Pseudo L-Fuzzy Cosets of an L – Fuzzy ℓ - Hx Group

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Abstract - *In this paper, we define a new algebraic structure of pseudo L-fuzzy cosets of an L – fuzzy ℓ - HX group and some related properties are investigated. We also discuss the properties of pseudo L-fuzzy cosets of an L – fuzzy sub ℓ - HX group under ℓ - HX group homomorphism and ℓ - HX group anti homomorphism.*

Keywords - *HX group, ℓ - HX group , fuzzy set in ℓ - HX group, L-fuzzy set in ℓ -HX group, L – fuzzy ℓ - HX group, pseudo L-fuzzy cosets , ℓ - HX group homomorphism, ℓ - HX group anti homomorphism.*

I. INTRODUCTION

The idea of L-fuzzy set was introduced by Goguen (1967) [1] as a extension of Zadeh's (1965) [8] fuzzy sets. Rosenfeld.A [6] introduced the concept of fuzzy group. Li Hongxing [3] introduced the concept of HX group and the authors Luo Chengzhong , Mi Honghai , Li Hongxing [3] introduced the concept of fuzzy HX group. Muthuraj et.al [2][5] defined the concept of Q – fuzzy HX group, intuitionistic Q-fuzzy HX group and study some of their related properties. G.S.V.Satya Saibaba [7] introduced the concept of fuzzy lattice ordered group.

In this paper we define a new concept of pseudo L-fuzzy cosets of an L-fuzzy sub ℓ -HX group and study some of their related properties.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition [3]

In 2^G-$\{\phi\}$, a non empty set $\vartheta \subset 2^G$-$\{\phi\}$ is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = $\{ab \,/\, a \in A \text{ and } b \in B\}$, and its unit element is denoted by E.

2.2 Definition

Let $\vartheta \subset 2^G$-$\{\phi\}$ be a HX group on group G. Define the relation $\leq$ on ϑ as for every A,B$\in\vartheta$, $A \leq B$ iff $a \leq b$, for every $a \in A$ and $b \in B$.

2.3 Definition [7]

A lattice ordered group or a ℓ - group is a system G = (G, $*$, $\leq$), where

i. (G, $\cdot$) is a group,
ii. (G, $\leq$) is a lattice.

2.4 Remark

Throughout this paper G = (G, $\cdot$, $\leq$) is a lattice ordered group or a ℓ - group, e is the identity element of G and xy we mean x$\cdot$y.

2.5 Definition

In 2^G-$\{\phi\}$, a non empty set ($\vartheta \subset 2^G$-$\{\phi\}$, $\cdot$, $\leq$) is called as an ℓ -HX group or lattice ordered HX group on G, if the following conditions are satisfied.

i. (ϑ , $\cdot$) is a HX group.
ii. (ϑ , $\leq$) is a Lattice.

2.6 Definition

Let ϑ and ϑ' be any two ℓ -HX groups on G and G′ respectively. The function f : $\vartheta \to \vartheta'$ is called an ℓ -HX group homomorphism if for all A, B in ϑ ,

i. f(AB) = f(A)f(B),
ii. f($A \vee B$) = f(A) $\vee$ f(B),
iii. f($A \wedge B$) = f(A) $\wedge$ f(B).

2.7 Definition

Let ϑ and ϑ' be any two ℓ -HX groups on G and G′ respectively (not necessarily commutative). The function f : $\vartheta \to \vartheta'$ is called an ℓ - HX group anti homomorphism if for all A, B in ϑ ,

i. f(AB) = f(B)f(A),
ii. f($A \vee B$) = f(A) $\vee$ f(B),
iii. f($A \wedge B$) = f(A) $\wedge$ f(B).

2.8 Definition [6]

Let G be a group. The mapping $\mu : G \to [0, 1]$ is called a fuzzy set defined on G.

2.9 Definition

Let G be a group and a non-empty set $\vartheta \subset 2^G$ – $\{\phi\}$ be a HX group. The mapping $\lambda^\mu : \vartheta \to L$ is called a L – fuzzy set defined on ϑ, where, (L, $\leq$) be a lattice with an involutive order reversing operation N: L $\to$ L.

2.10 Definition

Let G be a group. Let μ be an L – fuzzy set defined on G. Let $\vartheta \subset 2^G$-$\{\phi\}$,a non-empty set, be an ℓ-HX group on G. An L-fuzzy subset λ^μ of an ℓ-HX group ϑ is said to be an L - fuzzy sub ℓ-HX group of an ℓ-HX group ϑ if the following conditions are satisfied. For all A and B $\in \vartheta$,

i. $\lambda^\mu(AB) \geq \lambda^\mu(A) \wedge \lambda^\mu(B)$,
ii. $\lambda^\mu(A) = \lambda^\mu(A^{-1})$,
iii. $\lambda^\mu(A \vee B) \geq \lambda^\mu(A) \wedge \lambda^\mu(B)$,
iv. $\lambda^\mu(A \wedge B) \geq \lambda^\mu(A) \wedge \lambda^\mu(B)$,

where $\lambda^\mu(A) = \max\{\mu(x) \ / \ \text{for all } x \in A \subseteq G\}$.

2.11 Definition

Let G_1 and G_2 be any two groups. Let $\vartheta_1 \subset 2^{G_1} - \{\phi\}$ and $\vartheta_2 \subset 2^{G_2} - \{\phi\}$ be any two ℓ-HX groups defined on G_1 and G_2 respectively. Let μ and α be any two L - fuzzy subsets in G_1 and G_2 respectively. Let λ^μ and η^α be an L - fuzzy subsets defined on ϑ_1 and ϑ_2 respectively. Let f: $\vartheta_1 \to \vartheta_2$ be a mapping then the image of λ^μ denoted as $f(\lambda^\mu)$ is an L - fuzzy subset of ϑ_2 defined as for each U$\in \vartheta_2$,

$$(f(\lambda^\mu))(U) = \begin{cases} \sup\{\lambda^\mu(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

Also the pre-image of η^α denoted as $f^{-1}(\eta^\alpha)$ under f is a fuzzy subset of ϑ_1 defined as for each X $\in \vartheta_1$, $(f^{-1}(\eta^\alpha))(X) = \eta^\alpha(f(X))$.

2.12 Definition

Let G_1 and G_2 be any two groups. Let $\vartheta_1 \subset 2^{G_1} - \{\phi\}$ and $\vartheta_2 \subset 2^{G_2} - \{\phi\}$ be any two ℓ- HX groups defined on G_1 and G_2 respectively. Let λ^μ be an L-fuzzy HX sub ℓ- HX group of an ℓ- HX group in ϑ_1. Let f: $\vartheta_1 \to \vartheta_2$ be a function. Then λ^μ is called f-invariant if f(A) = f(B) implies that $\lambda^\mu(A) = \lambda^\mu(B)$.

III. Properties of pseudo L-fuzzy cosets of an L-fuzzy sub ℓ-HX group of an ℓ-HX group

In this section, we discuss some of the properties of pseudo L-fuzzy cosets of an L-fuzzy sub ℓ-HX group of an ℓ-HX group.

3.1 Definition

Let G be a group. Let μ be an L – fuzzy set defined on G. Let $\vartheta \subset 2^G$-$\{\phi\}$,a non-empty set, be an ℓ-HX group on G. Let λ^μ be an L - fuzzy sub ℓ-HX group of an ℓ-HX group ϑ and A $\in\vartheta$. Then the pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group of an ℓ-HX group determined by the element A $\in\vartheta$ is defined as $(A\lambda^\mu)^p(X) = p(A)\,\lambda^\mu(X)$, for every X $\in \vartheta$ and for some p$\in$ P, where P = {p(X) / p(X) $\in$ [0,1] for all X$\in\vartheta$}.

3.2 Theorem

Let G be a group. Let μ be an L – fuzzy set defined on G. Let a non-empty set $\vartheta \subset 2^G$-$\{\phi\}$ be an ℓ-HX group on G. If λ^μ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ, then $(A\lambda^\mu)^p$ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ, for all A $\in\vartheta$.

Proof: Let λ^μ be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ. Let A $\in\vartheta$.

For any X, Y $\in\vartheta$,

i. $(A\lambda^\mu)^p(XY) = p(A)\,\lambda^\mu(XY)$
$\geq p(A)\{\lambda^\mu(X) \wedge \lambda^\mu(Y)\}$
$\geq p(A)\lambda^\mu(X) \wedge p(A)\,\lambda^\mu(Y)$
$\geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.
$(A\lambda^\mu)^p(XY) \geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.

ii. $(A\lambda^\mu)^p(X^{-1}) = p(A)\,\lambda^\mu(X^{-1})$
$= p(A)\lambda^\mu(X)$.
$(A\lambda^\mu)^p(X^{-1}) = (A\lambda^\mu)^p(X)$.

iii. $(A\lambda^\mu)^p(X \vee Y) = p(A)\,\lambda^\mu(X \vee Y)$
$\geq p(A)\{\lambda^\mu(X) \wedge \lambda^\mu(Y)\}$
$\geq p(A)\lambda^\mu(X) \wedge p(A)\,\lambda^\mu(Y)$
$\geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.
$(A\lambda^\mu)^p(X \vee Y) \geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.

iv. $(A\lambda^\mu)^p(X \wedge Y) = p(A)\,\lambda^\mu(X \wedge Y)$
$\geq p(A)\{\lambda^\mu(X) \wedge \lambda^\mu(Y)\}$
$\geq p(A)\lambda^\mu(X) \wedge p(A)\,\lambda^\mu(Y)$
$\geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.
$(A\lambda^\mu)^p(X \wedge Y) \geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.

Hence, $(A\lambda^\mu)^p$ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ.

3.3 Theorem

Let G be a group. Let μ be an L – fuzzy set defined on G. Let a non-empty set $\vartheta \subset 2^G$-$\{\phi\}$ be an ℓ-HX group on G. If λ^μ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ, then $(A\lambda^\mu)^p$ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ, for all A $\in\vartheta$.

Proof :Let λ^μ be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ. Let A $\in\vartheta$.

For any X, Y $\in\vartheta$,

i. $(A\lambda^\mu)^p(XY) = p(A)\,\lambda^\mu(XY)$
$\geq p(A)\{\lambda^\mu(X) \wedge \lambda^\mu(Y)\}$
$\geq p(A)\lambda^\mu(X) \wedge p(A)\,\lambda^\mu(Y)$
$\geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.
$(A\lambda^\mu)^p(XY) \geq (A\lambda^\mu)^p(X) \wedge (A\lambda^\mu)^p(Y)$.

ii. $(A\lambda^\mu)^p(X^{-1}) = p(A)\,\lambda^\mu(X^{-1})$

$$= p(A)\lambda^{\mu}(X).$$
$$(A\lambda^{\mu})^{p}(X^{-1}) = (A\lambda^{\mu})^{p}(X).$$

iii. $(A\lambda^{\mu})^{p}(X \vee Y) = p(A)\,\lambda^{\mu}(X \vee Y)$
$$\geq p(A)\,\{\, \lambda^{\mu}(X) \wedge \lambda^{\mu}(Y)\,\}$$
$$\geq p(A)\lambda^{\mu}(X) \wedge p(A)\,\lambda^{\mu}(Y)$$
$$\geq (A\lambda^{\mu})^{p}(X) \wedge (A\lambda^{\mu})^{p}(Y).$$
$$(A\lambda^{\mu})^{p}(X \vee Y) \geq (A\lambda^{\mu})^{p}(X) \wedge (A\lambda^{\mu})^{p}(Y).$$

iv. $(A\lambda^{\mu})^{p}(X \wedge Y) = p(A)\,\lambda^{\mu}(X \wedge Y)$
$$\geq p(A)\,\{\, \lambda^{\mu}(X) \wedge \lambda^{\mu}(Y)\,\}$$
$$\geq p(A)\lambda^{\mu}(X) \wedge p(A)\,\lambda^{\mu}(Y)$$
$$\geq (A\lambda^{\mu})^{p}(X) \wedge (A\lambda^{\mu})^{p}(Y).$$
$$(A\lambda^{\mu})^{p}(X \wedge Y) \geq (A\lambda^{\mu})^{p}(X) \wedge (A\lambda^{\mu})^{p}(Y).$$

Hence, $(A\lambda^{\mu})^{p}$ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ.

3.4 Theorem

Let G be a group. Let μ be an L – fuzzy set defined on G. Let a non-empty set $\vartheta \subset 2^{G}$-$\{\phi\}$ be an ℓ-HX group on G. Let λ^{μ} is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ. Let $(A\lambda^{\mu})^{p}$ and $(B\lambda^{\mu})^{p}$ be pseudo L-fuzzy cosets of an L-fuzzy sub ℓ-HX group of an ℓ-HX group determined by the element A, B $\in\vartheta$. Then, their intersection, $(A\lambda^{\mu})^{p} \cap (B\lambda^{\mu})^{p}$ is an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ, for all A, B $\in\vartheta$.

Proof : It is clear.

IV. Properties of pseudo L-fuzzy cosets of an L-fuzzy sub ℓ-HX group of an ℓ-HX group under ℓ-HX group homomorphism and ℓ-HX group anti homomorphism

In this section, we discuss some of the properties of pseudo L-fuzzy cosets of an L – fuzzy sub ℓ-HX group of an ℓ-HX group under ℓ-HX group homomorphism and ℓ-HX group anti homomorphism.

4.1 Theorem

Let ϑ and ϑ' be any two ℓ-HX groups on G and G′.Let λ^{μ} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ. Let f: $\vartheta \to \vartheta'$ be an onto ℓ-HX group homomorphism. Let $(A\lambda^{\mu})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group λ^{μ} of an ℓ-HX group determined by the element A $\in\vartheta$. Then $f(A\lambda^{\mu})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f(\lambda^{\mu})$ of an ℓ-HX group determined by the element f(A) $\in\vartheta'$ and $f((A\lambda^{\mu})^{p}) = (f(A)f(\lambda^{\mu}))^{p}$ if λ^{μ} has supremum property and λ^{μ} is f - invariant.

Proof: Let λ^{μ} be an L-fuzzy sub ℓ-HX group of G and $(A\lambda^{\mu})^{p}$ be an pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group λ^{μ} of an ℓ-HX group G determined by the element A $\in\vartheta$. Clearly, $f(\lambda^{\mu})$ is an L-fuzzy sub ℓ-HX group of ϑ' and $f((A\lambda^{\mu})^{p})$ is an pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f(\lambda^{\mu})$ of an ℓ-HX group ϑ' determined by the element f(A)$\in\vartheta'$.

Now, for any X$\in\vartheta$, f(X)$\in\vartheta'$ and

$$(f(A)f(\lambda^{\mu}))^{p}(f(X)) = p(f(A))f(\lambda^{\mu})f(X)$$
$$= p(A)\lambda^{\mu}(X)$$
$$= (A\lambda^{\mu})^{p}(X)$$
$$= f((A\lambda^{\mu})^{p})f(X),$$
$$(f(A)f(\lambda^{\mu}))^{p}(f(X)) = f((A\lambda^{\mu})^{p})f(X)$$

Hence, $(f(A)f(\lambda^{\mu}))^{p} = f((A\lambda^{\mu})^{p})$.

4.2 Theorem

Let ϑ and ϑ' be any two ℓ-HX groups on G and G′.Let η^{α} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ'. Let f: $\vartheta \to \vartheta'$ be an ℓ-HX group homomorphism. Let $(B\eta^{\alpha})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group η^{α} of an ℓ-HX group determined by the element B $\in\vartheta'$. Then $f^{-1}(B\eta^{\alpha})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f^{-1}(\eta^{\alpha})$ of an ℓ-HX group determined by the element $f^{-1}(B)$ $\in\vartheta$ and $f^{-1}((B\eta^{\alpha})^{p}) = (f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}$.

Proof: Let η^{α} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ'. Let $(B\eta^{\alpha})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group η^{α} of an ℓ-HX group determined by the element B $\in\vartheta'$. Clearly, $f^{-1}(\eta^{\alpha})$ is an L – fuzzy sub ℓ-HX group of an ℓ-HX group ϑ and $f^{-1}(B\eta^{\alpha})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f^{-1}(\eta^{\alpha})$ of an ℓ-HX group determined by the element $f^{-1}(B)$ $\in\vartheta$.

Now, for any X$\in\vartheta$, f(X)$\in\vartheta'$ and

$$(f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}(X) = p(f^{-1}(B))\,(f^{-1}(\eta^{\alpha})(X))$$
$$= p(B)(\eta^{\alpha}(f(X))$$
$$= (B\eta^{\alpha})^{p}(f(X))$$
$$= f^{-1}((B\eta^{\alpha})^{p})(X).$$
$$(f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}(X) = f^{-1}((B\eta^{\alpha})^{p})(X).$$

Hence, $f^{-1}((B\eta^{\alpha})^{p}) = (f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}$.

4.3 Theorem

Let ϑ and ϑ' be any two ℓ-HX groups on G and G′.Let λ^{μ} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ. Let f: $\vartheta \to \vartheta'$ be an onto ℓ-HX group anti homomorphism. Let $(A\lambda^{\mu})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group λ^{μ} of an ℓ-HX group determined by the element A $\in\vartheta$. Then $f(A\lambda^{\mu})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f(\lambda^{\mu})$ of an ℓ-HX group determined by the element f(A) $\in\vartheta'$ and

$f((A\lambda^{\mu})^{p}) = (f(A)f(\lambda^{\mu}))^{p}$ if λ^{μ} has supremum property and λ^{μ} is f - invariant.

Proof: Let λ^{μ} be an L-fuzzy sub ℓ-HX group of G and $(A\lambda^{\mu})^{p}$ be an pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group λ^{μ} of an ℓ-HX group G determined by the element $A \in \vartheta$. Clearly, $f(\lambda^{\mu})$ is an L-fuzzy sub ℓ-HX group of ϑ' and $f((A\lambda^{\mu})^{p})$ is an pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f(\lambda^{\mu})$ of an ℓ-HX group ϑ' determined by the element $f(A) \in \vartheta'$.

Now, for any $X \in \vartheta$, $f(X) \in \vartheta'$ and

$$
\begin{aligned}
(f(A)f(\lambda^{\mu}))^{p}(f(X)) &= p(f(A))f(\lambda^{\mu})f(X) \\
&= p(A)\lambda^{\mu}(X) \\
&= (A\lambda^{\mu})^{p}(X) \\
&= f((A\lambda^{\mu})^{p})f(X), \\
(f(A)f(\lambda^{\mu}))^{p}(f(X)) &= f((A\lambda^{\mu})^{p})f(X) \\
\text{Hence, } (f(A)f(\lambda^{\mu}))^{p} &= f((A\lambda^{\mu})^{p}).
\end{aligned}
$$

4.4 Theorem

Let ϑ and ϑ' be any two ℓ-HX groups on G and G′.Let η^{α} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ'. Let $f: \vartheta \to \vartheta'$ be an ℓ-HX group anti homomorphism. Let $(B\eta^{\alpha})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group η^{α} of an ℓ-HX group determined by the element $B \in \vartheta'$. Then $f^{-1}(B\eta^{\alpha})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f^{-1}(\eta^{\alpha})$ of an ℓ-HX group determined by the element $f^{-1}(B) \in \vartheta$ and $f^{-1}((B\eta^{\alpha})^{p}) = (f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}$.

Proof: Let η^{α} be an L-fuzzy sub ℓ-HX group of an ℓ-HX group ϑ'. Let $(B\eta^{\alpha})^{p}$ be a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group η^{α} of an ℓ-HX group determined by the element $B \in \vartheta'$. Clearly, $f^{-1}(\eta^{\alpha})$ is an L – fuzzy sub ℓ-HX group of an ℓ-HX group ϑ and $f^{-1}(B\eta^{\alpha})$ is a pseudo L-fuzzy coset of an L-fuzzy sub ℓ-HX group $f^{-1}(\eta^{\alpha})$ of an ℓ-HX group determined by the element $f^{-1}(B) \in \vartheta$.

Now, for any $X \in \vartheta$, $f(X) \in \vartheta'$ and

$$
\begin{aligned}
(f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}(X) &= p(f^{-1}(B))\,(f^{-1}(\eta^{\alpha})(X)) \\
&= p(B)(\eta^{\alpha}(f(X)) \\
&= (B\eta^{\alpha})^{p}(f(X)) \\
&= f^{-1}((B\eta^{\alpha})^{p})(X). \\
(f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}(X) &= f^{-1}((B\eta^{\alpha})^{p})(X). \\
\text{Hence, } f^{-1}((B\eta^{\alpha})^{p}) &= (f^{-1}(B)f^{-1}(\eta^{\alpha}))^{p}.
\end{aligned}
$$

V. Conclusion

The new algebraic structure of pseudo L-fuzzy coset of an L-fuzzy ℓ-HX group of an ℓ-HX group is introduced and discussed its properties. The image and pre-image of a pseudo L-fuzzy coset of an L-fuzzy ℓ-HX group of an ℓ-HX group is discussed under ℓ-HX group homomorphism and ℓ-HX group anti homomorphism.

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