Infinite integrals and generalized multivariable Aleph-function

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
The aim of this paper is to introduce a generalization of multivariable Aleph-function by means of a multiple Mellin-Barnes contour integral and then establish integrals which are products of Aleph-function and generalized multivariable Aleph-function with the help of Mellin transform. The generalized Aleph-function of several variables contains all the functions of seveal variables defined so far. On specializing the parameters of the function involved in the relations, various known and unknown results may be derived as particulars cases.

Keywords :generalized multivariable Aleph-function, Aleph-function, Mellin-transform, generalized hypergeometric function, infinite integral,.

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1. Introduction and preliminaries.

Recently, Agrawal [1] has evaluated two infinite integrals involving the I-function of one variable and the generalized I-function of two variables.In this paper we evaluate two infinite integrals involving the generalized multivariable Aleph-function and the Aleph-function of one variable.
The Aleph- function , introduced by Südland [7] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z\ \middle|\ \begin{matrix}(a_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r}\\ \cdots\\ (b_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r}^{M,N}(s)z^{-s}\mathrm{d}s \quad (1.1)$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r'}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.2)$$

With :

$$|argz|<\frac{1}{2}\pi\Omega \quad \text{Where } \Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i\Big(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji}\Big)>0 \quad \text{with } i=1,\cdots,r'$$

For convergence conditions and other details of Aleph-function , see Südland et al [7]. The generalized hypergeometric function serie is defined as follows :

$${}_{p'}F_{q'}(y)=\sum_{s'=0}^{\infty}\frac{[(a_{p'})]_{s'}}{[(b_{q'})]_{s'}}y^{s'} \quad (1.3)$$

Here $[(a_{p'})]_{s'}=(a_1)_{s'}\cdots(a_{p'})_{s'}\,;[(b_{q'})]_{s'}=(b_1)_{s'}\cdots(b_{q'})_{s'}.$

The serie (1.5) converge if $p'\leqslant q' and\,|y|<1.$

The generalized Aleph-function of several variables generalize the multivariable I-function defined by H.M. Sharma and Ahmad [5] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1,\cdots,z_r)=\aleph_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}^{m,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}\left(\begin{matrix}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{matrix}\ \middle|\right.$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}],[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$$
$$[(\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,m}],[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$$

$$\left.\begin{matrix}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\;;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.4}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [2].
The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\sum_{j=1}^{m}\beta_j^{(k)}-\tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)}-\tau_i\sum_{j=m+1}^{q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)}-\tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\ \text{with}\ k=1\cdots,r,i=1,\cdots,R\ ,i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1}\ldots|z_r|^{\alpha_r}\,),max(\,|z_1|\ldots|z_r|\,)\rightarrow 0$$

$$\aleph(z_1,\cdots,z_r)=0(\,|z_1|^{\beta_1}\ldots|z_r|^{\beta_r}\,),min(\,|z_1|\ldots|z_r|\,)\rightarrow\infty$$

We will use these following notations in this paper

$$U=p_i,q_i,\tau_i;R\ ;\ V=m_1,n_1;\cdots;m_r,n_r \tag{1.6}$$

$$\mathrm{W}=p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.7}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.8}$$

$$B=\{(b_j;\beta_j,\cdots,\beta_j)_{1,m}\},\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.9}$$

$$C=\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\},\cdots,\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.10}$$

$$D=\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\},\cdots,\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.11}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r)=\aleph_{U:W}^{m,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\middle|\begin{array}{c}\mathrm{A}:\mathrm{C}\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{array}\right) \tag{1.12}$$

Throughout this paper, we will use the notations.

$$P_1=\min_{1\leqslant j\leqslant m,1\leqslant \mathrm{j}\leqslant m_1}\left(\sum_{j=1}^{m}\frac{b_j}{\beta_j^{(1)}}+\frac{d_{\mathrm{j}}^{(1)}}{\delta_{\mathrm{j}}^{(1)}}\right),\cdots,P_r=\min_{1\leqslant j\leqslant m,1\leqslant \mathrm{j}\leqslant m_r}\left(\sum_{j=1}^{m}\frac{b_j}{\beta_j^{(r)}}+\frac{d_{\mathrm{j}}^{(r)}}{\delta_{\mathrm{j}}^{(r)}}\right)$$

$$Q_1=\max_{1\leqslant j\leqslant n,1\leqslant \mathrm{j}\leqslant n_1}\left(\sum_{j=1}^{n}\frac{a_j-1}{\alpha_j^{(1)}}+\frac{c_{\mathrm{j}}^{(1)}-1}{\gamma_{\mathrm{j}}^{(1)}}\right),\cdots,$$

$$Q_r=\max_{1\leqslant j\leqslant n,1\leqslant \mathrm{j}\leqslant n_r}\left(\sum_{j=1}^{n}\frac{a_j-1}{\alpha_j^{(r)}}+\frac{c_{\mathrm{j}}^{(r)}-1}{\gamma_{\mathrm{j}}^{(r)}}\right)\text{ and } p=\min_{1\leqslant j\leqslant M}\frac{b'_j}{B'_j},\ q=\max_{1\leqslant j\leqslant N}\frac{a'_j-1}{A'_j}$$

2. Infinite integrals

The integrals to be established are :

a) $\int_0^\infty x^{\eta-1}\aleph_{P_{\mathrm{i}},Q_{\mathrm{i}},c_{\mathrm{i}};r}^{M,N}(zx^\rho)\aleph_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},\cdots,z_rx^{\sigma_r})\mathrm{d}x$

$$=\rho^{-1}z^{-\eta/\rho}\aleph_{U_{M+Q_i,N+P_i}:W}^{m+N,\mathfrak{n}+M:V}\left(\begin{array}{c}\mathrm{z}^{-\sigma_1/\rho}z_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}^{-\sigma_r/\rho}z_r\end{array}\middle|\begin{array}{c}\left[\left(1-b_j-\frac{\eta}{\rho}B_j;\frac{\sigma_1}{\rho}B_j,\cdots,\frac{\sigma_r}{\rho}B_j\right)\right]_{1,M}\\ \cdots\\ \left[\left(1-a_j-\frac{\eta}{\rho}A_j;\frac{\sigma_1}{\rho}A_j,\cdots,\frac{\sigma_r}{\rho}A_j\right)\right]_{1,N}\end{array}\right.$$

$$\left.\begin{array}{c},\left[c_{\mathrm{i}}(1-b_{q_j}-\frac{\eta}{\rho}B_{q_j};\frac{\sigma_1}{\rho}B_{q_j},\cdots,\frac{\sigma_r}{\rho}B_{q_j})\right]_{M+1,Q_i},A:C\\ \cdots\\ ,\left[c_{\mathrm{i}}(1-a_{p_j}-\frac{\eta}{\rho}A_{p_j};\frac{\sigma_1}{\rho}A_{p_j},\cdots,\frac{\sigma_r}{\rho}A_{p_j})\right]_{N+1,P_i},B:D\end{array}\right) \tag{2.1}$$

where $U_{M+Q_i,N+P_{\mathrm{i}}}=p_i+M+Q_{\mathrm{i}},q_i+N+P_{\mathrm{i}},\tau_i;R$ with $\mathrm{i}=1,\cdots,r';i=1\cdots,r$

provided that

$$Re(\eta+\sigma+\sum_{i=1}^{r}\sigma_iP_1+\rho p)>0,\cdots,Re(\eta+\sigma+\sum_{i=1}^{r}\sigma_iP_r+\rho p)>0$$

$$Re(\eta+\sigma+\sum_{i=1}^{r}\sigma_iQ_1+\rho p)<0,\cdots,Re(\eta+\sigma+\sum_{i=1}^{r}\sigma_iQ_r+\rho q)<0$$

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4) and $|argz| < \frac{1}{2}\pi\Omega$

b) $\int_0^\infty x^{\eta-1} {}_{p'}F_{q'}(y)(-zx^\rho)\aleph_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},\cdots,z_rx^{\sigma_r})\mathrm{d}x = s^{-\eta}\frac{[(b_{q'})]_{s'}}{[(a_{p'})]_{s'}}$

$$\aleph_{U_{q'+1,p'}:W}^{m+p',\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}^{-\sigma_1/\rho}z_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}^{-\sigma_r/\rho}z_r \end{array}\middle| \begin{array}{c} [(\mathrm{d}_j-\eta;\sigma_1,\cdots,\sigma_r)]_{1,q'},(1-\eta;\sigma_1,\cdots,\sigma_r),A:C \\ \cdots \\ [(\mathrm{c}_j-\eta;\sigma_1,\cdots,\sigma_r)]_{1,p'},B:D \end{array}\right) \quad (2.2)$$

where $U_{q'+1,p'} = p_i+q'+1, q_i+p', \tau_i; R$ with $i = 1\cdots,r$

provided that

$$Re(\eta+\sigma+\sum_{i=1}^r\sigma_iP_1)>0,\cdots,Re(\eta+\sigma+\sum_{i=1}^r\sigma_iP_r+p)>0$$

$$Re(\eta+\sigma+\sum_{i=1}^r\sigma_iQ_1)<0,\cdots,Re(\eta+\sigma+\sum_{i=1}^r\sigma_iQ_r)<0$$

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4) and $|argz| < \frac{1}{2}\pi(p'-q'+1)$

Proof of (2.1) Let $M = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}$

First, expressing the generalized multivariable Aleph-function on the L.H.S. as multiple Mellin-Barnes contour integral with the help of (1.4) and interchanging the order of integration, (which is permissible under the conditions stated), we get :

$$M\left(\int_0^\infty x^{\eta+\sum_{i=1}^r\sigma_is_i-1}\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(\mathrm{z}\middle|\begin{array}{c}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ \cdots \\ (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{array}\right)\mathrm{d}x\right)\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (2.3)$$

Now in the inner integral taking x for zx^ρ, we see that the transformed integral under the conditions stated in (2.1) and Aleph-function satisfies the remaining conditions of validity for the applications of Mellin-Barnes transform [3,page.408(51)]. Hence applying Mellin transform to x-integral and using the definition of the Aleph-function[ ] , we get the value of the inner-integral. Finally reinterpreting the Mellin-Barnes contour integral, we get the desired result.

The Aleph-function of one variable generalize the hypergeometric function of one variable , the formula (2.2) is obtained.

3. Multivariable I-function

If $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}},c_i\to 1; i=1,\cdots,R; i'=1,\cdots,r'$, the generalized Aleph-function of several variables degenere in the generalized I-function of several variables itself is a generalization of the multivariable I-function defined by Sharma [5]. The Aleph-function of one variable degenere in I-function of one variable defined by Saxena [4] and we have the following infinite integrals.

a) $\int_0^{\infty} x^{\eta-1} I_{P_i,Q_i;r}^{M,N}(zx^{\rho}) I_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},\cdots,z_rx^{\sigma_r})\mathrm{d}x$

$$= \rho^{-1} z^{-\eta/\rho}\, I_{U_{M+Q_i,N+P_i}:W}^{m+N,\mathfrak{n}+M:V}\left(\begin{array}{c|c} \mathrm{z}^{-\sigma_1/\rho}z_1 & \left[\left(1-b_j-\frac{\eta}{\rho}B_j;\frac{\sigma_1}{\rho}B_j,\cdots,\frac{\sigma_r}{\rho}B_j\right)\right]_{1,M} \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}^{-\sigma_r/\rho}z_r & \left[\left(1-a_j-\frac{\eta}{\rho}A_j;\frac{\sigma_1}{\rho}A_j,\cdots,\frac{\sigma_r}{\rho}A_j\right)\right]_{1,N} \end{array}\right.$$

$$\left.\begin{array}{c} ,\left[\left(1-b_{q_j}-\frac{\eta}{\rho}B_{q_j};\frac{\sigma_1}{\rho}B_{q_j},\cdots,\frac{\sigma_r}{\rho}B_{q_j}\right)\right]_{M+1,Q_i},A:C \\ \cdots \\ ,\left[\left(1-a_{p_j}-\frac{\eta}{\rho}A_{p_j};\frac{\sigma_1}{\rho}A_{p_j},\cdots,\frac{\sigma_r}{\rho}A_{p_j}\right)\right]_{N+1,P_i},B:D \end{array}\right) \quad (3.1)$$

where $U_{M+Q_i,N+P_i} = p_i + M + Q_{\mathfrak{i}}, q_i + N + P_{\mathfrak{i}}; R$ **with** $\mathfrak{i} = 1,\cdots,r'; i = 1\cdots,r$

under the same conditions and the validity conditions that (2.1) with $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}},c_i \rightarrow 1$

b) $\int_0^{\infty} x^{\eta-1}\,{}_{p'}F_{q'}(y)(-zx^{\rho}) I_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},\cdots,z_rx^{\sigma_r})\mathrm{d}x = s^{-\eta}\frac{[(b_{q'})]_{s'}}{[(a_{p'})]_{s'}}$

$$I_{U_{q'+1,p'}:W}^{m+p',\mathfrak{n}+1:V}\left(\begin{array}{c|c} \mathrm{z}^{-\sigma_1/\rho}z_1 & [(\mathrm{d}_j-\eta;\sigma_1,\cdots,\sigma_r)]_{1,q'},(1-\eta;\sigma_1,\cdots,\sigma_r),A:C \\ \cdot & \cdots \\ \cdot & [(\mathrm{c}_j-\eta;\sigma_1,\cdots,\sigma_r)]_{1,p'},B:D \\ \mathrm{z}^{-\sigma_r/\rho}z_r & \end{array}\right) \quad (3.2)$$

under the same conditions and the validity conditions that (2.2)

4. Aleph-function of two variables

If $r=2$, the generalized multivariable Aleph-function degenere in the generalized Aleph-function of two variables , itself is a generalization of Aleph-function defined by K.Sharma [6], and we have the following infinite integrals.

a) $\int_0^{\infty} x^{\eta-1} \aleph_{P_i,Q_i,c_i;r'}^{M,N}(zx^{\rho}) \aleph_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},z_2x^{\sigma_2})\mathrm{d}x$

$$= \rho^{-1} z^{-\eta/\rho}\, \aleph_{U_{M+Q_i,N+P_i}:W}^{m+N,\mathfrak{n}+M:V}\left(\begin{array}{c|c} \mathrm{z}^{-\sigma_1/\rho}z_1 & \left[\left(1-b_j-\frac{\eta}{\rho}B_j;\frac{\sigma_1}{\rho}B_j,\frac{\sigma_2}{\rho}B_j\right)\right]_{1,M} \\ \cdot & \cdots \\ \mathrm{z}^{-\sigma_2/\rho}z_2 & \left[\left(1-a_j-\frac{\eta}{\rho}A_j;\frac{\sigma_1}{\rho}A_j,\frac{\sigma_2}{\rho}A_j\right)\right]_{1,N} \end{array}\right.$$

$$\left.\begin{array}{c} ,\left[c_{\mathfrak{i}}(1-b_{q_j}-\frac{\eta}{\rho}B_{q_j};\frac{\sigma_1}{\rho}B_{q_j},\frac{\sigma_2}{\rho}B_{q_j}\right)\right]_{M+1,Q_i},A:C \\ \cdots \\ ,\left[c_{\mathfrak{i}}(1-a_{p_j}-\frac{\eta}{\rho}A_{p_j};\frac{\sigma_1}{\rho}A_{p_j},\frac{\sigma_2}{\rho}A_{p_j}\right)\right]_{N+1,P_i},B:D \end{array}\right) \quad (4.1)$$

under the same conditions and the validity conditions that (2.1)

b) $$\int_0^{\infty} x^{\eta-1} {}_{p'}F_{q'}(y)\big(-zx^{\rho}\big)\aleph_{U:W}^{m,\mathfrak{n}:V}(z_1x^{\sigma_1},z_2x^{\sigma_2})\mathrm{d}x = s^{-\eta}\frac{[(b_{q'})]_{s'}}{[(a_{p'})]_{s'}}$$

$$\aleph_{U_{q'+1,p'}:W}^{m+p',\mathfrak{n}+1:V}\left(\begin{array}{c} \mathrm{z}^{-\sigma_1/\rho}z_1 \\ \cdot \\ \cdot \\ \mathrm{z}^{-\sigma_2/\rho}z_2 \end{array}\middle| \begin{array}{c} [(\mathrm{d}_j-\eta;\sigma_1,\sigma_2)]_{1,q'},(1-\eta;\sigma_1,\sigma_2),A:C \\ \cdots \\ [(\mathrm{c}_j-\eta;\sigma_1,\sigma_2)]_{1,p'},B:D \end{array}\right) \quad (4.2)$$

under the same conditions and the validity conditions that (2.2)

5. Conclusion

In this paper, we have established two general infinite integrals involving the generalized multivariable Aleph-function and the Aleph-function of one variable. Due to general nature of the generalized multivariable aleph-function involving here, our formulas are capable to be reduced into many known and news infinite integrals formulas involving the special functions of one and several variables.

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Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE