# Primitive Idempotents of Abelian Codes of Length $4p^nq^m$

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**Abstract**

Let p, q and $l$ be distinct odd primes ($l$ is of the type 4k+1and p is of the type 4k+3) and $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m} - 1)$. If $o(l)_{2p^n} = \frac{\phi(2p^n)}{2}$, (n ≥ 1) and $o(l)_{2q^m} = \phi(2q^m), (m \geq 1)$ with gcd ($\phi(2p^n)/2$, $\phi(2q^m)$) =1, then the explicit expressions for the complete set of 8mn+4n+8m+4 primitive idempotents in the ring $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m} - 1)$ are obtained.

**Keywords:** primitive idempotents; primitive root; cyclotomic cosets.

MSC: 94B05, 20C05, 16S34, 12E20.

## 1. Introduction

Let F = GF($l$) be a field of odd prime order $l$.Let $\eta \geq 1$ be an integer with gcd($l,\eta$)=1.Let $R_\eta = GF(l)[x]/(x^\eta - 1)$ .The minimal cyclic codes of length $\eta$ over GF($l$) are the ideals of the ring $R_\eta$ generated by the primitive idempotents.For $\eta = 2$, 4, $p^n$, $2p^n$, p an odd prime and $l$ *is* primitive root mod($\eta$) the primitive idempotent in $R_\eta$ have been obtained by Arora and Pruthi [1,2] .When $m = p^nq$ where p, q are distinct odd primes and $l$ is a primitive root mod $p^n$ and q both with gcd ($\phi(p^n)/2$, $\phi(q)/2$) = 1, the primitive idempotent in $R_\eta$ have been obtained by, G.K.Bakshi and Madhu Raka [4]. When $\eta = 2p^nq^m$ ,where p,q are distinct odd primes and $o(l)_{2p^n} = \frac{\phi(2p^n)}{2}$ , $o(l)_{q^m} = \frac{\phi(q^m)}{2}$, gcd $(\frac{\phi(2p^n)}{2}, \frac{\phi(q^m)}{2})$=1.Then the complete set of 8mn+4n+4m+2 Cyclotomic Cosets modulo $2p^nq^m$ are obtained by, Ranjeet Singh. In this paper , we consider the case when $\eta = 4p^nq^m$ where p, q are distinct odd primes $o(l)_{2p^n} = \frac{\phi(2p^n)}{2}$, (n ≥ 1) and $o(l)_{2q^m} = \phi(2q^m), (m \geq 1)$ with gcd ($\phi(2p^n)/2, \phi(2q^m)$)=1.We obtain explicit expressions for all the 8mn+4n+8m+4 primitive idempotents in $R_{4p^nq^m}$ (see theorem 2.3).

## 2. Primitive Idempotents in $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m} - 1)$

2.1. For $0 \leq s \leq \eta - 1$ , let $C_s = \{s, sl, sl^2, ..., sl^{t_s - 1}\}$, where $t_s$ is the least positive integer such that $sl^{t_s} \equiv s$ (mod $\eta$) be the cyclotomic coset containing s, if $\alpha$ denotes a primitive $\eta th$ root of unity in some extension field of GF($l$) then the polynomial $M^s(x) = \prod_{i \in C_s}(x - \alpha^i)$ is the minimal polynomial of $\alpha^s$ over

GF($l$).Let $M_s$ be the minimal ideal in $R_\eta$ generated by $\frac{x^\eta - 1}{M^s(x)}$ and $\theta_s$ be the primitive idempotent of $M_s$ then we know by (Theorem1, [4]) the primitive idempotent $\theta_s$ corresponding to the cyclotomic coset $C_s$ containing s in $R_{4p^nq^m}$ is given by $\theta_s = \sum_{i=0}^{4p^nq^m-1} \varepsilon_i x^i$, where $\varepsilon_i = \frac{1}{4p^nq^m} \sum_{j \in C_s} \alpha^{-ij}$ $\forall i \geq 0$.Thus to describe $\theta_s$ it becomes necessary to compute $\varepsilon_i$ .To compute $\varepsilon_i$ numerically, we consider the case when $-C_1 = C_3$ and we get that

$$\varepsilon_i = \frac{1}{4p^nq^m} \sum_{j \in C_s} \alpha^{-ij} = \frac{1}{4p^nq^m} \sum_{j \in C_{3s}} \alpha^{ij} \quad \forall i \geq 0 .$$

**Lemma2.2.** Let $p, q, l$ be distinct odd primes ($l$ is of the type 4k+1 and p is of the type 4k+3) and $n \geq 1$, $m \geq 1$ are integers, $o(l)_{2p^n} = \frac{\phi\ 2p^n}{2}$, $o(l)_{2q^m} = \phi\ 2q^m$ and $\gcd\left(\frac{\phi(2p^n)}{2}, \phi(2q^m)\right)$=1.Then

$0(l)_{4p^{n-j}q^{m-k}} = \frac{\phi(4p^{n-j}q^{m-k})}{4}$ , for all j, k, $0 \leq j \leq n-1$, $0 \leq k \leq m-1$.

**Theorem2.3.** The 8mn+4n+8m+4 primitive idempotents corresponding to cyclotomic cosets

$C_0, C_{p^nq^m}, C_{2p^nq^m}, C_{3p^nq^m}, C_{p^nq^j}, C_{2p^nq^j}, C_{3p^nq^j}, C_{4p^nq^j}, C_{p^iq^m}, C_{2p^iq^m}, C_{3p^iq^m}, C_{4p^iq^m}, C_{ap^iq^m},$
$C_{2ap^iq^m}, C_{3ap^iq^m}, C_{4ap^iq^m}, C_{p^iq^j}, C_{2p^iq^j}, C_{3p^iq^j}, C_{4p^iq^j}, C_{ap^iq^j}, C_{2ap^iq^j}, C_{3ap^iq^j}, C_{4ap^iq^j}$
$0 \leq j \leq n-1, 0 \leq k \leq m-1$ in $R_{4p^nq^m}$ are

(i) $\theta_0(x) = \frac{1}{4p^nq^m}(1 + x + x^2 + \ldots + x^{4p^nq^m-1})$.

(ii)
$$\theta_{p^nq^m}(x) = \frac{1}{4p^nq^m}\{ \sum_{(i,r)=(0,0)}^{(n,m)} \sigma_{4(i,r)}(x) + \sigma_{4a(i,r)}(x) - \sigma_{2(i,r)}(x) - \sigma_{2a(i,r)}(x) +$$
$$\&\sigma_{3(i,r)}(x) + \sigma_{3a(i,r)}(x) - \sigma_{(i,r)}(x) - \sigma_{a(i,r)}(x))\}$$

(iii) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&\sigma_{3(i,r)}(x)$ by $-\sigma_{3(i,r)}(x)$ and $-\&\sigma_{(i,r)}(x)$ by $-\sigma_{(i,r)}(x)$ and we get the required expression for $\theta_{3p^nq^m}(x)$.

(iv) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&\sigma_{3(i,r)}(x)$ by $-\&\sigma_{3(i,r)}(x)$ and $-\&\sigma_{(i,r)}(x)$ by $\&\sigma_{(i,r)}(x)$ and we get the required expression for $\theta_{3p^nq^m}(x)$.

(v) For $0 \leq k \leq m-1$,

$$\theta_{p^nq^k}(x) = \frac{\phi(q^{m-k})}{4p^nq^m}\{ \sum_{i,r\ =\ 0,m-k}^{n-1,m-1} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + \&\sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x))]$$

$$+ \sum_{i,r\ =\ 0,m-k}^{n-1,m-1} [\sigma_{4a(i,r)}(x) - \sigma_{2a(i,r)}(x) + \&\sigma_{3a(i,r)}(x) - \sigma_{a(i,r)}(x))]$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+\sum_{(i,r)=(0,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+\sum_{(i,r)=(0,m)}^{(n-1,m)}[\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+\&\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))]+$$

$$(\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x))+\&\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))\}$$

(vi) Similarly (v), Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&_{3(i,r)}(x)$ by $-\sigma_{3(i,r)}(x)$ and $-\&_{(i,r)}(x)$ by $-\sigma_{(i,r)}(x)$ and we get the required expression for $\theta_{2p^n q^k}(x)$.

(vii) Similarly (v) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&_{3(i,r)}(x)$ by $-\&_{3(i,r)}(x)$ and $-\&_{(i,r)}(x)$ by $\&_{(i,r)}(x)$ and we get the required expression for $\theta_{3p^n q^k}(x)$.

(viii) Similarly (v) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&_{3(i,r)}(x)$ by $\sigma_{3(i,r)}(x)$ and $-\&_{(i,r)}(x)$ by $\sigma_{(i,r)}(x)$ and we get the required expression for $\theta_{4p^n q^k}(x)$.

(ix) For $0\le j\le n-1$

$$\theta_{p^j q^m}(x)=\frac{}{4p^{j+1}q^m}\{\sum_{r=0}^{m-1}[\eta_0(\sigma_{(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x))+\xi_1(\sigma_{2(n-j-i,r)}(x)+\sigma_{4(n-j-i,r)}(x))]$$

$$\sum_{r=0}^{m-1}[\eta_1(\sigma_{a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x))+\xi_0(\sigma_{2a(n-j-i,r)}(x)+\sigma_{4a(n-j-i,r)}(x))]\}$$

$$+\frac{\phi(p^{n-j})}{8p^n q^m}\{\sum_{i,r\ =\ n-j,0}^{n-1,m-1}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+\sum_{i,r\ =\ n-j,0}^{n-1,m-1}[\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+\&\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))]$$

$$+\sum_{r=0}^{m-1}[\sigma_{4(n,r)}(x)-\sigma_{2(n,r)}(x)+\&\sigma_{3(n,r)}(x)-\sigma_{(n,r)}(x))]$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}[\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+\&\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))]$$

$$+(\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+\&\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))\}$$

(x) Similarly (ix), Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&\sigma_{3(i,r)}(x)$ by $-\sigma_{3(i,r)}(x)$ and $-\&\sigma_{(i,r)}(x)$ by $-\sigma_{(i,r)}(x)$ and $\xi_0 = \xi_1, \eta_1 = \xi_1, \eta_0 = \xi_0, \xi_1 = \xi_0, \eta_1 = \xi_0$, we get the required expression for $\theta_{2p^jq^m}(x)$.

(xi) Similarly (ix) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&\sigma_{3(i,r)}(x)$ by $-\&\sigma_{3(i,r)}(x)$ and $-\&\sigma_{(i,r)}(x)$ by $\&\sigma_{(i,r)}(x)$ and $\xi_0 = \xi_1, \xi_1 = \xi_0$, we get the required expression for $\theta_{3p^jq^m}(x)$.

(xii) Similarly (ix) Replacing $-\sigma_{2(i,r)}(x)$ by $\sigma_{2(i,r)}(x)$, $\&\sigma_{3(i,r)}(x)$ by $\sigma_{3(i,r)}(x)$ and $-\&\sigma_{(i,r)}(x)$ by $\sigma_{(i,r)}(x)$ and $\xi_0 = \xi_1, \eta_1 = \xi_1, \eta_0 = \xi_0, \xi_1 = \xi_0, \eta_1 = \xi_1$ we get the required expression for $\theta_{4p^jq^m}(x)$.

(xiii) Similarly (ix) Replacing $\eta_0 = \eta_1, \xi_1 = \xi_0$, we get the required expression for $\theta_{ap^jq^m}(x)$.

(xiv) Similarly (x) Replacing $\xi_0 = \xi_1, \xi_1 = \xi_0$, we get the required expression for $\theta_{2ap^jq^m}(x)$.

(xv) Similarly (xi) Replacing $\eta_0 = \eta_1, \xi_1 = \xi_0$, $\xi_0 = \xi_1, \eta_1 = \eta_0 = \xi_0, \xi_1 = \xi_0$, we get the required expression for $\theta_{3ap^jq^m}(x)$.

(xvi) Similarly (xii) Replacing $\xi_0 = \xi_1, \xi_1 = \xi_0$, we get the required expression for $\theta_{4ap^jq^m}(x)$.

(xvii) For $0 \le j \le n-1,\ 0 \le k \le m-1$

$$\theta_{p^jq^k}(x) = \frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{i,r=0,0}^{(n-j-1,m-k-1)}[C_{(i+j,r+k)}\sigma_{(i,r)}(x) + B^*_{(i+j,r+k)}\sigma_{2(i,r)}(x) + A_{(i+j,r+k)}\sigma_{3(i,r)}(x) + D^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{i,r=0,0}^{(n-j-1,m-k-1)}[D_{(i+j,r+k)}\sigma_{a(i,r)}(x) + A^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x) + B_{(i+j,r+k)}\sigma_{3a(i,r)}(x) + C^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{i,r=n-j,m-k-1}^{n-1,m-k-1}(\sigma_{(i,m-k-1)}(x) + \sigma_{2(i,m-k-1)}(x) + \sigma_{3(i,m-k-1)}(x) + \sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{i,r=n-j,m-k-1}^{n-1,m-k-1}(\sigma_{a(i,m-k-1)}(x) + \sigma_{2a(i,m-k-1)}(x) + \sigma_{3a(i,m-k-1)}(x) + \sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{i,r=n-j-1,m-k}^{n-j-1,m-1}[\sigma_{(n-j-1,r)}(x) + \sigma_{2(n-j-1,r)}(x) + \sigma_{3(n-j-1,r)}(x) + \sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{i,r=n-j-1,m-k}^{n-j-1,m-1}[\sigma_{a(n-j-1,r)}(x) + \sigma_{2a(n-j-1,r)}(x) + \sigma_{3a(n-j-1,r)}(x) + \sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{i,r=n-j,m-k}^{n-1,m-1}(\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + \&\sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)))$$

$$+\sum_{i,r\,=\,n-j,m-k}^{n-1,m-1}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+\&\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x)))$$

$$+\sum_{i,r\,=\,n,m-k}^{n,m-1}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{i,r\,=\,n-j,m}^{n-1,m}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+\&\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{i,r\,=\,n-j,m}^{n-1,m}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+\&\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x)))$$

$$+\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+\&\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))]\}$$

(xviii) On the similar lines as in (xvii), we can find $\theta_{2p^jq^k}(x)$ by replacing $C_{(i+j,r+k)}=A^*_{(i+j,r+k)}$, $B^*_{(i+j,r+k)}=C^*_{(i+j,r+k)}$, $A_{(i+j,r+k)}=B^*_{(i+j,r+k)}$, $D_{(i+j,r+k)}=B^*_{(i+j,r+k)}$, and $A^*_{(i+j,r+k)}=D^*_{(i+j,r+k)}$, $B_{(i+j,r+k)}=A^*_{(i+j,r+k)}$ $\&\frac{\phi(4p^{n-j}q^{m-k})}{4}=-\frac{\phi(p^{n-j}q^{m-k})}{4}$ and vice versa in $\theta_{p^jq^k}(x)$ .

(xix) On the similar lines as in (xvii), we can find $\theta_{3p^jq^k}(x)$ by replacing $C_{(i+j,r+k)}=A_{(i+j,r+k)}$, $B_{(i+j,r+k)}=D_{(i+j,r+k)}$, $B^*_{(i+j,r+k)}=A^*_{(i+j,r+k)}$, $C^*_{(i+j,r+k)}=D^*_{(i+j,r+k)}$ and $-\&\frac{\phi(4p^{n-j}q^{m-k})}{4}=\&\frac{\phi(4p^{n-j}q^{m-k})}{4}$ vice versa in $\theta_{p^jq^k}(x)$.

(xx) On the similar lines as in (xvii), we can find $\theta_{4p^jq^k}(x)$ by replacing $C_{(i+j,r+k)}=C^*_{(i+j,r+k)}$, $A_{(i+j,r+k)}=B^*_{(i+j,r+k)}$, $B_{(i+j,r+k)}=C^*_{(i+j,r+k)}$, $D_{(i+j,r+k)}=D^*_{(i+j,r+k)}$, $A^*_{(i+j,r+k)}=C^*_{(i+j,r+k)}$ and $\&\frac{\phi(4p^{n-j}q^{m-k})}{4}=\frac{\phi(4p^{n-j}q^{m-k})}{4}$ and viceversa in $\theta_{p^jq^k}(x)$.

(xxi) On the similar lines as in (xvii), we can find $\theta_{ap^jq^k}(x)$ by replacing $C_{(i+j,r+k)}=D_{(i+j,r+k)}$, $B^*_{(i+j,r+k)}=A^*_{(i+j,r+k)}$, $A_{(i+j,r+k)}=B_{(i+j,r+k)}$, $D^*_{(i+j,r+k)}=C^*_{(i+j,r+k)}$, and viceversa in $\theta_{p^jq^k}(x)$.

(xxii) On the similar lines as in (xviii), we can find $\theta_{2ap^jq^k}(x)$ by replacing $A^*_{(i+j,r+k)}=B^*_{(i+j,r+k)}$, $C^*_{(i+j,r+k)}=D^*_{(i+j,r+k)}$, and viceversa in $\theta_{2p^jq^k}(x)$.

(xxiii) On the similar lines as in (xix), we can find $\theta_{3ap^jq^k}(x)$ by replacing $A_{(i+j,r+k)}=B_{(i+j,r+k)}$, $C_{(i+j,r+k)}=D_{(i+j,r+k)}$, $A^*_{(i+j,r+k)}=B^*_{(i+j,r+k)}$, $C^*_{(i+j,r+k)}=D^*_{(i+j,r+k)}$, and viceversa in $\theta_{3p^jq^k}(x)$.

(xiv) On the similar lines as in (xx), we can find $\theta_{4ap^jq^k}(x)$ by replacing $C^*_{(i+j,r+k)} = D^*_{(i+j,r+k)}$ and Viceversa in $\theta_{4p^jq^k}(x)$.

where $A_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{-1+r+\gamma+\delta}{4})$, $B_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{-1+r-\delta-\gamma}{4})$

$C_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{-1-r-\gamma+\delta}{4})$, $D_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{-1-r+\gamma-\delta}{4})$

$A^*_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{1+r+\gamma+\delta}{4})$, $B^*_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{1+r-\delta-\gamma}{4})$

$C^*_{(n-1,m-1)} = p^{n-1}q^{m-1}(\frac{1-r-\gamma+\delta}{4})$, $D^*_{(n-1,m-1)} = p^{n-1}q^{m1}(\frac{1-r+\gamma-\delta}{4})$

$$\eta_0 = \sum_{s=0}^{\frac{\phi(p)}{2}-1} (\alpha^{4p^{n-1}q^m})^{l^s}, \quad \eta_1 = \sum_{s=0}^{\frac{\phi(p)}{2}-1} (\alpha^{4p^{n-1}q^m})^{al^s}, \quad \xi_0 = \sum_{s=0}^{\frac{\phi(p)}{2}-1} (\alpha^{p^{n-1}q^m})^{l^s},$$

$$\xi_1 = \sum_{s=0}^{\frac{\phi(p)}{2}-1} (\alpha^{p^{n-1}q^m})^{al^s}$$ where $r^2 = -q$, $\gamma^2 = -p$, $\delta^2 = pq$.

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