# Effects of Axially Symmetric Stenosis on the Blood Flow in an Artery Having Mild Stenosis

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***Abstract*** — *Blood flow is responsible for nutrient and waste transport within the closed loop i.e. cardiovascular networks. A common problem of the cardiovascular network is the narrowing of arteries due to the development of atherosclerotic plagues or other types of abnormal tissue development. As this growth project into the lumen of the artery, the flow is distributed and develops a potential coupling between the growth of the stenosis and the blood flow through the artery. It is noted that very small growths leading to mild stenotic obstructions, may be important in triggering biological mechanism such as intimal cell proliferation or changes in vessel caliber. An analysis of the effect of an axially symmetric, time-dependent growth into the lumen of a tube of constant cross section, through which a Newtonian fluid is steadily flowing, is made through this work. This chapter is based on a simplified model in which the connective acceleration terms in the Nervier-Stokes equations are neglected. Finally the governing equations are then solved numerically and obtain the expression for pressure gradient and impedance ratio with respect to stenosis height.*

**Keywords—***Stenosis height, pressure gradient, impedance ratio.*

## I. Introduction

Arteries are blood vessels that carry oxygen from heart to the rest of the body therefore cardiovascular network is extremely important for sustaining life. The cardiovascular network is the higher pressure portion of the circulatory system and the pressure variation (systolic and diastolic) within the artery produces the pulse which is observable in any artery, and reflects heart activity. Stenosis in cardiovascular network is a common occurrence and hemodynamics factor play a very important role in the formation and proliferation of this disease. It is well known that at various locations in the cardiovascular network, stenosis may develop due to abnormal intravascular growths.

Several models have been developed to predict the pressure drop, caused by a given stenotic area and it state that the minimum lumen areas created in stenosed tube is about 65% and 90%, including a model without stenosis, respectively. Ischemic heart disease, which results from high grade stenosis, is the single most common cause of death all over the world. Approximately 35 % of all deaths are resulted by this cause. High grade stenosis increases flow resistance in arteries, which forces the body to raise the blood pressure in order to maintain the necessary blood supply. Both high pressure and narrowing vessels cause high flow velocity, high shear stress and low or even negative pressure, at the throat of the stenosis.

These may be related to thrombus formation, atherosclerosis growth and plaque cap rupture, leads directly to stroke and heart attack. The exact mechanism of this complicated process is still not well understood. A more comprehensive study in this physiological process is of great importance for diagnosis, prevention and treatment of stenosis related diseases. A considerable number of experimental and numerical researches have been conducted to study the flow dynamics and stresses in collapsible elastic tube (Tang, et al (2003)).

There have been a number of studies for the flow of blood in stenosed artery like as Young, (1968), Shukla et al. (1980), Perkkio and Keskinen (1981) etc, using various types of mathematical models. The effects of viscosity concentration dependence and of the concentration profile of blood flow through a vessel with stenosis have been done, Perkkio and Keskinen (1983). A steady of axisymmetric flow in a constricted rigid tube in which a shear-thinning fluid modeling the deformation dependent viscosity of the blood is proposed and the flow pattern with the distributions of pressure and shear stress at the wall are computed (Pontrelli (2001)). Further Most et al. (2003) showed the effect of a reduction in blood viscosity on maximal myocardial oxygen delivery distal to a moderate coronary stenosis.

The effects of pulsatality, stenosis and non-Newtonian behavior of blood have been simultaneously studied by Mandal et. al. (2007) and Sankar et al (2009), where results for the rate of flow, the resistive impedance have been obtained. Padmanabhan et al (2006) presented closed-form solutions for governing equations of the pulsatile flow of blood through models of mild axisymmetric arterial stenosis, taking into account the effect of arterial distencibility. Some experimental studies have also been conducted in last few decades with and without bypass, Rodkiewicz et al. (1988) and Qiao et al. (2007). In particular Rodkiewicz et al. (1988) showed analytically and experimentally that within the scope of a surgery, the effects of variations in the position of the transplant- aorta contact point in the transplant length and in the transplant curvature are relatively insignificant regarding mean flow resistance. They concluded that it is not important how the transplant will be situated and that the space convenience should be surgical determining factor. It has been shown that the rate of blood flow to the kidney may be significantly curtailed if the selected transplant diameter is too small.

A numerical study of non-Newtonian blood flow in stenosed coronary artery bypass, have also done by various researcher, in which they investigated the effects of it by using different bypass angles on the flow pattern. Their result shows that the proper choice of diameter of the graft might improve the balance of inflow and outflow in the coronary artery. The influence of graft diameter on the hemodynamics of femoral bypass graft, the pulsatile blood flows in three bypass models with different graft diameters were studied by Qiao et al. (2007). Further as far as the effects of micro-polar parameter on any stenotic artery is concern, Kumar et al (2009) designed a mathematical model to observe for the effect of micro-polar parameter in a stenotic artery

Kumar et al. (2010) also worked in this area be taking a mathematical model, solved numerically and observed the effects of multi-stenosis and post stenosis dilations in small artery. Here a fully developed one-dimensional non-Newtonian fluid known as, Herschel-Bulkley fluid through a single vessel is made. A recent work in this direction is also done by Kumar et al. (2011) in which they prepare a mathematical model for blood flow and cross-sectional area of an artery, because the cross sectional area plays an important role in order for the blood to flow smoothly through the blood vessels. A system of non-linear partial differential equations for blood flow and cross sectional area of the artery was discussed and observed the various effects. Recently Kumar and Diwakar (2012) worked on a biomagentic fluid dynamic model for the MHD Couette flow between two infinite horizontal parallel porous plates and give that the main flow component decreases with the increase of Hartmann number and the velocity decreases with the increases of the injection suction parameter. Again Biswas and Paul (2013) are developed a mathematical model with steady flow of blood (Newtonian fluid) through an inclined tapered constricted artery with an axial slip in velocity at the tapered vessel wall. They investigate the combined influence of tapering angle, slip at wall, inclined artery, Newtonian nature and non-symmetrical stenosis, for blood flow through constricted arteries. And they obtained analytic solutions in closed form, and there analytic expressions for impedance to flow and wall shear stress are obtained and their variations with several flow parameters, are presented graphically. Also Alana et al. (2013) discussed about the two-layered blood flow through a composite stenosis in the presence of a magnetic field and they investigated by examining the effects that an external, uniform, transverse magnetic field has on its velocity, flow rate and three flow characteristics (impedance, wall shear stress and shear stress at the stenosis throat). This two-layered model, consisting of a cell -free peripheral plasma layer and a particle-fluid suspension in the core region, was used to represent blood flowing through a composite stenosis in an artery. The effects of introducing the magnetic field, as well as increasing its intensity were examined. The fluid's velocity and flow rate were reduced when the magnetic field was introduced as well as when its intensity was increased. All three of the flow characteristics increased when the magnetic field was introduced and as its intensity increased. These effects make it more difficult for the blood to flow and may even lead to wall vessel damage and plaque rupture. When hematocrit, stenosis height and tube length were varied the same trends were observed in the presence of the external magnetic field as they observed in the absence of the magnetic field. The two-fluid model flow characteristics were lower than those of the one-fluid model. This knowledge can aid in the improvement of existing diagnostic tools, especially those used in diagnosing cardiovascular diseases. Again Bhatnagar and Shrivastav (2014) worked on analysis of MHD flow of blood through a multiple stenosed artery in the presence of slip velocity, they given a mathematical model that the flow of blood in a multiple stenosed artery employing velocity slip condition under the externally applied transverse magnetic field. Blood is modeled as Herschel-Bulkley fluid to represent the non-Newtonian character of blood in small blood vessels.

The expressions for wall shear stress, volumetric flow rate, axial velocity, and core velocity have been derived analytically and graphically. These expressions reveal considerable alterations in flow characteristics due to slip velocity and stenosis. The magnetic field perpendicular to the flow of blood is incorporated which significantly controls the flow patterns. The study provides an insight into the effects of magnetic field and slip velocity on flow rate of blood, wall shear stress, axial and core velocities of blood. Recently Hazarika and Sharma (2015) is discussed about the oscillatory flow of blood through a stenosed artery. They investigated the results that work in presence of magnetic field. The laminar, incompressible, fully developed, Newtonian flow of blood in an artery having mild stenosis is numerically studied under the action of transverse magnetic field and the numerical results are obtained for the oscillatory flow of blood using Shoot-ing method. It is assumed that the surface roughness is cosine-shaped and the maximum height of the roughness is very small compared with the radius of the unconstricted tube. Numerical solutions are obtained for the flow velocity, flow rate, wall shear stress and impedance. The results are shown graphically. It is observed that fluid velocity decreases for the increase in the val-ues of the Hartmann number as well as the frequency parameter while increases for the increasing in the values of the Reynolds number. Volumetric flow rate and wall shear stress increase with increase in the values of the Reynolds number and decrease with the increasing values of the Hartmann number. The effect of the frequency parameter on impedance to the flow is discussed graphically and observed that it enhance the impedance. Again R bali et al. (2016) discussed about a mathematical model for blood flow through an axially symmetric blood capillary with peripheral layer and slip at the wall. They longitudinal transport of nanoparticles in blood vessels has been analyzed with blood as a power law fluid in a central core region of suspension of all the erythrocytes and a Newtonian fluid in a peripheral layer of plasma. They expressed for velocity profile, flow rate, mean velocity and concentration of the solute have been obtained and results have been discussed through graphs.

Therefore on the basis of above information we are going to find the effects of stenosis height on the pressure gradient and impedance ratio for the blood flow in an artery having mild stenosis.

## II. MATHEMATICAL MODEL

The blood flow in healthy arteries is laminar in nature; but the presence of abnormal flow conditions can promote the development of cardiovascular disease such as arthrosclerosis and thrombus formation. In fact healthy arteries are flexible and elastic, but over the time with age, arteries become non-elastic and its lumen get constricted due to calcium deposits.

If $R\left(=R(z)\right)$ is the radius of stenotic artery, $R_0$ is the radius of non-stenotic artery, □ is the height of stenosis growth, $2z_0$ is the length of the stenosis and $R_e$ is the Reynolds number of fluid flow, then consider the time dependent, steady flow of a Newtonian fluid past an axially-symmetric stenosis into the lumen of a tube of constant cross section, whose surface is given by (Young, 1968)

$$\frac{R}{R_0}=1-\frac{\delta}{2R_0}\left(1+\cos\frac{\pi z}{z_0}\right) \qquad \text{...(i)}$$

with the following conditions:

$$\frac{\delta}{R_0}<<1,\ \frac{R_0}{z_0}\approx O(T),\ R_e\frac{\delta}{z_0}<<1. \qquad \text{...(ii)}$$

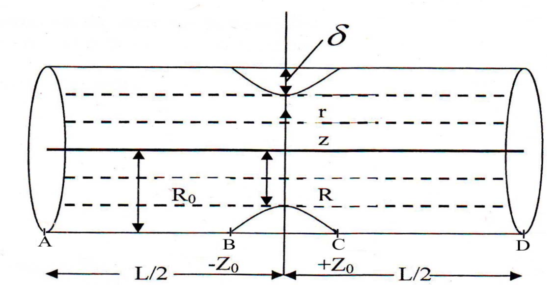


Fig. 1 The geometry of the model

By carrying out an order of magnitude analysis on these basic equations of motion in cylindrical polar coordinates, it can be shown that the radial velocity can be neglected in relation to axial velocity *v* which is determined by:

$$0=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right) \qquad \text{...(iii)}$$

$$0=-\frac{\partial p}{\partial r} \qquad \text{...(iv)}$$

Now if we assume that $P\left(=P(z)\right)=-\frac{\partial p}{\partial z}$, then the above equation may be expressed as:

$$-P=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \qquad \text{...(v)}$$

The no-slip condition on the stenosis surface gives:

$$\begin{aligned} v&=0,\ at\ r=R, \quad -z_0\le z\le z_0 \\ v&=0,\ at\ r=R_0, \quad |z|\ge z_0 \end{aligned} \qquad \text{...(vi)}$$

## III *SOLUTIONS OF THE PROBLEM*

Blood flow and pressure are generally unsteady in nature, and the cyclic nature of the heart pump creates pulsatile conditions in all arteries. Arterial pressure varies between the peak pressure during heart contraction, called the systolic pressure and the minimum or diastolic pressure between contractions, when the heart expands and refills. This pressure variation within the artery produces the pulse which is observable in any artery, and reflects heat activity. Therefore, the pressure effects are very much important to observe during the problem of mild stenosis.

For a mild stenosis, the main difference from the usual Poiseuille flow is that the pressure gradient and axial velocity are functions of $z$ also. Solving equation (v) and using the boundary conditions (vi) we get,

$$v=-\frac{P}{4\mu}\left[r^2-R^2\right] \qquad \text{...(vii)}$$

If Q is the flux through the tube, then

$$Q=\int_0^R v.2\pi r dr=\frac{\pi P}{8\mu}R^4 \qquad \text{...(viii)}$$

The pressure gradient can be obtained from equation (viii), as follows:

$$P=\frac{8\mu Q}{\pi R^4} \qquad \text{...(ix)}$$

Now, the pressure gradient without the stenosis region, where $R=R_0$

$$P=\frac{8\mu Q}{\pi R_0^4} \qquad \text{...(x)}$$

The pressure drop across A to B (fig. 8(a)) is given by,

$$\Delta p_{AB}=\int_{-L/2}^{-z_0}\frac{8\mu Q}{\pi R_0^4}dz\ \frac{8\mu Q}{\pi R_0^4}\left(-z_0+L/2\right) \qquad \text{...(xi)}$$

while the pressure drop across B to C is,

$$\Delta p_{BC}=\int_{-z_0}^{z_0}\frac{8\mu Q}{\pi R^4}dz=\frac{16\mu Q}{\pi}\int_0^{z_0}\frac{1}{R^4}dz \qquad \text{...(xii)}$$

Now integrating the above equation along with the equation (i), we get:

$$\Delta p_{BC}=\frac{16\mu Q}{\pi R_0^{\ 4}}f(\alpha) \qquad \text{...(xiii)}$$

Where

$$f(\alpha)=\left(1-\frac{\alpha}{2}\right)\left(1-\alpha+\frac{5}{8}\alpha^2\right)(1-\alpha)^{-7/2} \quad \text{...(xiv)}$$

when $\alpha = \delta / R_0$

Similarly the pressure drop across C to D is:

$$\Delta p_{CD} = \int_{z_0}^{L/2} \frac{8\mu Q}{\pi R_0^4} dz = \frac{8\mu Q}{\pi R_0^4}(L/2 - z_0) \quad \text{...(xv)}$$

Now if there is no stenosis in the artery i.e. δ=0 or in other words $f\left(\frac{\delta}{R_0}\right) = 1$, then the impedance ratio $k$ may be expressed as:

$$k = \left(\frac{\Delta p}{(\Delta p)_P}\right) = \frac{8\mu Q(L-2z_0)+16\mu Q z_0 f(\alpha)}{8\mu Q(L-2z_0)+16\mu Q z_0} \quad \text{...(xvi)}$$

or it may be written as:

$$k = 1-\beta+\alpha\,\beta\, f(\alpha) \quad \text{...(xvii)}$$

where $\beta = 2z_0 / L$

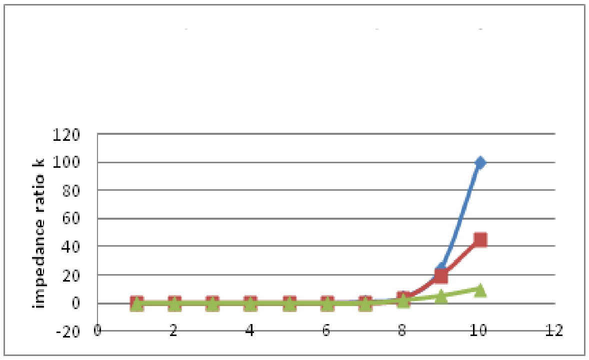


Fig. 2 Impedance ratio with size of stenosis $\delta / R_0$

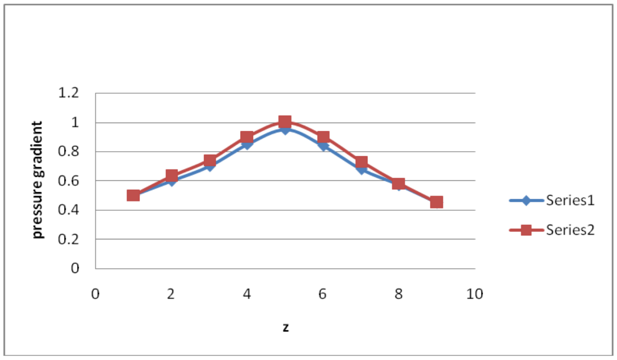


Fig. 3 Pressure gradient for different values of z and for different viscosity

## III. Conclusions

The curve labeled $\beta = 1$ shows directly the influence of the stenosis on the resistance. It is noted that for $\alpha = 0.2$, the impedance has increased. Here it is noted that, if the resistance over a long segment of artery is considered, the effect of the stenosis is very small until a certain value of $\alpha$ (height of stenosis to radius ratio) is exceeded. Beyond this critical value of $\alpha$, the presence of the stenosis rapidly becomes significant. It should be emphasized that for the mild stenosis under consideration the change in the actual pressure at a point in the artery due to the stenosis will still be small in comparison to the mean arterial pressure. The curve labeled $\beta = 0.1$ highlights the effects of the stenosis is very small until a certain value of $\alpha$ is exceeded. Here it is also observed that for the mild stenosis, beyond this critical value of $\alpha$, the pressure of the stenosis becomes significant. Now as far as the pressure gradient is concerned, it is minimum in the beginning and at the end while it is maximum in the stenotic region.

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