

# Common fixed point theorem in intuitionistic fuzzy metric space using an implicit relation

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**Abstract.** The aim of this paper is to prove some common fixed point theorems for weakly compatible mappings in intuitionistic fuzzy metric spaces satisfying an integral type implicit relation using property (E.A) which generalize results of Aliouche [3], Aliouche and Popa [4], Pathak et al. [11], Badshah and Pariya [5], Imdad et al. [7], Manro et al. [9] and references mentioned therein from metric spaces, fuzzy metric spaces to intuitionistic fuzzy metric spaces.

**Keywords -** Intuitionistic fuzzy metric space, weakly compatible mapping, implicit relation.

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**1. Introduction.** Several authors proved common fixed point theorems for contractive type mappings in recent years. Branciari [6] gave a fixed point result for a single mapping satisfying Banach's contraction principle for an integral type inequality. Since proving fixed point theorems using an implicit relation is a good idea because it covers several contractive conditions rather than one contractive condition, recently, Imdad et al [7], Pathak et al. [11], Aliouche [3], Aliouche and Popa [4], Badshah and Pariya [5], proved many fixed point theorems satisfying implicit relation in metric and symmetric spaces.

## 2. Basic Definitions and Preliminaries.

**Definition 2.1[2]** A binary operation  $*$ :  $[0,1] \times [0,1] \rightarrow [0,1]$  is called a *t-norm*  $*$  satisfies the following conditions:

- i.  $*$  is continuous,
- ii.  $*$  is commutative and associative,
- iii.  $a * 1 = a$  for all  $a \in [0, 1]$ ,
- iv.  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0,1]$ .

Examples of *t-norm* -  $a * b = ab$  and  $a * b = \min\{a, b\}$ .

**Definition 2.2[2]** A binary operation  $\diamond$ :  $[0,1] \times [0,1] \rightarrow [0,1]$  is said to be continuous *t-conorm* if it satisfied the following conditions:

- i.  $\diamond$  is associative and commutative,
- ii.  $a \diamond 0 = a$  for all  $a \in [0,1]$ ,
- iii.  $\diamond$  is continuous,
- iv.  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for each  $a, b, c, d \in [0,1]$

Examples of *t-conorm* -  $a \diamond b = \min(a+b, 1)$  and  $a \diamond b = \max(a, b)$

**Remark 2.1 [2]** The concept of triangular norms (*t-norm*) and triangular conorms (*t-conorm*) are known as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

**Definition 2.3 [2]** A 5- tuple  $(X, M, N, *, \diamond)$  is called intuitionistic fuzzy metric space if  $X$  is an arbitrary non empty set,  $*$  is a continuous *t-norm*,  $\diamond$  continuous *t-conorm* and  $M, N$  are fuzzy sets on  $X^2 \times [0, \infty]$  satisfying the following conditions:

For each  $x, y, z, \in X$  and  $t, s > 0$

$$(IFM-1) \quad M(x, y, t) + N(x, y, t) \leq 1,$$

$$(IFM-2) \quad M(x, y, 0) = 0, \text{ for all } x, y \text{ in } X,$$

$$(IFM-3) \quad M(x, y, t) = 1 \text{ for all } x, y \text{ in } X \text{ and } t > 0 \text{ if and only if } x=y,$$

$$(IFM-4) \quad M(x, y, t) = M(y, x, t), \text{ for all } x, y \text{ in } X \text{ and } t > 0,$$

$$(IFM-5) \quad M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),$$

$$(IFM-6) \quad M(x, y, .): [0, \infty] \rightarrow [0, 1] \text{ is left continuous,}$$

$$(IFM-7) \quad \lim_{t \rightarrow \infty} M(x, y, t) = 1,$$

$$(IFM-8) \quad N(x, y, 0) = 1, \text{ for all } x, y \text{ in } X,$$

$$(IFM-9) \quad N(x, y, t) = 0, \text{ for all } x, y \text{ in } X \text{ and } t > 0 \text{ if and only if } x = y,$$

$$(IFM-10) \quad N(x, y, t) = N(y, x, t), \text{ for all } x, y \text{ in } X \text{ and } t > 0,$$

$$(IFM-11) \quad N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s),$$

$$(IFM-12) \quad N(x, y, .): [0, \infty] \rightarrow [0, 1] \text{ is right continuous,}$$

$$(IFM-13) \quad \lim_{t \rightarrow \infty} N(x, y, t) = 0, \text{ for all } x, y \text{ in } X \text{ and } t > 0.$$

Then  $(M, N)$  is called an intuitionistic fuzzy metric on  $X$ . The function  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and degree of non nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

**Example 2.1. [10]** Let  $(X, d)$  be a metric space. Define  $a * b = ab$  and  $a \diamond b = \min\{1, a+b\}$ , for all  $a, b \in [0, 1]$  and let  $M$  and  $N$  be fuzzy sets on  $X^2 \times (0, \infty)$  defined as follows:

$$M(x, y, t) = \frac{t}{t+d(x,y)} \text{ and } N(x, y, t) = \frac{d(x,y)}{t+d(x,y)} \quad \text{for all } x, y \in X \text{ and all } t > 0.$$

then  $(M, N)$  is called an intuitionistic fuzzy metric space on  $X$ . We call this intuitionistic fuzzy metric induced by a metric  $d$  the standard intuitionistic fuzzy metric.

**Remark 2.3.** Note that the above examples holds even with the t- norm  $a * b = \min\{a, b\}$  and t- conorm  $a \diamond b = \max\{a, b\}$  and hence  $(M, N)$  is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

**Lemma 2.1.[12]** Let  $(X, M, N, *, \diamond)$  Intuitionistic fuzzy metric space, If there exists  $k \in (0, 1)$  such that for all  $x, y \in X$ ,  $M(x, y, kt) \geq M(x, y, t)$  and  $N(x, y, kt) \leq N(x, y, t)$

for all  $t > 0$ , then  $x = y$ .

**Definition 2.4[2].** A sequence  $\{x_n\}$  in intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be:

(i) Cauchy sequence if for each  $t > 0, p > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0.$$

(ii) Convergent to a point  $x \in X$  if for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0.$$

An Intuitionistic Fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be complete if every Cauchy sequence in it converges to a point in it.

**Definition 2.5.[2]** A pair of self mapping  $(A, S)$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be commuting if  $M(ASx, Sx, t) = 1$  and  $N(ASx, Sx, t) = 0$  for all  $x \in X$ .

**Definition 2.6.[2]** A pair of self mapping  $(A, S)$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be weakly commuting if

$$M(ASx, Sx, t) \geq M(Ax, Sx, t) \text{ and } N(ASx, Sx, t) \leq N(Ax, Sx, t) \\ \text{for all } x \in X \text{ and } t > 0.$$

**Definition 2.7.[1]** A pair of self mapping  $(A, S)$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be compatible if  $\lim_{n \rightarrow \infty} M(ASx_n, Sx_n, t) = 1$  and  $\lim_{n \rightarrow \infty} N(ASx_n, Sx_n, t) = 0$  for all  $t > 0$ . Whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = u$  for some  $u \in X$ .

**Definition 2.8.[12]** A pair of self mapping  $(A, S)$  of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be point wise R-weakly commuting if given  $x \in X$ , there exists  $R > 0$  such that for all  $t > 0$

$$M(ASx, Sx, t) \geq M\left(Ax, Sx, \frac{t}{R}\right) \text{ and}$$

$$N(ASx, Sx, t) \leq N\left(Ax, Sx, \frac{t}{R}\right).$$

Clearly every pair of weakly commuting point wise R-weakly commuting with  $R=1$ .

**Definition 2.9.[9]** A pair  $(A, S)$  of self-mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to satisfy the *EA* like property if there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$  for some  $z \in A(X)$  or  $z \in S(X)$ .

**Definition 2.9.[9]** Two pairs  $(A, S)$  and  $(B, T)$  of self-mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  are said to satisfy the common (*EA*) like property if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z$  for some  $z \in S(X) \cap T(X)$  or  $z \in A(X) \cap B(X)$ .

**Definition 2.10.[8]** A pair  $(f, g)$  of self-mappings of a metric space  $(X, d)$  is said to be **weakly compatible mappings** if the mappings commute at all of their coincidence points, i.e.,  $fx = gx$  for some  $x \in X$  implies  $fgx = gfx$ .

It is also well known that pointwise  $R$ -weak commutativity is equivalent to commutativity at coincidence points and in the setting of metric spaces this notion is equivalent to weak compatibility.

### Implicit Relation

Let  $M_6$  denotes the set of all real valued continuous function  $\phi$  and  $\psi: [0,1]^6 \rightarrow \mathbb{R}$  satisfying the following conditions:

- (A)  $\int_0^{\phi(u,1,u,1,1,u)} \phi(t)dt < 0$  implies  $u \geq 1$
- (B)  $\int_0^{\phi(u,1,1,u,u,1)} \phi(t)dt < 0$  implies  $u \geq 1$
- (C)  $\int_0^{\phi(u,u,1,1,u,u)} \phi(t)dt < 0$  implies  $u \geq 1$
- (D)  $\int_0^{\psi(v,0,v,0,0,v)} \psi(t)dt > 0$  implies  $v \leq 0$
- (E)  $\int_0^{\psi(v,0,0,v,v,0)} \psi(t)dt > 0$  implies  $v \leq 0$
- (F)  $\int_0^{\psi(v,v,0,0,v,v)} \psi(t)dt > 0$  implies  $v \leq 0$

**Example 2.2[9]** – Define  $\phi, \psi: [0,1]^6 \rightarrow \mathbb{R}$  as  $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \phi_1(\min\{t_2, t_3, t_4, t_5, t_6\})$  where  $\phi_1: [0,1] \rightarrow [0,1]$  is a continuous increasing function such that  $\phi_1(s) > s$  for all  $s \in (0,1)$  and  $\psi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi_1(\max\{t_2, t_3, t_4, t_5, t_6\})$  where  $\psi_1: [0,1] \rightarrow [0,1]$  is a continuous decreasing function such that  $\psi_1(k) < k$  for all  $k \in (0,1)$

Clearly  $\phi$  and  $\psi$  satisfy conditions (A) to (F). Therefore  $\phi, \psi \in M_6$ .

### 3. Main Results

**Theorem 3.1.** Let  $A, B, S$  and  $T$  be self mappings of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the following conditions:

$$A(X) \subset T(X) \text{ and } B(X) \subset S(X) \quad (3.1)$$

$$\int_0^{\phi(M(Ax,By,t), M(Sx,Ty,t), M(Ax,Sx,t), M(By,Ty,t), M(Sx,By,t), M(Ax,Ty,t))} \phi(t)dt \geq 0$$

and

$$\int_0^{\psi(N(Ax,By,t), N(Sx,Ty,t), N(Ax,Sx,t), N(By,Ty,t), N(Sx,By,t), N(Ax,Ty,t))} \psi(t)dt \leq 0 \quad (3.2)$$

for all  $x, y$  in  $X$  and  $\phi, \psi \in M_6$ , where  $\varphi: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is a Lebesgue – integrable mapping which is summable, nonnegative and such that

$$\int_0^\varepsilon \varphi(t)dt > 0 \text{ for each } \varepsilon > 0.$$

Suppose that (A,S) or (B,T) satisfies property (E.A) and the pairs (A,S) and (B,T) are weakly compatible. If one of the  $A(X)$ ,  $B(X)$ ,  $S(X)$ , and  $T(X)$  is closed subset of  $X$ , then  $A$ ,  $B$ ,  $S$  and  $T$  have a common unique fixed point in  $X$ .

**Proof:** Suppose that (B, T) satisfies property (E.A), then there exists a sequence  $\{x_n\}$  in  $X$  such that

$\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Tx_n = z$  for some  $z \in X$ , therefore, we have  $\lim_{n \rightarrow \infty} M(Bx_n,$

$Tx_n, t) = 1$  and  $\lim_{n \rightarrow \infty} N(Bx_n, Tx_n, t) = 0$ . Since  $B(X) \subset S(X)$ , there exists in  $X$  a sequence  $\{y_n\}$  such that

$Bx_n = Sy_n$ . Hence  $\lim_{n \rightarrow \infty} Sy_n = z$ . Let us show that  $\lim_{n \rightarrow \infty} Ay_n = z$ . Indeed, in view of (3.2), we have

$$\int_0^{\phi(M(Ay_n, Bx_n, t), M(Sy_n, Tx_n, t), M(Ay_n, Sy_n, t), M(Bx_n, Tx_n, t), M(Sy_n, Bx_n, t), M(Ay_n, Tx_n, t))} \varphi(t) dt \geq 0$$

$$\int_0^{\phi(M(Ay_n, Bx_n, t), M(Bx_n, Tx_n, t), M(Ay_n, Bx_n, t), M(Bx_n, Tx_n, t), 1, M(Ay_n, Tx_n, t))} \varphi(t) dt \geq 0$$

Suppose that  $\lim_{n \rightarrow \infty} \sup M(Ay_n, Bx_n, t) = l$ . Taking limit  $n \rightarrow \infty$  we get

$$\int_0^{\phi(l, l, l, l, l, l)} \varphi(t) dt \geq 0$$

a contradiction of (A). So  $l = 1$

and

$$\begin{aligned} \int_0^{\psi(N(Ay_n, Bx_n, t), N(Sy_n, Tx_n, t), N(Ay_n, Sy_n, t), N(Bx_n, Tx_n, t), N(Sy_n, Bx_n, t), N(Ay_n, Tx_n, t))} \varphi(t) dt \\ = \int_0^{\psi(N(Ay_n, Bx_n, t), N(Bx_n, Tx_n, t), N(Ay_n, Bx_n, t), N(Bx_n, Tx_n, t), 0, N(Ay_n, Tx_n, t))} \varphi(t) dt \leq 0 \end{aligned}$$

Suppose that  $\lim_{n \rightarrow \infty} \inf N(Ay_n, Bx_n, t) = u$ . Taking limit  $n \rightarrow \infty$  we get

$$\int_0^{\psi(u, 0, u, 0, 0, u)} \varphi(t) dt \leq 0$$

a contradiction of (D). So  $u = 0$  i.e.  $\lim_{n \rightarrow \infty} Ay_n = z$ .

Suppose  $S(X)$  is a closed subspace of  $X$ . then  $z = Su$  for some  $u \in X$ . If  $z \neq Au$

Using (3.2) we obtain

$$\int_0^{\phi(M(Au, Bx_n, t), M(Su, Tx_n, t), M(Au, Su, t), M(Bx_n, Tx_n, t), M(Su, Bx_n, t), M(Au, Tx_n, t))} \varphi(t) dt \geq 0$$

Letting limit  $n \rightarrow \infty$  we have

$$\int_0^{\phi(M(Au, z, t), 1, M(Au, z, t), 1, 1, M(Au, z, t))} \varphi(t) dt \geq 0$$

which is contradiction of (A) and

$$\int_0^{\psi(N(Au, Bx_n, t), N(Su, Tx_n, t), N(Au, Su, t), N(Bx_n, Tx_n, t), N(Su, Bx_n, t), N(Au, Tx_n, t))} \varphi(t) dt \leq 0$$

Letting limit  $n \rightarrow \infty$  we have

$$\int_0^{\psi(N(Au, z, t), 0, N(Au, z, t), 0, 0, N(Au, z, t))} \varphi(t) dt \leq 0$$

which is contradiction of (D). Hence  $z = Au = Su$ .

Since  $A(X) \subset T(X)$ , there exists  $v \in X$  such that  $z = Au = Tv$ . If  $z \neq Bv$

Using (3.2) we obtain

$$\int_0^{\infty} \phi(M(Au, Bv, t), M(Su, Tv, t), M(Au, Su, t), M(Bv, Tv, t), M(Su, Bv, t), M(Au, Tv, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \phi(M(z, Bv, t), 1, 1, M(z, Bv, t), M(Au, z, t), 1) \varphi(t) dt \geq 0$$

which is contradiction of (B) and

$$\int_0^{\infty} \psi(M(Au, Bv, t), M(Su, Tv, t), M(Au, Su, t), M(Bv, Tv, t), M(Su, Bv, t), M(Au, Tv, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \psi(M(z, Bv, t), 0, 0, M(z, Bv, t), M(Au, z, t), 0) \varphi(t) dt \leq 0$$

which is contradiction of (E). and therefore  $Au = Su = z = Bv = Tv$ .

Since the pair (A,S) is weakly compatible , we have  $ASu = SAu$  i. e.  $Az = Sz$ .

If  $Az \neq z$ , using (3.2) we obtain

$$\int_0^{\infty} \phi(M(Az, Bv, t), M(Sz, Tv, t), M(Az, Sz, t), M(Bv, Tv, t), M(Sz, Bv, t), M(Az, Tv, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \phi(M(Az, z, t), M(Az, z, t), 1, 1, M(Az, z, t), M(Az, z, t)) \varphi(t) dt \geq 0$$

which is contradiction of (C) and

$$\int_0^{\infty} \psi(N(Az, Bv, t), N(Sz, Tv, t), N(Az, Sz, t), N(Bv, Tv, t), N(Sz, Bv, t), N(Az, Tv, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \psi(N(Az, z, t), N(Az, z, t), 1, 1, N(Az, z, t), N(Az, z, t)) \varphi(t) dt \leq 0$$

which is contradiction of (F). Then  $Az = Sz = z$ .

Since (B,T) is weakly compatible we have  $BTv = TBv$  i. e.  $Bz = Tz$ .

If  $Bz \neq z$ , using (3.2) we obtain

$$\int_0^{\infty} \phi(M(Az, Bz, t), M(Sz, Tz, t), M(Az, Sz, t), M(Bz, Tz, t), M(Sz, Bz, t), M(Az, Tz, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \phi(M(z, Bz, t), M(z, Bz, t), 1, 1, M(z, Bz, t), M(z, Bz, t)) \varphi(t) dt \geq 0$$

which is contradiction of (C) and

$$\int_0^{\infty} \psi(N(Az, Bz, t), N(Sz, Tz, t), N(Az, Sz, t), N(Bz, Tz, t), N(Sz, Bz, t), N(Az, Tz, t)) \varphi(t) dt =$$

$$\int_0^{\infty} \psi(N(z, Bz, t), N(z, Bz, t), 0, 0, N(z, Bz, t), N(z, Bz, t)) \varphi(t) dt \leq 0$$

which is contradiction of (F). Then  $z = Bz = Tz = Az = Sz$ .

Therefore z is a common fixed point of A, B, S and T.

To prove uniqueness of z let w be another common fixed point of A, B, S and T then

using (3.2) we obtain

$$\int_0^{\infty} \phi(M(Az, Bw, t), M(Sz, Tw, t), M(Az, Sz, t), M(Bw, Tw, t), M(Sz, Bw, t), M(Az, Tw, t)) \varphi(t) dt =$$

$$\int_0^{\phi(M(z,w,t),M(z,w,t),1,1,M(z,w,t),M(z,w,t))} \varphi(t) dt \geq 0$$

which is contradiction of (C) and

$$\int_0^{\Psi(N(Az,Bw,t),N(Sz,Tw,t),N(Az,Sz,t),N(Bw,Tw,t),N(Sz,Bw,t),N(Az,Tw,t))} \varphi(t) dt =$$

$$\int_0^{\Psi(N(z,w,t),N(z,w,t),0,0,N(z,w,t),N(z,w,t))} \varphi(t) dt \leq 0$$

which is contradiction of (F). Then  $z = w$ .

Hence  $z$  is common fixed point of  $A, B, S$  and  $T$ .

**Corollary 3.1.** Let  $A, B, S$  and  $T$  be self mappings of a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the following conditions:

$$A(X) \subset T(X) \text{ and } B(X) \subset S(X) \quad (3.1)$$

$$\phi(M(Ax, By, t), M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Sx, By, t), M(Ax, Ty, t)) \geq 0$$

and

$$\psi(N(Ax, By, t), N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), N(Sx, By, t), N(Ax, Ty, t)) \leq 0 \quad (3.2)$$

for all  $x, y$  in  $X$  and  $\phi, \psi \in M_6$ . Suppose that  $(A, S)$  or  $(B, T)$  satisfies property (E.A) and the pairs  $(A, S)$  and  $(B, T)$  are weakly compatible. If one of the  $A(X), B(X), S(X)$ , and  $T(X)$  is closed subset of  $X$ , then  $A, B, S$  and  $T$  have a common unique fixed point in  $X$ .

**Proof.** If we put  $\varphi(t) = 1$  in theorem 3.1, the result follows.

**Remark.** Our main result in theorem 3.1, improve and generalization of several known results of Aliouche [3], Aliouche and Popa[4], Pathak et al [10], Badshah and pariya [5], Imdad et al [7], Manro et al[8] and references mentioned therein from metric spaces, fuzzy metric spaces to intuitionistic fuzzy metric spaces.

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