Parametric Analysis on the MCDM using TOPSIS

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Abstract

Multi-Criteria Decision Making (MCDM) is a branch of a general class of Operations Research models and management science, which deals with decision problems under the presence of a number of decision criteria. In this paper, we study the stability of (MCDM) problems with parameters in the decision matrix, by using the hybridization between Simple Multi Attribute Rating Technique(SMART) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods, to explain the effect of the parameters in the decision matrix. The SMART method is applied to determinate the weights for each of the criteria to reflect its relative importance. In TOPSIS method alternatives are ranked based on their distance from an ideal solution and a negative-ideal solution. Parametric analysis allows us to choose parameters for evaluation, refine the parameters until satisfied with the results. Finally, an illustrative numerical example is given to explain the parametric analysis on the MCDM problem by using the hybridization between SMART and TOPSIS method, where the results are obtained using the MATLAB program.

Keywords: *MCDM, SMART, TOPSIS, Parametric analysis*

I. Introduction

The multiple criteria decision making (MCDM) can be generally described as the process of selecting one from among a finite set of alternatives or ranking alternatives , based on a set of the multiple usually conflicting and different units criteria. In these cases, we use normalization to transform the various criterion dimensions into non-dimensional criteria [1].

The SMART method is applied to determinate the weights for each of the criteria to reflect its relative importance. The TOPSIS method, developed by Hwang & Yoon (1981), is one of the MCDM methods. TOPSIS method is based on the concept that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS) and the farthest distance from the Negative Ideal Solution (NIS). The PIS is a set composing all the best values of each criterion, while the NIS is a set composing all the worst values of each criterion [2],[3].

"Reference [4]" reviewed and compared the application of four popular MCDM methods in maintenance decision making. These methods are analytic hierarchy process(AHP), elimination and choice expressing reality(ELECTRE), simple additive weighting (SAW), and technique for order preference by similarity to ideal solution (TOPSIS). The comparisons were based on the aspects of consistency, problem structure, concept, core process, and accuracy of final results.

"Reference [5]" provided decision methods for project managers in construction companies. The methodology is combined into three methods consisting of Delphi method, Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). As the result, the criteria for selection are determined by expert opinions, and then AHP is used to determine the weights of the decision criteria and TOPSIS is used to rank the alternatives. According to the result, all of methods provide the systematic approach for group decision making that can help project manager prioritize project and this information can help them provide master plan in project management and can be applied in other companies which tend to decide for project selection problem.

"Reference [6]" created multi criteria decision making, that discussed about important mechanism that provided guideline to decision maker for evaluation of material mobilization and material utilization using TOPSIS and AHP methods.

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Formulation of MCDM problem

A standard feature of a MCDM methodology is the decision matrix as shown in Table1,

$$\begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \dots & C_n \end{array} \\ \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

Table 1. The decision matrix

Where $A_1, A_2, ..., A_m$ are decision alternatives and $C_1, C_2, ..., C_n$ are decision criteria. We assume that all alternatives score with respect to all criteria are known by the decision maker, x_{ij} indicates the performance of alternative A_i with respect to criterion c_j, (for $i = 1,2,\dots,m$, and $j = 1,2,\dots,n$).

The main steps of multiple criteria decision making are the following:

(a) Establishing system evaluation criteria that relate system capabilities to goals,

(b) Developing alternative systems for attaining the goals (generating alternatives),

(c) Evaluating alternatives in terms of criteria (the values of the criterion functions),

(d) Selection of appropriate multi-criteria decision making method,

(e) Accepting one alternative as ''optimal'',

(f) If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization [7].

A parametric study consists of constant term and variable term**,** we add parameter variable λ to each element in the decision matrix by different coefficients, define the parameter range. An illustrative numerical example is given to explain the parametric analysis on the MCDM problem**.** We apply the hybridization between Simple Multi Attribute Rating Technique (SMART) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods to explain the effect of the parameters in the decision matrix.

II. A Brief Overview of SMART and TOPSIS MCDM Methods

A. SMART Method

Simple Multi Attribute Rating Technique (SMART) is a simple method to assess weights for each of the criteria to reflect its relative importance to the decision. The weights are obtained by rank the importance of the changes in the criteria from the worst criteria levels to the best levels. Then 10 points are assigned to the least important criteria, Then, the next-least-important criterion is chosen, more points are assigned to it, and so on, to reflect their relative importance [8], [9].

B. TOPSIS Method

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method, developed by Hwang & Yoon (1981), is one of the MCDM methods. TOPSIS method is based on the concept that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS) and the farthest distance from the Negative Ideal Solution (NIS). The PIS is a set composing all the best values of each criterion, while the NIS is a set composing all the worst values of each criterion [2], [3].

The procedure of TOPSIS can be expressed in a series of steps: [3], [7]

Step1: Calculate the normalized decision matrix.

To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes all the x_{ij} values in the decision matrix $(x_{ij})_{mxn}$ have to be normalized to form the matrix $R = (r_{ij})_{mxn}$. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad , \quad j = 1,2,\dots,n$$

Step 2: Calculate the weighted normalized decision matrix. Calculate the weighted normalized decision matrix: by multiplying the normalized matrix by the weight w_j of the j^{th} criterion. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_j r_{ij} \ , \ i = 1,2,\dots,m \ , \ j = 1,2,\dots,n \ ,.$$

Where w_j is the weight of the j^{th} criterion, and $\sum_{j=1}^{n} w_j = 1$

Step 3: Determine the Positive ideal and negative-ideal solutions.

Ideal alternative: the one which has the best level for all criteria considered. Negative ideal alternative: the one which has the worst criterion values. The Positive ideal solution $A^* = \{v_1^*, \dots, v_n^*\}$, where , $v^* = \{ max\,(v_{ij})$ if $j \epsilon J$, $min\,(v_{ij})$ if $j \epsilon J'$ **}.** The negative-ideal solution $A' = \{v_1' \dots v_n'\}$, where $v' = \{min\,(v_{ij})$ if $j \epsilon J$; $max\,(v_{ij})$ if $j \epsilon J'\}$, where J is associated with benefit criteria (more is better), and J' is associated with cost criteria (less is better).

Step 4: Calculate the separation measures, using the n dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as:

$$D_i^* = [\sum_{j=1}^{n} (v_j^* - v_{ij})^2]^{1/2} \ , \quad i = 1,2,\dots,m$$

Similarly, the separation from the negative ideal solution is given as:

$$D_i' = [\sum_{j=1}^{n} (v_j' - v_{ij})^2]^{1/2} \ , \quad i = 1,2,\dots,m$$

Step 5: Calculate the relative closeness to the ideal solution. The relative closeness of the alternative a_i with respect to A^* is defined as following:

$$C_i^* = D_i' / (D_i^* + D_i'), \quad 0 < C_i^* < 1, \quad i = 1,2,\dots,m$$

If $C_i^* = 1$, then $a_i = A^*$ and if $C_i^* = 0$, then $a_i = A'$. Therefore, the conclusion is that a_i is closer to A^* if the C_i^* is closer to value 1.

Step 6: Rank the preference order.

The best (optimal) alternative can now be decided according to the preference rank order of C_i^*, meaning that the bigger C_i is the better the alternative. Therefore, the best alternative is the one that has the shortest distance to the ideal solution.

III. Parametric analysis

We study the stability of MCDM problems with parameters in the decision matrix. Parametric studies allow us to nominate (choose) parameters for evaluation, refine the parameters until satisfied with the results. A parametric study consists of constant term and variable term. We add parameter variable λ to each element in the decision matrix, Define the parameter range, the values for these parameters are specified by a starting value, and an ending value. We apply the hybridization between SMART and TOPSIS methods to explain the effect of the parameters in the decision matrix. We get the limit of parameter λ in the decision matrix that keeps the ordering of them according to the rank r given before $[T_{i1}, T_{i2}, \dots, T_{im}]$ in the original decision matrix. Consider the following decision matrix $D(\lambda) = (d_{ij}(\lambda))$, as shown in Table 2,

$$D = \begin{pmatrix} d_{11}(\lambda) & d_{12}(\lambda) & \dots & d_{1n}(\lambda) \\ d_{21}(\lambda) & d_{22}(\lambda) & \dots & d_{2n}(\lambda) \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}(\lambda) & d_{m2}(\lambda) & \dots & d_{mn}(\lambda) \end{pmatrix}$$

Table2. Decision matrix by adding parameter

where $i = 1,2,\dots,m,$ $j = 1,2,\dots,n$ which represents m decision alternatives and n decision criteria, $\lambda \in R^+ \cup \{0\}$ is a real parameter such that $\lambda \geq 0$, and assume that $W = (w_1, w_2, \dots, w_n)$ is the weight vector of the criteria such that $\sum_{j=1}^{n} w_j = 1,\ w_j \geq 0\ \ \forall j.$

Assume that for $\lambda = \bar{\lambda}$, $w = \bar{w}$, the TOPSIS method results in a ranking r of the alternatives, then the following stability sets of the first kind can be defined as:

$S_1(r) = \{ w \in R^n \mid r$ is kept as the rank, $\lambda = \bar{\lambda} \}$

$S_2(r) = \{ \lambda \in R^+ \cup \{0\} \mid r$ is keptas the rank, $w = \bar{w}\}$

$S_3(r) = \{ (w, \lambda) \in R^n \times R^+ \cup \{0\} \mid r$ is kept as the rank$\}$

A. Some basic definitions of stability sets of the first kind

1) Definition of $S_1(r)$

In this case w is the only parameter and after computing T_i, $i = 1,2,\dots,m,$ and in order to keep the same rank as before in the form $\{T_{i_1}, T_{i_2}, \dots, T_{i_m}\}$, we get

$$S_1(r) = \{ w \in R^n \mid T_{i_1} \geq T_{i_2} \geq \dots T_{i_m} \}$$

2) Definition of $S_2(r)$

Consider $\propto_{ij}(\lambda)$, for each $j = 1,2,\dots,n$ find the intersection of lines $\propto_{rj}(\lambda)$, $\propto_{sj}(\lambda)$, $\forall r, s = 1,2,\dots,m$, $r \neq s$ to get set of points $\{\lambda_1, \lambda_2, \dots, \lambda_k\}$,such that $\lambda_i \in [0, \infty[$ $i = 1,2,\dots,k$. There points decompose the parametric space $[0, \infty[$ into a set of intervals $\{I_1, I_2, \dots, I_l\}$ where $I_t \subset [0, \infty[$, $t = 1,\dots,l$. In each interval I_t find T_{it} using TOPSIS and then keep the ordering of them according to the rank r given before $[T_{i1}, T_{i2}, \dots, T_{im}]$ and compute $S_{2t}(r)$ as:

$$S_{2t}(r) = \{ \lambda \in I_t \mid T_{i_1 t} \geq T_{i_2 t} \geq \dots T_{i_m t} \}$$

$$S_2(r) = \cup_{t=1}^{l} S_{2t}(r)$$

3) Definition of $S_3(r)$

Same steps as for $S_2(r)$, the only difference in that w is considered as a parameter.

B. Remarks

1- The number of points k on the parametric space $[0, \infty[$ is at most $\frac{n\,m\,(m-1)}{2}$ and therefore the number of interval l is at most $\frac{n\,m\,(m-1)}{2} + 1$

2- The complexity of the algorithms Alg1, Alg2, Alg3, increase against m more than against n.

3- If the parametric space is restricted in the form $[a, b]$, the treatment of the algorithms Alg2, Alg3 will be easier.

4- The efficiency of the algorithms Alg1, Alg2, Alg3 lie in the fact that if we have another rank for the alternatives say, $r = \{J_1, J_2, \dots, J_m\}$, then in the algorithms Alg1, Alg2 for ex. compute $S_{2t}(r_1)$ can be attains in the form $S_{2t}(r_1) = \{ x \in I_t \mid J_{j_1 t} \geq J_{j_2 t} \geq \dots \geq J_{j_m t} \}$ and therefore $S_2(r_1) = \cup_{t=1}^{l} S_{2t}(r_1)$

5- In Alg2, the intersection I_i $i = 1,\dots,l$, and closed at its begging and open at its end.

IV. The algorithms of the stability sets of the first kind as follow:

A. The algorithm of the stability set $S_1(r)$ (ALG1)

Step 1: Apply the TOPSIS method to rank the alternatives

a) Determine the weight of each of the criteria by using SMART method

1- Rank the importance of the changes in the criteria from the worst criterion levels to the best levels.

2- Assign 10 points to the least important criterion.

3- The relative importance of the other criteria are then evaluated by giving them points from 10 upwards.

4- Normalized to get the weights, weights summing is 1.

5- Test of consistency until Consistency Rate reaches to less than 0.1.

Finally we get the suitable weights of criteria w_j .

b) Calculate the normalized decision matrix by $n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$

c) Calculate the weighted normalized decision matrix $v_{ij} = n_{ij} \times w_j$

d) Determine the ideal and negative-ideal solution.

e) Calculate the separation of each alternative from the ideal solution and from the negative ideal solution

f) Calculate the relative closeness to the ideal solution.

g) Rank the preference order, we get the ranking r , $\{ T_{i_1}, T_{i_2}, \dots, T_{i_m} \}$

Step 2: **Determination of** $\boldsymbol{S_1(r)}$

In this case w is the only parameter**,** $w = \{w_1, w_2, \dots, w_n\}$ **,** where w_j is the weight of the j^{th} criterion, and $\sum_{j=1}^{n} w_j = 1.$ $w_j \geq 0 \;\; \forall j.$

a- Applying the TOPSIS method on the weighted normalized matrix **(**with the only parameter w **)** as in step 1.

b- Computing T_i , $i = 1,2, \dots, m$, and in order to keep the same rank as before in (step 1-g) we get the condition on the weights

$$\boldsymbol{S_1(r)} = \{ w \in R^n \mid T_{i_1} \geq T_{i_2} \geq \dots T_{i_m} \}$$

B. The algorithm of the stability set $S_2(r)$ (ALG2)

Step 1: Determine the weight of each of the criteria by using SMART method as in ALG1 (step1-a)

Step 2: Determination of $S_2(r)$

In this case λ is the only parameter in the decision matrix.

a) Apply the TOPSIS method **as** in ALG1 (step1-from b to g) to get set of points$\{ \lambda_1, \lambda_2, \dots, \lambda_k \}$, such that $\lambda_i \in [0, \infty[$ $i = 1,2, \dots, k$.

b) There points decompose the parametric space $[0, \infty[$ into a set of intervals $\{ I_1, I_2, \dots, I_l \}$ where $I_t \subset [0, \infty[$, $t = 1, \dots, l$.

c) We get a set of intervals that keeping the ordering of alternatives according to the rank r given before $[T_{i1}, T_{i2}, \dots, T_{im}]$.

d) Compute $\boldsymbol{S_{2t}(r)}$ as:

$\boldsymbol{S_{2t}(r)} = \{ \lambda \in I_t \mid T_{i_1 t} \geq T_{i_2 t} \geq \dots T_{i_m t} \}$, the intervals closed at its begging and open at its end.

e) Compute $\boldsymbol{S_2(r)} = \cup_{t=1}^{l} \boldsymbol{S_{2t}(r)}$

C. The algorithm of the stability set $S_3(r)$ (ALG3)

In this case λ **and** w are the parameters**,** λ is the parameter in the decision matrix, and

w is the another parameter**,** $w = \{w_1, w_2, \dots, w_n\}$ **,** where w_j is the weight of the j^{th} criterion, and $\sum_{j=1}^{n} w_j = 1.$ $w_j \geq 0 \;\; \forall j.$

Step 1: Determine the weighted normalized matrix, each element is function in two parameters λ and w**.**

Step 2: Determination of $S_3(r)$

a) Following the same steps as in (ALG2 step2) for decomposing the range of λ

b) in order to keep the same rank as before in (ALG1 step1-g) we get the condition on the parameter**s** λ and w**.**

$S_3(r) = \{ (w, \lambda) \in R^n \times R^+ \cup \{0\} \mid r$ is kept as the rank$\}$

V. *Numerical Example*

Let us assume that multi-criteria decision problem as shown in [10]. By using the set of three alternatives: A_1, A_2, A_3, together with the set of four criteria: C_1 , C_2 , C_3 , C_4 , where C_1 , C_2 , C_3 , are benefit criteria and C_4 is cost criterion.

The weights of criteria have been computed by using SMART method. Data was gathered by using scale values of 1-9 as shown in Table 3 and the decision matrix shown in Table 4.

	C_1	C_2	C_3	C_4
C_1	1	2	3	9
C_2	0.5	1	2	3
C_3	1/3	0.5	1	2
C_4	1/9	1/3	0.5	1

Table 3. The comparison matrix of the criteria

Table 4. The decision matrix

A. Applying the hybrid between SMART and TOPSIS methods on the original decision matrix

	C_1	C_2	C_3	C_4
A	10	2	20	1
A	20	10	3	3
A	15	8	7	4

The weight of each of the criteria by SMART method is

	C_1	C_2	C_3	C_4
W	0.25	0.125	0.063	0.562

Then applying the TOPSIS method to get the relative closeness to the ideal solution

$$C_i^* = \begin{matrix} A_1 & A_2 & A_3 \\ [0.2899 & 0.7175 & 0.53\ ] \end{matrix}$$

Finally, the ranking of alternatives is A_2, A_3, A_1.

B. Determination of $S_1(r)$

w is the only parameter, $w = \{w_1, w_2, w_3, w_4\}$

Applying the TOPSIS method on the following weighted normalized matrix

$$= \begin{pmatrix} \frac{10w_1}{\propto_1} & \frac{2w_2}{\propto_2} & \frac{20w_3}{\propto_3} & \frac{w_4}{\propto_4} \\ \frac{20w_1}{\propto_1} & \frac{10w_2}{\propto_2} & \frac{3w_3}{\propto_3} & \frac{3w_4}{\propto_4} \\ \frac{15w_1}{\propto_1} & \frac{8w_2}{\propto_2} & \frac{7w_3}{\propto_3} & \frac{4w_4}{\propto_4} \end{pmatrix}$$

$\propto_1 = 26.9258$, $\propto_2 = 12.9615$, $\propto_3 = 21.4009$, $\propto_4 = 5.0990$ then ,to get the same ranking $r\,(A_2, A_3, A_1)$, we must have

$C_2^* > C_3^* > C_1^*$ $\quad 0.138\ w_1^2 + 0.381\ w_2^2 > 0.631\ w_3^2 + 0.115\ w_4^2$

i.e.

$S_1(r) = \{w \in R^4 \mid 0.138\ w_1^2 + 0.381\ w_2^2 \geq 0.631\ w_3^2 + 0.115\ w_4^2\ ,\ w_1, w_2, w_3, w_4 \geq 0\ ,\ w_1 + w_2 + w_3 + w_4 = 1\}$

C. Determination of $S_2(r)$

Applying the TOPSIS method on decision matrix after adding the parameter λ, and the weights of criteria are (0.562 0.25 0.125 0.063)

Consider the following decision matrix is $D(\lambda) = (d_{ij}(\lambda))$

$$D(\lambda) = \begin{pmatrix} 3\lambda + 10 & 5\lambda + 2 & \lambda + 20 & 5\lambda + 1 \\ \lambda + 20 & 2\lambda + 10 & 4\lambda + 3 & 4\lambda + 3 \\ 2\lambda + 15 & 3\lambda + 8 & 2\lambda + 7 & 3\lambda + 4 \end{pmatrix}$$

Consider the following weighted normalized matrix is

$$V(\lambda) = \begin{pmatrix} \frac{0.562(3\lambda+10)}{\propto_1} & \frac{0.25(5\lambda+2)}{\propto_2} & \frac{0.125(\lambda+20)}{\propto_3} & \frac{0.063(1+5\lambda)}{\propto_4} \\ \frac{0.562(\lambda+20)}{\propto_1} & \frac{0.25(2\lambda+10)}{\propto_2} & \frac{0.125(4\lambda+3)}{\propto_3} & \frac{0.063(4\lambda+3)}{\propto_4} \\ \frac{0.562(2\lambda+15)}{\propto_1} & \frac{0.25(3\lambda+8)}{\propto_2} & \frac{0.125(2\lambda+7)}{\propto_3} & \frac{0.063(4+3\lambda)}{\propto_4} \end{pmatrix}$$

Where, $\propto_1(\lambda) = \sqrt{14\lambda^2 + 160\lambda + 725}$

$\propto_2(\lambda) = \sqrt{38\lambda^2 + 108\lambda + 168}$

$\propto_3(\lambda) = \sqrt{21\lambda^2 + 92\lambda + 458}$

$\propto_4(\lambda) = \sqrt{50\lambda^2 + 58\lambda + 26}$

In order to determine the maximum and the minimum over each column the range of λ must be divided as follows:

1- $0 \leq \lambda < 1$, $1 \leq \lambda < 1.5$, $1.5 \leq \lambda < 2$, $2 \leq \lambda < 2.5$, and $2.5 \leq \lambda < 2.9$, the rank r is (A_2, A_3, A_1).

2- $2.9 \leq \lambda < 3$, the rank r is (A_2, A_1, A_3).

3- $3 \leq \lambda < 3.75$, the rank r is (A_2, A_1, A_3).

4- $3.75 \leq \lambda < 4.508$, the rank r is (A_1, A_2, A_3).

5- $4.508 \leq \lambda < 5$, the rank r is (A_1, A_3, A_2).

6- $5 \leq \lambda < 5.67$, $5.67 \leq \lambda < 13$, and $13 \leq \lambda$, the rank r is (A_1, A_3, A_2).

We get the same ranking $r\,(A_2, A_3, A_1)$, for the intervals $0 \leq \lambda < 1$, $1 \leq \lambda < 1.5$, $1.5 \leq \lambda < 2$, $2 \leq \lambda < 2.5$ and $2.5 \leq \lambda < 2.9$.

Similar treatment must be alone for the other ranges.

D. Determination of $S_3(r)$

In this case λ **and** w are the parameters**.** Consider the following weighted normalized matrix is

$$= \begin{pmatrix} \frac{w_1(10+3\lambda)}{\propto_1} & \frac{w_2(2+5\lambda)}{\propto_2} & \frac{w_3(20+\lambda)}{\propto_3} & \frac{w_4(1+5\lambda)}{\propto_4} \\ \frac{w_1(20+\lambda)}{\propto_1} & \frac{w_2(10+2\lambda)}{\propto_2} & \frac{w_3(3+4\lambda)}{\propto_3} & \frac{w_4(3+4\lambda)}{\propto_4} \\ \frac{w_1(15+2\lambda)}{\propto_1} & \frac{w_2(8+3\lambda)}{\propto_2} & \frac{w_3(7+2\lambda)}{\propto_3} & \frac{w_4(4+3\lambda)}{\propto_4} \end{pmatrix}$$

Following the same steps as in (5.3) for decomposing the range of λ and applying the TOPSIS method, we find that in order to keep r for the ranges $0 \leq \lambda < 1$, $1 \leq \lambda < 1.5$, $1.5 \leq \lambda < 2$, $2 \leq \lambda < 2.5$ and $2.5 \leq \lambda < 2.9$, we must have the condition

$$\lambda^2\left(3\frac{w_1^2}{\propto_1^2} + 5\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) - 2\lambda\left(15\frac{w_1^2}{\propto_1^2} + 12\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) + \left(75\frac{w_1^2}{\propto_1^2} + 28\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) > \frac{w_3^2}{\propto_3^2}(2\lambda - 4)^2$$

i.e.

$S_3(r) = \{(w, \lambda) \in R^4 \times R^+ \cup \{0\} \mid \lambda^2\left(3\frac{w_1^2}{\propto_1^2} + 5\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) - 2\lambda\left(15\frac{w_1^2}{\propto_1^2} + 12\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) + \left(75\frac{w_1^2}{\propto_1^2} + 28\frac{w_2^2}{\propto_2^2} + \frac{w_4^2}{\propto_4^2}\right) > \frac{w_3^2}{\propto_3^2}(2\lambda - 4)^2,\ \ w_1, w_2, w_3, w_4 \geq 0\ ,\ w_1 + w_2 + w_3 + w_4 = 1$.

VI. CONCLUSION

Parametric studies allow us to choose parameters for evaluation, refine the parameters until satisfied with the results. These describe sets of input values, define the parameter range. The values for these parameters are specified by a starting value, and an ending value. The hybridization between Simple Multi Attribute Rating Technique and Technique for Order Preference by Similarity to Ideal Solution (SMART and TOPSIS) methods has been applied to explain the effect of the parameters in the decision matrix on the numerical example. We add parameter variable λ to each element in the original decision matrix, to determine which variables have the greatest effect on model output The limits of parameter in the decision matrix that satisfied the same ranking solution (A_2, A_3, A_1) stability sets of the first kind $S_1(r)$, $S_2(r)$, $S_3(r)$

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