{n}+4:V}_{p_i+5,q_i+3;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\middle|\begin{array}{ll}\text{(1-a-m };\theta_1,\theta_2), & \text{(1-c+a-n-p };\delta_1-\theta_1,\delta_2-\theta_2),\\ \cdots & \cdots\\ \text{(1-c-m-n };\delta_1,\delta_2), & \text{(b-a };\theta_1-\phi_1,\theta_2-\phi_2),\end{array}\right.$$

$$\left.\begin{array}{ll}\text{(1+d };\delta_1-\theta_1-\phi_1,\cdots,\delta_r-\theta_r-\phi_r),\text{(1+b-c-n };\delta_1-\phi_1,\cdots,\delta_r-\phi_r),A:C\\ \cdots & \cdots\\ \text{(d; }\delta_1-\theta_1-\phi_1,\cdots,\delta_r-\theta_r-\phi_r), & \text{B : D}\end{array}\right) \tag{3.3}$$

with the validity conditions : $\delta_i > \phi_i > 0; i = 1,\cdots,r$ and $|arg z_k| < \frac{1}{2}B_i^{(k)}\pi,$ where $B_i^{(k)}$ is defined by (3.1) and $d = a + b - c$

Formula 3

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(b)_v}{u!v!t!}I^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\middle|\begin{array}{l}\text{(1-a-v -t;}\theta_1,\cdots,\theta_r),\\ \cdots\\ \text{(1-c-u-v-t };\delta_1,\cdots,\delta_r),\end{array}\right.$$

$$\left.\begin{array}{ll}\text{(1-c+a-u;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\text{(1-c+b-u-t };\delta_1,\cdots,\delta_r),\text{(1+d-n+v;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),A:C\\ \cdots \quad \cdots & \cdots\\ \text{(d+m-u; }\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\ \text{(b+p-a-t };\theta_1,\cdots,\theta_r), & \text{B : D},\end{array}\right)$$

$$=(-)^{m+n+p}(b)_{m+p}I^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\middle|\begin{array}{ll}\text{(1-a-m;}\theta_1,\cdots,\theta_r), & \text{(1+b-c-n };\delta_1,\cdots,\delta_r),\\ \cdots & \cdots\\ \text{(1-c-m-n-p };\delta_1,\cdots,\delta_r), & \text{(b-a };\theta_1,\cdots,\theta_r),\end{array}\right.$$

$$\left.\begin{array}{ll}\text{(1+d };\delta_1-\theta_1,\cdots,\delta_r-\theta_r),\text{(1-c+a-n-p;}\delta_1-\theta_1,\cdots,\delta_r-\theta_r),A:C\\ \cdots & \cdots\\ \text{(d; }\delta_1-\theta_1,\cdots,\delta_r-\theta_r), & \text{B : D},\end{array}\right) \tag{3.4}$$

with the validity conditions : $\delta_i > \theta_i > 0; i = 1, \cdots, r$ and $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

Formula 4

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(c-b)_{v+t}(c-a)_u}{(1+a-b-p)_t u!v!t!} I_{p_i+3,q_i+2;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle| \begin{array}{c} (1\text{-b-v};\delta_1,\cdots,\delta_r), \\ \cdots \\ (\text{n-d-v};\delta_1,\cdots,\delta_r), \end{array}\right.$$

$$\left.\begin{array}{cc} (1\text{-d-m+u};\delta_1,\cdots,\delta_r),(1\text{-a-v-t};\delta_1,\cdots,\delta_r), & A : C \\ \cdots & \cdots \\ (1\text{-c-u-v-t};\delta_1,\cdots,\delta_r), & \mathrm{B : D}, \end{array}\right)$$

$$=(-)^{m+n}\frac{(c-a)_{n+p}(c-b)_n}{(b-a)_p} I_{p_i+3,q_i+2;R:W}^{m,\mathfrak{n}+3:V}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle| \begin{array}{cc} (1\text{-a-m};\delta_1,\cdots,\delta_r),(1\text{+d};\delta_1,\cdots,\delta_r), \\ \cdots & \cdots \\ (\text{-d};\delta_1,\cdots,\delta_r), & \cdots, \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-b-m-p};\delta_1,\cdots,\delta_r), A : C \\ \cdots \\ (1\text{-c-m-n-p};\delta_1,\cdots,\delta_r), B : D \end{array}\right) \tag{3.5}$$

with the validity conditions : $\delta_i > 0; i = 1, \cdots, r$ and $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

4. Aleph-function of two variables

If $r = 2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following relations.

Formula 1

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t}{u!v!t!}\aleph_{p_i+5,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+4:V}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_2 \end{array}\middle| \begin{array}{c} (1\text{-a-v-t};\theta_1,\theta_2), \\ \cdots \\ (1\text{-c-u-v-t};\delta_1,\delta_2), \end{array}\right.$$

$$\begin{array}{cc} (1\text{-c+a-u};\delta_1-\theta_1,\delta_2-\theta_2), & (1\text{-c+b-u -t};\delta_1-\phi_1,\delta_2-\phi_2), \\ \cdots & \cdots \\ (\text{d+m-u};\delta_1-\phi_1-\theta_1,\delta_2-\phi_2-\theta_2), & (\text{b+p-a-t};\theta_1-\phi_1,\theta_2-\phi_2), \end{array}$$

$$\left.\begin{array}{c} (1\text{+d-n+v};\delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2),(1-b-v;\phi_1,\phi_2), A : C \\ \cdots \\ \mathrm{B : D} \end{array}\right)$$

$$=(-)^{m+n+p}\aleph_{p_i+5,q_i+3,\tau_i;R:W}^{m,\mathfrak{n}+4:V}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_2 \end{array}\middle| \begin{array}{cc} (1\text{-a-m};\theta_1,\theta_2), & (1\text{-c+a-n-p};\delta_1-\theta_1,\delta_2-\theta_2), \\ \cdots & \cdots \\ (1\text{-c-m-n};\delta_1,\delta_2), & (\text{b-a};\theta_1-\phi_1,\theta_2-\phi_2), \end{array}\right.$$

$$\begin{array}{lll} (1+d\ ;\delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2), & (1+b-c-n\ ;\delta_1-\phi_1,\delta_2-\phi_2), & (1-b-m-p\ ;\phi_1,\phi_2), A:C \\ \cdots & \cdots & \cdots \\ (d;\ \delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2), & \cdots, & B:D \end{array} \Bigg) \quad (4.1)$$

with the validity conditions : $\delta_i,\theta_i,\phi_i>0,\delta_i>\theta_i+\phi_i; i=1,2$ and $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2) and $d=a+b-c$

Formula 2

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(a)_{v+t}}{u!v!t!}\aleph^{m,n+3:V}_{p_i+5,q_i+2,\tau_i;R:W}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_2 \end{array}\middle| \begin{array}{l} (1-c+a-u\ ;\delta_1,\delta_2), \\ \cdots \\ (1-c-u-v-t\ ;\delta_1,\delta_2), \end{array}\right.$$

$$\begin{array}{lll} (1-b-v\ ;\phi_1,\phi_2), & (1-c+b-u-t\ ;\delta_1-\phi_1,\delta_2-\phi_2), & (1+a-b-p+t\ ;\phi_1,\phi_2), \\ \cdots & \cdots & \cdots \\ (d+m-u\ ;\delta_1-\phi_1,\delta_2-\phi_2), & B:D, & \cdots \end{array}$$

$$\left.\begin{array}{c} (1+d-n+s\ ;\delta_1-\phi_1,\delta_2-\phi_2), A:C \\ \cdots \\ \cdots \end{array}\right)$$

$$=(-)^{m+n+p}(a)_m\aleph^{m,n+3:V}_{p_i+5,q_i+2,\tau_i;R:W}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_2 \end{array}\middle| \begin{array}{ll} (1-c+a-n-p\ ;\delta_1,\delta_2), & (1-b-m-p\ ;\phi_1,\phi_2), \\ \cdots & \cdots \\ (1-c-m-n-p\ ;\delta_1,\delta_2), & (1+d-n\ ;\delta_1-\phi_1,\delta_2-\phi_2), \end{array}\right.$$

$$\begin{array}{lll} (1+d\ ;\delta_1-\phi_1,\delta_2-\phi_2), & (1+a-b\ ;\phi_1,\phi_2), & (1+b-n-c\ ;\delta_1-\phi_1,\delta_2-\phi_2), A:C \\ \cdots & \cdots & \cdots \\ B:D, & \cdots & \cdots \end{array}\Bigg) \quad (4.2)$$

with the validity conditions : $\delta_i>\phi_i>0; i=1,2$ and $|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2) and $d=a+b-c$

Formula 3

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(b)_v}{u!v!t!}\aleph^{m,n+3:V}_{p_i+4,q_i+3,\tau_i;R:W}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_2 \end{array}\middle| \begin{array}{l} (1-a-v-t;\theta_1,\theta_2), \\ \cdots \\ (1-c-u-v-t\ ;\delta_1,\delta_2), \end{array}\right.$$

$$\begin{array}{lll} (1-c+a-u;\delta_1-\theta_1,\delta_2-\theta_2), & (1-c+b-u-t\ ;\delta_1,\delta_2), & (1+d-n+v;\delta_1-\theta_1,\delta_2-\theta_2), A:C \\ \cdots & \cdots & \cdots \\ (d+m-u;\ \delta_1-\theta_1,\delta_2-\theta_2), & (b+p-a-t\ ;\theta_1,\theta_2), & B:D, \end{array}\Bigg)$$

$$=(-)^{m+n+p}(b)_{m+p}\aleph^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3,\tau_i;R:W}\left(\begin{array}{c|c} z_1 & (1\text{-a-m};\theta_1,\theta_2), \quad (1\text{+b-c-n };\delta_1,\delta_2), \\ \cdot & \cdots \qquad \cdots \\ \cdot & (1\text{-c-m-n-p };\delta_1,\delta_2), \quad (\text{b-a };\theta_1,\theta_2), \\ z_2 & \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{+d };\delta_1-\theta_1,\delta_2-\theta_2),(1\text{-c+a-n-p};\delta_1-\theta_1,\delta_2-\theta_2), A:C \\ \cdots \qquad \cdots \\ (\text{d}; \delta_1-\theta_1,\delta_2-\theta_2), \qquad \text{B : D}, \end{array}\right) \tag{4.3}$$

with the validity conditions : $\delta_i > \theta_i > 0; i=1,2$ and $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

Formula 4

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(c-b)_{v+t}(c-a)_u}{(1+a-b-p)_t u!v!t!}\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} z_1 & (1\text{-b-v };\delta_1,\delta_2), \\ \cdot & \cdots \\ \cdot & (\text{n-d-v };\delta_1,\delta_2), \\ z_2 & \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-d-m+u };\delta_1,\delta_2),(1\text{-a-v-t};\delta_1,\delta_2), A:C \\ \cdots \qquad \cdots \\ (1\text{-c-u-v-t };\delta_1,\delta_2), \qquad \text{B : D}, \end{array}\right)$$

$$=(-)^{m+n}\frac{(c-a)_{n+p}(c-b)_n}{(b-a)_p}\aleph^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} z_1 & (1\text{-a-m };\delta_1,\delta_2),(1\text{+d };\delta_1,\delta_2), \\ \cdot & \cdots \qquad \cdots \\ \cdot & (\text{-d };\delta_1,\delta_2), \qquad \cdots, \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{c} (1\text{-b-m-p };\delta_1,\delta_2), A:C \\ \cdots \\ (1\text{-c-m-n-p };\delta_1,\delta_2), B:D \end{array}\right) \tag{4.4}$$

with the validity conditions : $\delta_i > 0; i=1,2$ and $|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.2)

5. I-function of two variables

If $\tau_i, \tau_{i'}, \tau_{i''} \rightarrow 1$, then the Aleph-function of two variables degenere in the I-function of two variables defined by sharma et al [4] and we obtain

Formula 1

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t}{u!v!t!}I^{m,\mathfrak{n}+4:V}_{p_i+5,q_i+3;R:W}\left(\begin{array}{c|c} z_1 & (1\text{-a-v-t };\theta_1,\theta_2), \\ \cdot & \cdots \\ \cdot & (1\text{-c-u-v-t };\delta_1,\delta_2), \\ z_2 & \end{array}\right.$$

$$\begin{array}{c} (1\text{-c+a-u };\delta_1-\theta_1,\delta_2-\theta_2), \qquad (1\text{-c+b-u -t};\delta_1-\phi_1,\delta_2-\phi_2), \\ \cdots \qquad \cdots \\ (\text{d+m-u };\delta_1-\phi_1-\theta_1,\delta_2-\phi_2-\theta_2), \ (\text{b+p-a-t };\theta_1-\phi_1,\theta_2-\phi_2), \end{array}$$

$$\left. \begin{matrix} (1+d-n+v;\delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2),(1-b-v;\phi_1,\phi_2),A:C \\ \cdots \\ B:D \end{matrix} \right)$$

$$=(-)^{m+n+p} I^{m,\mathfrak{n}+4:V}_{p_i+5,q_i+3;R:W} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_2 \end{matrix} \middle| \begin{matrix} (1-a-m\ ;\theta_1,\theta_2),\ (1-c+a-n-p\ ;\delta_1-\theta_1,\delta_2-\theta_2), \\ \cdots \qquad \cdots \\ (1-c-m-n\ ;\delta_1,\delta_2),\quad (b-a\ ;\theta_1-\phi_1,\theta_2-\phi_2), \end{matrix} \right.$$

$$\left. \begin{matrix} (1+d\ ;\delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2),(1+b-c-n\ ;\delta_1-\phi_1,\delta_2-\phi_2),(1-b-m-p\ ;\phi_1,\phi_2),A:C \\ \cdots \qquad \cdots \qquad \cdots \\ (d;\ \delta_1-\theta_1-\phi_1,\delta_2-\theta_2-\phi_2), \qquad \cdots, \qquad B:D \end{matrix} \right) \tag{5.1}$$

with the validity conditions : $\delta_i,\theta_i,\phi_i>0,\delta_i>\theta_i+\phi_i;i=1,2$ and $|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1) and $d=a+b-c$

Formula 2

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(a)_{v+t}}{u!v!t!} I^{m,\mathfrak{n}+3:V}_{p_i+5,q_i+2;R:W} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_2 \end{matrix} \middle| \begin{matrix} (1-c+a-u\ ;\delta_1,\delta_2), \\ \cdots \\ (1-c-u-v-t\ ;\delta_1,\delta_2), \end{matrix} \right.$$

$$\begin{matrix} (1-b-v\ ;\phi_1,\phi_2), \qquad (1-c+b-u-t\ ;\delta_1-\phi_1,\delta_2-\phi_2),(1+a-b-p+t\ ;\phi_1,\phi_2), \\ \cdots \qquad \cdots \qquad \cdots \\ (d+m-u\ ;\delta_1-\phi_1,\delta_2-\phi_2), \qquad B:D, \qquad \cdots \end{matrix}$$

$$\left. \begin{matrix} (1+d-n+s\ ;\delta_1-\phi_1,\delta_2-\phi_2),A:C \\ \cdots \\ \cdots \end{matrix} \right)$$

$$=(-)^{m+n+p}(a)_m I^{m,\mathfrak{n}+3:V}_{p_i+5,q_i+2;R:W} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_2 \end{matrix} \middle| \begin{matrix} (1-c+a-n-p\ ;\delta_1,\delta_2),(1+d\ ;\delta_1-\phi_1,\delta_2-\phi_2), \\ \cdots \qquad \cdots \\ (1-c-m-n-p\ ;\delta_1,\delta_2), \qquad B:D, \end{matrix} \right.$$

$$\left. \begin{matrix} (1+a-b\ ;\phi_1,\phi_2),(1+b-n-c\ ;\delta_1-\phi_1,\delta_2-\phi_2),A:C \\ \cdots \qquad \cdots \\ \cdots \qquad \cdots \end{matrix} \right) \tag{5.2}$$

with the validity conditions : $\delta_i>\phi_i>0;i=1,2$ and $|argz_k|<\frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1) and $d=a+b-c$

Formula 3

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(b)_v}{u!v!t!}I^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ z_2\end{array}\middle|\begin{array}{l}(1\text{-a-v -t};\theta_1,\theta_2),\\ \cdots\\ (1\text{-c-u-v-t };\delta_1,\delta_2),\end{array}\right.$$

$$\left.\begin{array}{llll}(1\text{-c+a-u};\delta_1-\theta_1,\delta_2-\theta_2), & (1\text{-c+b-u-t };\delta_1,\delta_2), & (1\text{+d-n+v};\delta_1-\theta_1,\delta_2-\theta_2), & A:C\\ \cdots & \cdots & \cdots & \\ (\text{d+m-u};\ \delta_1-\theta_1,\delta_2-\theta_2), & (\text{b+p-a-t };\theta_1,\theta_2), & & \text{B : D},\end{array}\right)$$

$$=(-)^{m+n+p}(b)_{m+p}I^{m,\mathfrak{n}+3:V}_{p_i+4,q_i+3;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ z_2\end{array}\middle|\begin{array}{ll}(1\text{-a-m};\theta_1,\theta_2), & (1\text{+b-c-n };\delta_1,\delta_2),\\ \cdots & \cdots\\ (1\text{-c-m-n-p };\delta_1,\delta_2), & (\text{b-a };\theta_1,\theta_2),\end{array}\right.$$

$$\left.\begin{array}{lll}(1\text{+d };\delta_1-\theta_1,\delta_2-\theta_2), & (1\text{-c+a-n-p};\delta_1-\theta_1,\delta_2-\theta_2), & A:C\\ \cdots & \cdots & \\ (\text{d};\ \delta_1-\theta_1,\delta_2-\theta_2), & & \text{B : D},\end{array}\right) \tag{5.3}$$

with the validity conditions : $\delta_i > \theta_i > 0; i = 1,2$ and $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

Formula 4

$$\sum_{u=0}^{m}\sum_{v=0}^{n}\sum_{t=0}^{p}\frac{(-m)_u(-n)_v(-p)_t(c-b)_{v+t}(c-a)_u}{(1+a-b-p)_t u!v!t!}I^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ z_2\end{array}\middle|\begin{array}{l}(1\text{-b-v };\delta_1,\delta_2),\\ \cdots\\ (\text{n-d-v };\delta_1,\delta_2),\end{array}\right.$$

$$\left.\begin{array}{lll}(1\text{-d-m+u };\delta_1,\delta_2), & (1\text{-a-v-t};\delta_1,\delta_2), & A:C\\ \cdots & \cdots & \\ (1\text{-c-u-v-t };\delta_1,\delta_2), & & \text{B : D},\end{array}\right)$$

$$=(-)^{m+n}\frac{(c-a)_{n+p}(c-b)_n}{(b-a)_p}I^{m,\mathfrak{n}+3:V}_{p_i+3,q_i+2;R:W}\left(\begin{array}{c}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\middle|\begin{array}{ll}(1\text{-a-m };\delta_1,\delta_2), & (1\text{+d };\delta_1,\delta_2),\\ \cdots & \cdots\\ (\text{-d };\delta_1,\delta_2), & \cdots,\end{array}\right.$$

$$\left.\begin{array}{l}(1\text{-b-m-p };\delta_1,\delta_2), A:C\\ \cdots\\ (1\text{-c-m-n-p };\delta_1,\delta_2), B:D\end{array}\right) \tag{5.4}$$

with the validity conditions : $\delta_i > 0; i = 1,2$ and $|argz_k| < \frac{1}{2}B_i^{(k)}\pi$, where $B_i^{(k)}$ is defined by (3.1)

6. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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