Newton's Divided Difference Interpolation formula: Representation of Numerical Data by a Polynomial curve

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Abstract: *Due to the necessity of a formula for representing a given set of numerical data* on a pair of variables *by a suitable polynomial, in interpolation by the approach which consists of the representation of numerical data by a suitable polynomial and then to compute the value of the dependent variable from the polynomial corresponding to any given value of the independent variable, one such formula has been derived* from *Newton's divided difference interpolation formula. This paper describes the derivation of the formula with numerical example as its application.*

Keywords: *Interpolation, divided difference formula, polynomial curve, representation of numerical data.*

1. Introduction:

Interpolation, which is a tool for estimating the value of the dependent variable corresponding to a value of the independent variable lying between its two extreme values on the basis of the given values of the independent and the dependent variables {Hummel (1947), Erdos & Turan (1938) et al}. A number of interpolation formulas such as Newton's Forward Interpolation formula, Newton's Backward Interpolation formula, Lagrange's Interpolation formula, Newton's Divided Difference Interpolation formula, Newton's Central Difference Interpolation formula, Stirlings formula , Bessel's formula and some others are available in the literature of numerical analysis {Bathe & Wilson (1976), Jan (1930), Hummel (1947) et al}.

In case of the interpolation by the existing formulae, the value of the dependent variable corresponding to each value of the independent variable is to be computed afresh from the used formula putting the value of the independent variable in it. That is if it is wanted to interpolate the values of the dependent variable corresponding to a number of values of the independent variable by a suitable existing interpolation formula, it is required to apply the formula for each value separately and thus the numerical computation of the value of the dependent variable based on the given data are to be performed in each of the cases. In order to get rid of these repeated numerical computations from the given data, one can think of an approach which consists of the representation of the given numerical data by a suitable polynomial and then to compute the value of the dependent variable from the polynomial corresponding to any given value of the independent variable. However, a formula is necessary for representing a given set of numerical data on a pair of variables by a suitable polynomial. Due to this necessity, one such formula has been developed by Das & Chakrabarty (2016).They have been derived this formula from Lagrange's interpolation formula. In this study, another formula has been derived for the same purpose. The formula has here been derived from Newton's divided difference interpolation formula. This paper describes the derivation of the formula with numerical example as its application.

2. Newton's Divided Difference Interpolation Formula:

Newton's Divided Difference is a way of finding an interpolation polynomial (a polynomial that fits a particular set of points or data). Similar to Lagrange's method for finding an interpolation polynomial, it finds the same interpolation polynomial due to the uniqueness of interpolation polynomials. Newton's Divided Difference uses the following equation called the *divided difference* to accomplish this task:

$$f(x_0, x_1, x_2 \ldots\ldots = \frac{f(x_1, x_2, x_3 \ldots\ldots x_n) - f(x_0, x_1, x}{x_n - x_0}$$

and the following equation called *Newton's divided difference formula for the interpolation polynomial* is where the polynomial is derived from:

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + \ldots\ldots + (x - x_0)(x - x_1)(x - x_2) \ldots (x - x_{i-1}) f(x_0, x_1, x_2, x_3 \ldots x_{i-1}) + \ldots\ldots + (x - x_0)(x - x_1)(x - x_2) \ldots\ldots (x - x_{n-1}) f(x_0, x_1, x_2, x_3 \ldots x_n) \quad (2.1)$$

3. Representation of Numerical Data by Polynomial Curve:

By algebraic expansion, one can obtain that

$$(x - x_0)(x - x_1) = x^2 - (x_0 + x_1) x + x_0 x_1$$

$$= x^2 - \left(\sum_{i=0}^{1} x_i\right) x + x_0 x_1,$$

Also,

$$(x - x_0)(x - x_1)(x - x_2)$$

$$= x^3 - (x_0 + x_1 + x_2) x^2 + (x_0 x_1 + x_0 x_2 + x_1 x_2) x - x_0 x_1 x_2$$

$$= x^3 - \left(\sum_{i=0}^{2} x_i\right) x^2 + \left(\sum_{i=0}^{1} \sum_{j=1}^{2} x_i x_j\right) x - x_0 x_1 x_2,$$

Again,

$$(x - x_0)(x - x_1)(x - x_2)(x - x_3)$$

$$= x^4 - (x_0 + x_1 + x_2 + x_3) x^3 + (x_0 x_1 + x_0 x_2 + x_1 x_2 + x_0 x_3 + x_1 x_3 + x_2 x_3) x^2 - (x_1 x_2 x_3 + x_1 x_3 x_4 + x_2 x_3 x_4 + x_1 x_2 x_4) x + x_0 x_1 x_2 x_3$$

$$= x^4 - \left(\sum_{i=0}^{3} x_i\right) x^3 + \left(\sum_{i=0}^{2} \sum_{j=1}^{3} x_i x_j\right) x^2 - \left(\sum_{i=0}^{1} \sum_{j=1}^{2} \sum_{k=2}^{3} x_i x_j x_k\right) x + x_0 x_1 x_2 x_3,$$

In general, one can obtain that

$$(x - x_0)(x - x_1)(x - x_2)(x - x_3) \ldots (x - x_{n-1})$$

$$= x^n - \left(\sum_{i=0}^{n-1} x_i\right) x^{n-1} + \left(\sum_{i=0}^{n-2} \sum_{j=1}^{n-1} x_i x_j\right) x^{n-2} - \left(\sum_{i=0}^{n-3} \sum_{j=1}^{n-2} \sum_{k=2}^{n-1} x_i x_j x_k\right) x^{n-3} + \left(\sum_{i=0}^{n-3} \sum_{j=1}^{n-2} \sum_{k=2}^{n-1} \sum_{l=3}^{n} x_i x_j x_k x_l\right) x^{n-4} + \ldots\ldots + (-1)^n (x_0 x_1 x_2 x_3 \ldots\ldots x_{n-1})$$

Now, divided difference interpolation formula, described by equation (2.1), can be expressed as

$$f(x) = C_0 + C_1(x - x_0) + C_2\left\{x^2 - \left(\sum_{i=0}^{1} x_i\right) x + x_0 x_1\right\}$$
$$C_3\left\{x^3 - \left(\sum_{i=0}^{2} x_i\right) x^2 + \left(\sum_{i=0}^{1} \sum_{j=1}^{2} x_i x_j\right) x - x_0 x_1 x_2\right\} + C_4\left\{x^4 - \left(\sum_{i=0}^{3} x_i\right) x^3 + \left(\sum_{i=0}^{2} \sum_{j=1}^{3} x_i x_j\right) x^2 - \left(\sum_{i=0}^{1} \sum_{j=1}^{2} \sum_{k=2}^{3} x_i x_j x_k\right) x + x_0 x_1 x_2 x_3\right\}$$
$$+ \ldots\ldots + C_n\left\{x^n - \left(\sum_{i=0}^{n-1} x_i\right) x^{n-1} + \left(\sum_{i=0}^{n-2} \sum_{j=1}^{n-1} x_i x_j\right) x^{n-2} - \left(\sum_{i=0}^{n-3} \sum_{j=1}^{n-2} \sum_{k=2}^{n-1} x_i x_j x_k\right) x^{n-3} + \left(\sum_{i=0}^{n-3} \sum_{j=1}^{n-2} \sum_{k=2}^{n-1} \sum_{l=3}^{n} x_i x_j x_k x_l\right) x^{n-4} + \ldots\ldots + + (-1)^n (x_0 x_1 x_2 x_3 \ldots\ldots x_{n-1})\right\} \quad (3.1)$$

where $C_0 = f(x_0)$

$C_1 = f(x_0, x_1)$

$C_2 = f(x_0, x_1, x_2)$

$C_3 = f(x_0, x_1, x_2, x_3)$

....

....

$C_i = f(x_0, x_1, x_2, x_3 \ldots x_i)$

....

....

$$C_n = f(x_0, x_1, x_2, x_3 \ldots.. x_n)$$

Now, we have

Constant term = $C_0 - C_1 x_0 + C_2\, x_0 x_1 - C_3\, x_0 x_1 x_2 + C_4\, x_0 x_1 x_2 x_3 - \ldots\ldots\ldots\ldots\ldots\ldots + C_n\,(-1)^n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

Coefficient of x = $C_1 - C_2\,(\sum_{i=0}^{1} x_i) + C_3\,(\sum_{i=0}^{1}\sum_{j=1}^{2} x_i x_j) - C_4\,(\sum_{i=0}^{1}\sum_{j=1}^{2}\sum_{k=2}^{3} x_i x_j x_k) + \ldots\ldots\ldots\ldots(-1)^n C_n x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

Coefficient of x^2 = $C_2 - C_3\,(\sum_{i=0}^{2} x_i) + C_4\,\left(\sum_{i=0}^{2}\sum_{j=1}^{3} x_i x_j\right) - \ldots\ldots\ldots\ldots (-1)^n\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-3} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

Coefficient of x^3 = $C_3 - C_4\,\left(\sum_{i=0}^{3} x_i\right) + \ldots\ldots\ldots\ldots + (-1)^n\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-4} + (x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-3} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Coefficient of x^i = $C_i - C_{i+1}\,(\sum_{j=0}^{i} x_j) + \ldots\ldots\ldots\ldots\ldots + (-1)^{n-i}\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-i+1} \ldots\ldots\ldots\ldots + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Coefficient of x^n = C_n

$\therefore$ The equation (3.1), can be expressed as

$$f(x) = A_0 + A_1 x + A_2\, x^2 + A_3\, x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + A_n\, x^n \tag{3.2}$$

which is the required formula for representation of numerical data by a polynomial curve

where

$A_0 = C_0 - C_1 x_0 + C_2\, x_0 x_1 - C_3\, x_0 x_1 x_2 + C_4\, x_0 x_1 x_2 x_3 - \ldots\ldots\ldots\ldots\ldots\ldots + C_n\,(-1)^n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$A_1 = C_1 - C_2\,(\sum_{i=0}^{1} x_i) + C_3\,(\sum_{i=0}^{1}\sum_{j=1}^{2} x_i x_j) - C_4\,(\sum_{i=0}^{1}\sum_{j=1}^{2}\sum_{k=2}^{3} x_i x_j x_k) + \ldots\ldots\ldots\ldots + (-1)^n C_n (x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$A_2 = C_2 - C_3\,(\sum_{i=0}^{2} x_i) + C_4\,\left(\sum_{i=0}^{2}\sum_{j=1}^{3} x_i x_j\right) - \ldots\ldots\ldots(-1)^n\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-3} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$A_3 = C_3 - C_4\,\left(\sum_{i=0}^{3} x_i\right) + \ldots\ldots\ldots\ldots + (-1)^n\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-4} + (x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-3} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-2} + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$A_i = C_i - C_{i+1}\,(\sum_{j=0}^{i} x_j) + \ldots\ldots\ldots\ldots\ldots + (-1)^{n-i}\, C_n\,(x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-i+1} \ldots\ldots\ldots\ldots + x_0 x_1 x_2 x_3 \ldots\ldots\ldots\ldots\, x_{n-1})$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$A_n = C_n$

Equation (3.2), with the coefficients

$$A_0,\ A_1, A_2,\ A_3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, A_n$$

as defined above, is the required formula for representing a given set of numerical data on a pair of variables by a suitable polynomial we have aimed at.

4. Example of Application of the Formula:

The following table shows the data on total population of Assam corresponding to the years:

Year	*Total Population*
1971	14625152
1981	18041248
1991	22414322

2001	26638407
2011	31205576

Taking 1971 as origin and changing scale by 1/10, one can obtain the following table for independent variable x (representing time) and $f(x)$ (representing total population of Assam):

Year	x_i	***Total Population***
1971	0	14625152
1981	1	18041248
1991	2	22414322
2001	3	26638407
2011	4	31205576

Now here $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$

$f(x_0) = 14625152$, $f(x_1) = 18041248$,

$f(x_2) = 22414322$, $f(x_3) = 26638407$,

$f(x_4) = 31205576$

Difference Table

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3, x_4)$
0	14625152				
		3416096			
1	18041248		478489		
		4373074		$-$ 184327.83	
2	22414322		$-$74494.5		66584.99
		4224085		82012.16	
3	26638407		171542		
		4567169			
4	31205576				

Now, $C_0 = f(x_0) = 14625152$

$C_1 = f(x_0, x_1) = 3416096$

$C_2 = f(x_0, x_1, x_2) = 478489$

$C_3 = f(x_0, x_1, x_2, x_3) = -\ 184327.83$

$C_4 = f(x_0, x_1, x_2, x_3, x_4) = \ 66584.99$

$\therefore$ The polynomial is

$$f(x) = A_0 + A_1 x + A_2\, x^2 + A_3\, x^3 + A_4\, x^4 \qquad (4.1)$$

where $A_0 = C_0 - C_1 x_0 + C_2\, x_0 x_1 - C_3\, x_0 x_1 x_2 + C_4\, x_0 x_1 x_2\, x_3$

$= 14625152 - 3416096 \times 0 + 478489 \times 0 \times 1 + 184327.83 \times 0 \times 1 \times 2 + 66584.99 \times 0 \times 1 \times 2 \times 3$

$= 14625152$

$A_1 = C_1 - C_2 (x_0 + x_1) + C_3 (x_0 x_1 + x_1 x_2) - C_4 (x_0 x_1 x_2 + x_0 x_1\, x_3 + x_0 x_2\, x_3 + x_1 x_2\, x_3)$

$= 3416096 - 478489 \times (0 + 1) - 184327.83 \times (0 + 2) - 66584.99 \times (0 + 0 + 0 + 6)$

$= 3416096 - 478489 - 184327.83 \times 2 - 66584.99 \times 6$

$= 3416096 - 478489 - 368655.66 - 399509.94$

$= 3416096 - 1246654.6$

$= 2169441.4$

$A_2 = C_2 - C_3 (x_0 + x_1 + x_2) + C_4 (x_0 x_1 + x_0 x_2 + x_1 x_2 + x_0 x_3 + x_1\, x_3 + x_2\, x_3)$

$= 478489 + 184327.83 \times (0 + 1 + 2) + 66584.99\, (0 + 0 + 2 + 0 + 3 + 6)$

$= 478489 + 184327.83 \times 3 + 66584.99 \times 11$

$= 478489 + 552983.49 + 732434.89$

$= 1763907.38$

$A_3 = C_3 - C_4 (x_0 + x_1 + x_2 + x_3)$

$= -184327.83 - 66584.99\ (0 + 1 + 2 + 3)$

$= -184327.83 - 66584.99 \times 6$

$= -184327.83 - 399509.94$

$= -583837.77$

$A_4 = C_4 = 66584.99$

Thus, the polynomial that can represent the given numerical data is

$\therefore$ **(4.1)** $\Rightarrow$ $\boldsymbol{f(x) = 14625152 + 2169441.4\, x + 1763907.38}\, x^2$
$\boldsymbol{-\, 583837.77}\, x^3 + \boldsymbol{66584.99}\, x^4$

This polynomial yields the values of the function $f(x)$ corresponding to the respective observed values as follows:

$f(0) = 14625152 + 2169441.4 \times 0 + 1763907.38 \times 0 -$
$583837.77 \times 0 + 66584.99 \times 0$
$= 14625152$

$f(1) = 14625152 + 2169441.4 \times 1 + 1763907.38 \times 1 -$
$583837.77 \times 1 + 66584.99 \times 1$

$= 14625152 + 2169441.4 + 1763907.38 -$
$583837.77 + 66584.99$

$= 18625085.77 - 583837.77$
$= 18041248$

$f(2) = 14625152 + 2169441.4 \times 2 + 1763907.38 \times 4 -$
$583837.77 \times 8 + 66584.99 \times 16$

$= 14625152 + 4338882.8 + 7055629.52 -$
$4670702.16 + 1065359.84$

$= 27085024.16 - 4670702.16$
$= 22414322$

$f(3) = 14625152 + 2169441.4 \times 3 + 1763907.38 \times 9 -$
$583837.77 \times 27 + 66584.99 \times 81$
$= 14625152 + 6508324.2 + 15875166.42$

$-15763619.79 + 5393384.19$
$= 42402026.81 - 15763619.79$
$= 26638407$

$f(4) = 14625152 + 2169441.4 \times 4 + 1763907.38 \times 16$

$- 583837.77 \times 64 + 66584.99 \times 256$
$= 14625152 + 8677765.6 + 28222518.08$

$- 37365617.28 + 17045757.44$

$= 68571193.12 - 37365617.28$
$= 31205575.84$
$= 31205576$

Example (ii):

PROBLEM : (Total population of India)

The following table shows the data on total population of India corresponding to the years:

Year	***Total Population***
1971	548159652
1981	683329097
1991	846302688
2001	1027015247
2011	1210193422

Taking 1971 as origin and changing scale by 1/10, one can obtain the following table for independent variable x (representing time) and $f(x)$ (representing total population of India):

Year	x_i	***Total Population***
1971	0	548159652
1981	1	683329097
1991	2	846302688
2001	3	1027015247
2011	4	1210193422

Now here $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$f(x_0) = 548159652$, $f(x_1) = 683329097$,

$f(x_2) = 846302688$, $f(x_3) = 1027015247$,

$f(x_4)$ = 1210193422

Difference Table

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3, x_4)$
0	548159652				
		135169445			
1	683329097		13902073		
		162973591		−1677529.66	
2	846302688		8869484		– 217007.25
		180712559		−2545558.66	
3	1027015247		1232808		
		183178175			
4	1210193422				

Now, $C_0 = f(x_0) = 548159652$

$C_1 = f(x_0, x_1) = 135169445$

$C_2 = f(x_0, x_1, x_2) = 13902073$

$C_3 = f(x_0, x_1, x_2, x_3) = -1677529.66$

$C_4 = f(x_0, x_1, x_2, x_3, x_4) = -217007.25$

∴ The polynomial is

$$f(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 \quad \text{……..(4.2)}$$

Where

$$A_0 = C_0 - C_1 x_0 + C_2 x_0 x_1 - C_3 x_0 x_1 x_2 + C_4 x_0 x_1 x_2 x_3$$

$= 548159652 - 135169445 \times 0 + 13902073 \times 0 \times 1 + 1677529.66 \times 0 \times 1 \times 2 - 217007.25 \times 0 \times 1 \times 2 \times 3$

$= 548159652$

$$A_1 = C_1 - C_2 (x_0 + x_1) + C_3 (x_0 x_1 + x_1 x_2) - C_4 (x_0 x_1 x_2 + x_0 x_1 x_3 + x_0 x_2 x_3 + x_1 x_2 x_3)$$

$= 135169445 - 13902073 \times (0 + 1) - 1677529.66 \times (0 + 2) - (-217007.25)(0 + 0 + 0 + 6)$

$= 135169445 - 13902073 - 1677529.66 \times 2 + 217007.25 \times 6$

$= 135169445 - 13902073 - 3355059.32 + 1302043.5$

$= 136471488.5 - 17257132.32$

$= 119214356.18$

$$A_2 = C_2 - C_3 (x_0 + x_1 + x_2) + C_4 (x_0 x_1 + x_0 x_2 + x_1 x_2 + x_0 x_3 + x_1 x_3 + x_2 x_3)$$

$= 13902073 + 1677529.66 \times (0 + 1 + 2) - 217007.25 \times (0 + 0 + 2 + 0 + 3 + 6)$

$= 13902073 + 1677529.66 \times 3 - 217007.25 \times 11$

$= 13902073 + 5032588.98 - 2387079.75$

$= 18934661.98 - 2387079.75$

$= 16547582.23$

$$A_3 = C_3 - C_4 (x_0 + x_1 + x_2 + x_3)$$

$= -1677529.66 + 217007.25\,(0 + 1 + 2 + 3)$

$= -1677529.66 + 217007.25 \times 6$

$= -1677529.66 + 1302043.5$

$= -375486.16$

$A_4 = C_4 = -217007.25$

Thus, the polynomial that can represent the given numerical data is

∴ (4.2) ⇒ $\boldsymbol{f(x) = 548159652 + 119214356.18\, x + 16547582.23\, x^2 - 375486.16\, x^3 - 217007.25\, x^4}$

which is the required polynomial

This polynomial yields the values of the function $f(x)$ corresponding to the respective observed values as follows

$f(0) = 548159652 + 119214356.18 \times 0 + 16547582.23 \times 0 - 375486.16 \times 0 - 217007.25 \times 0$

$= 548159652$

$f(1) = 548159652 + 119214356.18 \times 1 + 16547582.23 \times 1 - 375486.16 \times 1 - 217007.25 \times 1$

$= 548159652 + 119214356.18 + 16547582.23$

$- 375486.16 \ - 217007.25$

$= 683921590.41 - 592493.41$

$= 683329097$

$f(2) = 548159652 + 119214356.18 \times 2 + 16547582.23 \times 4 - 375486.16 \times 8 - 217007.25 \times 16$

$= 548159652 + 238428712.36 + 66190328.92 - 3003889.28 - 34\,72116$

$= 852778693.28 - 6476005.28$

$= 846302688$

$f(3) = 548159652 + 119214356.18 \times 3 + 16547582.23 \times 9 - 375486.16 \times 27 - 217007.25 \times 81$

$= 548159652 + 357643068.54 + 148928240.07 - 10138126.32 - 17577587.25$

$= 1054730960.61 - 27715713.57$

$= 1027015247$

$f(4) = 548159652 + 119214356.18 \times 4 + 16547582.23 \times 16 - 375486.16 \times 64 - 217007.25 \times 256$

$= 548159652 + 476857424.72 + 264761315.68 - 24031114.24 - 55553856$

$= 1289778392.4 - 79584970.24$

$= \ 1210193422.16$

$= 1210193422$

5. Conclusion:

The formula described by equation (3.2) can be used to represent a given set of numerical data on a pair of variables, by a polynomial.

The degree of the polynomial is one less than the number of pairs of observations.

The polynomial that represents the given set of numerical data can be used for interpolation at any position of the independent variable lying within its two extreme values.

The approach of interpolation, described here, can be suitably applied in inverse interpolation also.

Newton's forward interpolation formula is valid for estimating the value of the dependent variable under the following two conditions:

(i) The given values of the independent variable are at equal interval.

(ii) The value of the independent variable corresponding to which the value of the dependent variable is to be estimated lies in the first half of the series of the given values of the independent variable.

However, Newton's divided difference interpolation formula is valid for estimating the value of the dependent variable beyond these two conditions. Therefore, the formula derived here is valid for representing a set of numerical data on a pair of variables by a polynomial beyond these two conditions.

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