k-cordial labeling of some prisms.

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Abstract — *We discuss here k-cordial labeling of prism for all odd k. We prove that prisms $P_m \times C_k$, $P_m \times C_{k+1}$, $P_m \times C_{k+3}$ are k-cordial for all odd k and $m \geq 2$. In addition to this we prove that all the Prisms $P_m \times C_{2k-1}$ are k-cordial for all odd k, $m \geq 2$ and $m \neq tk$; $t \geq 1$.*

Keywords—*Abelian Group; k-cordial Labeling; Prism.*

I. Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph G= (V (G), E (G)) of order |V (G)| and size |E (G)|.

Definition 1.1 A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s) If the domain of the mapping is the set of vertices(edges) then the labeling is called a *vertex labeling*(an *edge labeling.)*

Definition 1.2 Let $< A,* >$ be any Abelian group. A graph is said to be *A-cordial* if there is a mapping $f:V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e=uv$ is labeled as $f(u)*f(v)$

(i) $|v_f(a)-v_f(b)| \leq 1$; for all $a,b \in A$,

(ii) $|e_f(a)-e_f(b)| \leq 1$; for all $a,b \in A$,

Where,

$v_f(a)$=the number of vertices with label a;

$v_f(b)$=the number of vertices with label b;

$e_f(a)$=the number of edges with label a;

$e_f(b)$=the number of edges with label b.

Definition 1.3 The *prism* $P_m \times C_n$ is obtained by taking the cartesian product of path P_m with cycle C_n.

Seoud and Abdel Maqusoud[5] proved that the prism $C_n \times P_m$ is cordial except for the case $C_{4k+2} \times P_2$ for $m \geq 2$.

For any undefned term in graph theory we rely upon Clark and Holton [2].

II. Main Results

Theorem 2.1 All the Prisms $P_m \times C_n$ are k-cordial for all odd k and $n = k; k + 1; k + 3$, $m \geq 2$.

Proof We divide the proof in three cases.

Case I: For $n=k$, we prove that prism $G = P_m \times C_k$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_k$ be the prism. Let $v_1, v_2, ..., v_{mk}$ be the vertices of the prism. Let $v_1, v_2, ..., v_k$ be vertices of the outer most cycle. Start the labeling pattern from the vertices v_1 v_2, ...,v_k arranged in the clockwise direction. Let v_{k+1} be the vertex of first inner cycle which is adjacent to v_k. Next label the vertices $v_{k+1}, v_{k+2}, ..., v_{2k}$ in the clockwise direction. Continue this pattern up to the last inner cycle. Let $v_{(m-1)k+1}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)k}$. Finally label the vertices $v_{(m-1)k+1}, v_{(m-1)k+2}, v_{(m-1)k+3}, ..., v_{mk}$ of the last inner cycle in the clockwise direction.

Note that $|V(G)| = mk$ and $|E(G)| = (2m - 1)k$.

In this case we define k-cordial labeling $f: V(G) \rightarrow Z_k$ as follows.

$f(v_i) = p_i$, where $(2i - 1) \equiv p_i (mod\ k)$, for $i = 1,2,...,mk$.

Case II: For $n=k + 1$, we prove that prism $G = P_m \times C_{k+1}$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_{k+1}$ be the prism. Let v_1 v_2,..., $v_{m(k+1)}$ be the vertices of the prism. Let $v_1, v_2, ..., v_k, v_{k+1}$ be the vertices of the outer most cycle. Start the labeling pattern from the vertices v_1, v_2,..., v_k, v_{k+1} arranged in the clockwise direction. Let v_{k+2} be the vertex of first inner cycle which is adjacent to v_{k+1}. Next label the vertices, v_{k+2}, v_{k+3}, v_{k+4},..., v_{2k+1}, v_{2k+2} in the clockwise direction. Continue this pattern up to the last inner cycle. Let $v_{(m-1)k+m}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)k+(m-1)}$. Finally label the vertices $v_{(m-1)k+m}$, $v_{(m-1)k+m+1}$,..., $v_{m(k+1)}$ of the last inner cycle in the clockwise direction.

Note that $|V(G)| = m(k+1)$ and $|E(G)| = (2m-1)(k+1)$.

In this case we define k-cordial labeling $f: V(G) \rightarrow Z_K$ as follows.

$f(v_i)=p_i$, where $(2i - 1) \equiv p_i (mod\ k)$, for $i = 1, 2,...,m(k + 1)$.

Case III: For $n=k + 3$, we prove that prism $G = P_m \times C_{k+3}$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_{k+3}$ be the prism. Let $v_1, v_2, ..., v_{m(k+3)}$ be the vertices of the prism. Let $v_1, v_2, v_3, ..., v_{k+2}$, v_{k+3} be vertices of the outer most cycle. Start the labeling pattern from $v_1, v_2, v_3, ..., v_{k+2}$, v_{k+3} arranged in the clockwise direction. Let v_{k+4} be the vertex of the first inner cycle which is adjacent to v_{k+3}. Next label the vertices $v_{k+4}, v_{k+5}, v_{k+6}, ..., v_{2k+5}$, v_{2k+6} in the clockwise

direction. Continue this pattern up to the last inner cycle. Let $v_{(m-1)k+3m-2}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)k+3m-3}$. Finally label the vertices,
$v_{(m-1)k+3m-2}, v_{(m-1)k+3m-1}, v_{(m-1)k+3m}, v_{(m-1)k+3m+1}, \ldots, v_{m(k+3)-1}, v_{m(k+3)}$
of the last inner cycle in the clockwise direction.

Note that $|V(G)| = m(k+3)$ and $|E(G)| = (2m-1)(k+3)$.
In this case we define k-cordial labeling $f : V(G) \rightarrow Z_k$ as follows.

$f(v_i)=p_i$, where $(2i-1) \equiv p_i (mod\ k)$, for $i = 1, 2, \ldots, m(k+3)$.

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling.

Illustration 2.2(a): The prism $P_4 \times C_9$ and its 9-cordial labeling is shown in Fig 2.1.

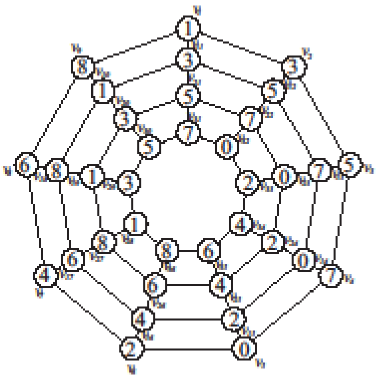

Fig 2.1: 9-Cordial labeling of prism $P_4 \times C_9$.

Illustration 2.2(b): The prism $P_5 \times C_6$ and its 5-cordial labeling is shown in Fig 2.2.

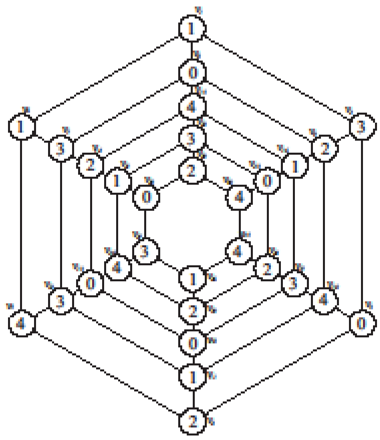

Fig 2.2: 5-*Cordial labeling of prism* $P_5 \times C_6$.

Illustration 2.2(c): The prism $P_3 \times C_{10}$ and its 7-cordial labeling is shown in Fig 2.3.

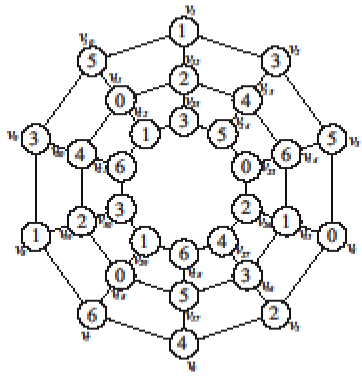

Fig 2.3: 7-Cordial labeling of prism $P_3 \times C_{10}$.

Theorem 2.3 All the Prisms $P_m \times C_n$ are k-cordial for all odd k and $n=2k-1$, $m \geq 2$ and $m \neq tk$, $t \geq 1$.

Proof: For $n=2k-1$.
Let $G = P_m \times C_{2k-1}$ be the prism. Let $v_1, v_2, \ldots, v_{m(2k-1)}$ be the vertices of the prism. Let $v_1, v_2, v_3, \ldots, v_{2k-1}$ be the vertices of outer most cycle. Start the labeling pattern from $v_1, v_2, v_3, \ldots, v_{2k-1}$ arranged in the clockwise direction. Let v_{2k} be the vertex of first inner cycle which is adjacent to v_{2k-1}. Next label $v_{2k}, v_{2k+1}, v_{2k+2}, \ldots, v_{2(2k-1)}$ vertices in the clockwise direction. Continue this pattern up to last inner cycle. Let $v_{(m-1)(2k-1)+1}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)(2k-1)}$. Finally label the vertices $v_{(m-1)(2k-1)+1}, v_{(m-1)(2k-1)+2}, v_{(m-1)(2k-1)+2}, \ldots, v_{m(2k-1)}$ of the last inner cycle in the clockwise direction.

We note that $|V(G)| = m(2k-1)$ and $|E(G)| = (2m-1)(2k-1)$.

Case I: $m \neq (tk+1)$
In this case we define k-cordial labelling $f : V(G) \longrightarrow Z_k$ as follows.

$f(v_i)=p_i$, where $(2i-1) \equiv p_i (mod\ k)$, for $i = 1, 2, \ldots, m(2k-1)$.

Case II: $m = (tk+1)$
In this case we define k-cordial labeling $f : V(G) \longrightarrow Z_k$ as follows.

$f(v_i)=p_i$, where $(2i-1) \equiv p_i (mod\ k)$, for $i = 1, 2, \ldots, (m(2k-1)-1)$.
$f(v_{m(2k-1)})=k-1$, where $i=(m(2k-1))$.

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling.

Illustration 2.4(a): The prism $P_6 \times C_9$ and its *5*-cordial labeling is shown in Fig 2.4.

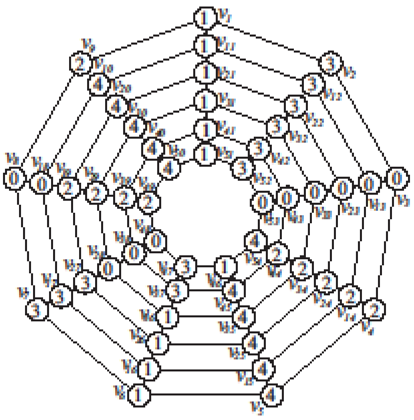

Fig 2.4: *5*-Cordial labeling of prism $P_6 \times C_9$.

Illustration 2.4(b): The prism $P_3 \times C_{13}$ and its *7*-cordial labeling is shown in Fig 2.5.

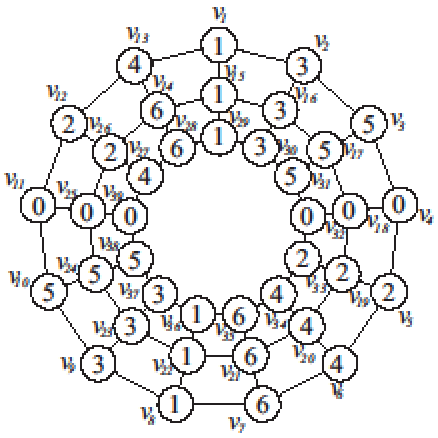

Fig 2.5: *7*-Cordial labeling of prism $P_3 \times C_{13}$.

III. CONCLUDING REMARKS

Here we have contributed two general results to the theory of k-cordial labeling. To derive similar results for other graph families is an open area of research.

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