# Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space via Compatible Mappings of Type (K)

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***Abstract:*** *In this article, we establish the concept of common fixed point theorem in intuitionistic fuzzy metric space via compatible mappings of type (K) with example. Our result generalizes and improves other similar results.*

**Keywords:** *Fixed point theorem, Intuitionistic fuzzy metric space, weakly compatible, compatible mappings of type (K)*

**1. Introduction:**

Fixed point theory is a central area of functional analysis. L.A Zade In 1965[11]. particularly vigorous field of research of mappings fulfilling contractive type of common fixed point conditions. The concept of fuzzy set various authors have obtained fixed point theorems in fuzzy metric space using these compatible ideas. Then fuzzy metric spaces been introduced by Kramosil and Michalek [13]. George and Veeramani [1] made to order the notion of fuzzy metric spaces among the help of continuous t-norms. In 1986, G. Jungck [5] introduced notion of compatible mappings. In 1993 overview of compatible mappings called compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions by G. Jungck, P. P. Murthy and Y. J. Cho [6]. Introduced the concept of compatible mappings in fuzzy metric space. Recently introduced the concept of compatible mappings of type (K) in metric space by , Jha et al. [10]and shows that the compatible mapping of type (K) is independent with other compatible. Also, Manandhar et al. [9] introduced compatible mapping of type (K) in fuzzy metric space. Many The reason of this paper is to create a common fixed point theorem for compatible mappings of type (K) in intuitionistic fuzzy metric spaces with example.

**2. Preliminaries**

**Definition 2.1.** [12] Let X is a any non empty set. A fuzzy set A in X is a function with domain X and values in [0,1].

**Definition 2.2.** [1] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $*$ is Satisfying the following conditions:

(a) $*$ is commutative and associative;

(b) $*$ is continuous;

(c) $a * 1 = a$ for all $a \in [0, 1]$;

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

**Definition 2.3.** [2] A binary operation $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions:

(a) $\Diamond$ is commutative and associative;

(b) $\Diamond$ is continuous;

(c) $a \lozenge 0 = a$ for all $a \in [0, 1]$;

(d) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

**Definition 2.4.** [3] A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, *is a continuous t-norm ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t>0$;

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$;

(IFM-2) $M(x, y, 0) = 0$;

(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$;

(IFM-4) $M(x, y, t) = M(y, x, t)$;

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(IFM-6) $M(x, y, .): [0, \infty) \to [0, 1]$ is left continuous;

(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$

(IFM-8) $N(x, y, 0) = 1$;

(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$;

(IFM-10) $N(x, y, t) = N(y, x, t)$;

(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ ;

(IFM-12) $N(x, y, .): [0, \infty) \to [0, 1]$ is right continuous;

(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N(x, y, t) denote the degree of closeness and degree of non-nearness between x and y with respect to t, respectively.

**remark 2.5.**[ 12] Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space if X of the form (X, M, 1 - M, *, ◊) such that t- norm * and t-conorm ◊ are associated, that is, $x \lozenge y = 1-((1 - x) * (1 - y))$ for any $x, y \in X$. But the converse is not true.

**Example 2.6.** [14] Let (X, d) be a metric space. Denote $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let $M_d$ and $N_d$ be fuzzy sets on $X^2 \times (0, \lozenge)$ defined as follows;

$$M_d(x,y,t) = \frac{t}{t+d(x,y)};\ N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then $(M_d, N_d)$ is an intuitionistic fuzzy metric on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

**Remark 2.7.** Note the above example holds even with the t-norm $a * b = \min \{a, b\}$ and the t-conorm $a \lozenge b = \max\{a, b\}$ and hence $(M_d, N_d)$ is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t- conorm.

**Definition 2.8.** [3] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space.A sequence $\{x_n\}$ in X is called

(i) cauchy sequence if for each $t > 0$ and $P > 0$, $\lim_{n\to\infty} M(x_{n+p},x_n, t) = 1$and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(ii) convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each $t >0$.

(iii). complete if every Cauchy sequence is convergent for intuitionistic fuzzy metric space .

**Definition 2.9** [4] The self mappings A and S is called weakly compatible mapping if they commutative in their coincident point.

**Definition 2.10.**[10] The self mappings A and S of a fuzzy metric space (X, M, ∗) are said to be compatible of type (E) iff $\lim_{n\to\infty} M(AAx_n, ASx_n, t) = 1$, $\lim_{n\to\infty} M(AAx_n, Sx, t) = 1$, $\lim_{n\to\infty} M(ASx_n, Sx, t) = 1$ and $\lim_{n\to\infty} M(SSx_n, SAx_n, t) = 1$, $\lim_{n\to\infty} M(SSx_n, Ax, t) = 1$,
$\lim_{n\to\infty} M(SSx_n, Ax, t)=1$.

**Definition 2.11.**[10] The self mappings A and S of a metric space (X, d) are said to be compatible of type (K) iff $\lim_{n\to\infty} AAx_n = Sx$ and $\lim_{n\to\infty} SSx_n = Ax$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X.

**Definition 2.12** The self mappings A and S of a intuitionistic fuzzy metric space (X,M,N,*,◊) are said to be compatible of type (K) iff $\lim_{n\to\infty} M(AAx_n, Sx, t) = 1$ and $\lim_{n\to\infty} M(SSx_n, Ax, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X and $t > 0$.

**Lemma 2.13.** [8] In an intuitionistic fuzzy metric space X, M(x, y, .) is non-decreasing and N (x,y,.) is non increasing for all x, y ∈X.

**Lemma 2.14.** [16] Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ (0, 1) such that,

(i) $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$,

(ii) $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

for every t > 0 and n =1, 2,... then $\{y_n\}$ is a Cauchy sequence in X .

**(ii).** $M(x, y, kt) \geq M(x, y, t)$, $N(x, y, kt) \leq N(x, y, t)$, for all x, y ∈X. Then x = y.

**Proposition 2.16.** If A and S be compatible mappings of type (K) on a intuitionistic fuzzy metric space (X, M, N, *, ◊) and if one of function is continuous .Then, we have

(a) A (x) = S(x) where $\lim_{n\to\infty} Ax_n = x$ $\lim_{n\to\infty} Sx_n = x$, for some point x ∈ X, and sequence $\{x_n\}$,

(b) If these exist u ∈ X such that Au = Su = x then, ASu = SAu.

**3.MAIN THEOREM:**

**Theorem 3.1 :** If A, B, S and are self mappings on a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) satisfying the condition :

(1) $A(X) \subseteq T(X)$; $B(X) \subseteq S(X)$

(2) There exists q ∈ (0, 1) such that for every x, y ∈ x and t > 0.

$M(Ax, By, qt) \geq \min\{Ty, By, t), M(Sx, Ax, t), M(Sx, Ty, t)\}$

$N(Ax, By, qt) \leq \max\{Ty, By, t), N(Sx, Ax, t), N(Sx, Ty, t)\}$ (3.1)

(3) B and T weakly compatible mappings if the pair of mappings (A, S) is compatible of type (k) and one of the mappings continuous then A, B, S and T have a unique common fixed point in X.

**Proof** : We define a sequence $\{x_n\}$ such that

$Ax_{2n+1} = Sx_{2n}$ and $Ax_{2n} = Tx_{2n+1}$, n = 1, 2, …

We shall prove that $\{Ax_{n+1}\}$ is a Cauchy sequence. For this suppose $x = x_{2n+1}$ and $y = x_{2n+2}$ in (3.1), we write

$$\begin{aligned} M(Ax_{2n+1}, Bx_{2n+2}, qt) \\ &\geq \min\{M(Tx_{2n+2}, Bx_{2n+2}, t), M(Sx_{2n+1}, Ax_{2n+1}, t), M(Sx_{2n+1}, Tx_{2n+2}, t)\} \\ &\geq \min\{M(Ax_{2n+1}, Bx_{2n+2}, t), M(Ax_{2n+2}, Ax_{2n+1}, t), M(Ax_{2n+2}, Ax_{2n+1}, t)\} \\ &\geq \min\{M(Ax_{2n}, Bx_{2n+1}, t/q), M(Ax_{2n+1}, Ax_{2n}, t/q), M(Ax_{2n+1}, Ax_{2n}, t/q)\} \end{aligned}$$

$$N\left(Ax_{2n+1}, Bx_{2n+2}, qt\right)$$
$$\leq \max\left\{N\left(Tx_{2n+2}, Bx_{2n+2}, t\right), N\left(Sx_{2n+1}, Ax_{2n+1}, t\right), N\left(Sx_{2n+1}, Tx_{2n+2}, t\right)\right\}$$
$$\leq \max\left\{N\left(Ax_{2n+1}, Bx_{2n+2}, t\right), N\left(Ax_{2n+2}, Ax_{2n+1}, t\right), N\left(Ax_{2n+2}, Ax_{2n+1}, t\right)\right\}$$
$$\leq \max\left\{N\left(Ax_{2n}, Bx_{2n+1}, t/q\right), N\left(Ax_{2n+1}, Ax_{2n}, t/q\right), N\left(Ax_{2n+1}, Ax_{2n}, t/q\right)\right\}$$

Therefore

$$M\left(Ax_{2n+1}, Bx_{2n+2}, qt\right) \geq M\left(Ax_{2n}, Ax_{2n+1}, t/q\right)$$
$$N\left(Ax_{2n+1}, Bx_{2n+2}, qt\right) \leq N\left(Ax_{2n}, Ax_{2n+1}, t/q\right)$$

By induction

$$M\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \geq M\left(Ax_{2m+1}, Ax_{2k}, t/q\right)$$
$$N\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \leq N\left(Ax_{2m+1}, Ax_{2k}, t/q\right)$$

For every k and m in N. Further if 2m + 1 > 2k, then

$$M\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \geq M\left(Ax_{2k}, Ax_{2m+1}, t/q\right) \ldots\ldots \geq M\left(Ax_{1}, Ax_{2m+2}, t/q^{2k}\right)$$
$$N\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \leq N\left(Ax_{2k}, Ax_{2m+1}, t/q\right) \ldots\ldots \leq N\left(Ax_{1}, Ax_{2m+2}, t/q^{2k}\right)$$

(3.2)

If 2k > 2m +1, then

$$M\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \geq M\left(Ax_{2k}, Bx_{2m+1}, t/q\right) \ldots\ldots \geq M\left(Ax_{2k+1(2m+2)}, Bx_{1}, t/q^{2m+2}\right)$$
$$N\left(Ax_{2k+1}, Bx_{2m+2}, qt\right) \leq N\left(Ax_{2k}, Bx_{2m+1}, t/q\right) \ldots\ldots \leq N\left(Ax_{2k+1(2m+2)}, Bx_{1}, t/q^{2m+2}\right)$$

(3.3)

By simple induction with (3.2) and (3.3) we have

$$M\left(Ax_{n+1}, Bx_{n+p}, qt\right) \geq M\left(Ax_{1}, Bx_{p+1}, t/q^{n}\right)$$
$$N\left(Ax_{n+1}, Bx_{n+p}, qt\right) \leq N\left(Ax_{1}, Bx_{p+1}, t/q^{n}\right)$$

For n = 2k, p = 2m +1 or n = 2k + 1, p = 2m + 1 and by (FM-4)

$$M\left(Ax_{n+1}, Bx_{n+p}, qt\right) \geq M\left(Ax_{1}, Bx_{2}, t/2q^{n}\right) * M\left(Ax_{2}, Bx_{p}, t/2q^{n}\right)$$
$$N\left(Ax_{n+1}, Bx_{n+p}, qt\right) \leq N\left(Ax_{1}, Bx_{2}, t/2q^{n}\right) * N\left(Ax_{2}, Bx_{p}, t/2q^{n}\right) \qquad (3.4)$$

If n = 2k, p = 2m or n = 2k + 1, p = 2m.

For every positive integer p and n in N, by noting that

$$M\left(Ax_{1}, Bx_{p}, t/q^{n}\right) \to 1 \text{ as } n \to \infty$$
$$N\left(Ax_{1}, Bx_{p}, t/q^{n}\right) \to 1 \text{ as } n \to \infty$$

Thus $\{Ax_n\}$ is a Cauchy sequence. Since the space x is complete there exists

$$z = \lim_{n\to\infty} Ax_{n} \text{ and } z = \lim_{n\to\infty} Sx_{2n-1} = \lim_{n\to\infty} Tx_{2n}$$

If follows that Az = Sz = Tz and

$$M\left(Az,B^2z,qt\right)\geq \min\{M(TBz,ABz,t),M(Sz,Bz,t),M(Sz,TBz,t)\}$$
$$N\left(Az,B^2z,qt\right)\leq \max\{N(TBz,ABz,t),N(Sz,Bz,t),N(Sz,TBz,t)\}$$

Therefore

$$\begin{aligned}M\left(Az,A^2z,qt\right)&\geq M(Sz,TAz,t)\\&\geq M(Sz,ATz,t)\\&\geq M\left(Az,B^2z,t\right)\\&\ldots\ldots\ldots\ldots\\&\geq M\left(Az,B^2z,t/q^n\right)\end{aligned}$$

$$\begin{aligned}N\left(Az,A^2z,qt\right)&\leq N(Sz,TAz,t)\\&\leq N(Sz,ATz,t)\\&\leq N\left(Az,B^2z,t\right)\\&\ldots\ldots\ldots\ldots\\&\leq N\left(Az,B^2z,t/q^n\right)\end{aligned}$$

Since $\lim_{n\to\infty} M\left(Az,B^2z,t/q^n\right)=1$, $\lim_{n\to\infty} N\left(Az,B^2z,t/q^n\right)=1$, so Az = $A^2z$.

Thus z is a common fixed point of A, S and T.

For uniqueness, let w(w $\neq$ z) be another common fixed point of S, t and A. By (3.1) we write

$$M(Az,Aw,qt)\geq \min\{M(Tw,Aw,t),M(Sz,Az,t),M(Sz,Tw,t)\}$$
$$N(Az,Aw,qt)\leq \max\{N(Tw,Aw,t),N(Sz,Az,t),N(Sz,Tw,t)\}$$

Which implies that

$$M(z,w,qt)\geq M(z,w,t)$$
$$N(z,w,qt)\leq N(z,w,t)$$

Therefore by lemma 2.12, we write z = w.

This completes the proof of theorem 3.1.

Now we prove theorem 1 for fuzzy 2-metric space. We prove the following :

**COROLLARY 3.2:** Let (X, M, *) be a complete fuzzy 2-metric space and let S and t be continuous mappings of X in X, then S and T have a common fixed point in X if there X exists continuous mapping A of X into S(X) ∩ T(X) which commute with s and T and

$$M(Ax,Ay,a,qt)\geq \min\{M(Ty,Ay,a,t),M(Sx,Ax,a,t),M(Sx,Ty,a,t)\}$$
$$N(Ax,Ay,a,qt)\leq \max\{N(Ty,Ay,a,t),N(Sx,Ax,a,t),N(Sx,Ty,a,t)\} \quad (3.1)$$

For all x, yu, a in X, t > 0 an d0 < q < 1,

$$\lim_{n\to\infty} M(x,y,z,t)=1,\ \lim_{n\to\infty} N(x,y,z,t)=1 \text{ for all x, y, z in X.}$$

Then S, T and A have a unique common fixed point.

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