Properties of supra N-open sets

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***Abstract**: The purpose of this paper is to introduce and study some of the properties of supra N-open sets via supra N-derived, supra N-border, supra N-frontier and supra N-exterior. Also we introduce some separation axioms using supra N- open set.*

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1. INTRODUCTION

In 1983, A.S.Mashhour et al [4] introduced supra topological spaces and studied, s-continuous functions and s*- continuous functions. The Authors have introduced the notion of supra N-closed set[6] in supra topological spaces.

In this paper, we bring out some of the concept of supra N-derived, supra N- border, supra N-frontier and supra N-exterior of a set and study their properties. And also brings out some of the separation axioms by using supra N-open sets.

2. PRELIMINARIES

Definition 2.1[4] A subfamily μ of X is said to be supra topology on X if

i) $X, \varphi \in \mu$

ii)If $A_i \in \mu$, $\forall i \in J$ then $\cup Ai \in \mu$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2.3[4] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if $\tau \subseteq \mu$.

Definition 2.4

A subset A of a space X is called

(i) supra semi-open set[3], if $A \subseteq cl^\mu(int^\mu(A))$.

(ii) supra α -open set[2], if $A \subseteq int^\mu(cl^\mu(int^\mu(A)))$.

(iii) supra Ω closed set[5], if $scl^\mu(A) \subseteq int^\mu(U)$,whenever $A \subseteq U$, U is supra open set.

(iv) supra N-closed set[6] if $\Omega cl^\mu(A) \subseteq U$, whenever $A \subseteq U$, U is supra α open set.

(v) supra regular closed if $A= cl^\mu(int^\mu(A))$.

The complement of the above mentioned sets are their respective open and closed sets.

Remark 2.5

The supra closure of a set A is denoted by $cl^\mu(A)$, and is defined as supra $cl(A) = \cap \{B : B$ is supra closed and $A \subseteq B\}$.

The supra interior of a set A is denoted by $int^\mu(A)$, and is defined as supra $int(A) = \cup\{B:B$ is supra open and $A \supseteq B\}$.

The supra N-closure of a set A is denoted by N $cl^\mu(A)$, and is defined as supra $Ncl(A) = \cap \{B : B$ is supra N-closed and $A \subseteq B\}$.

The supra N-interior of a set A is denoted by $Nint^\mu(A)$, and is defined as supra $Nint(A) = \cup\{B:B$ is supra N-open and $A \supseteq B\}$.

3. SUPRA N-DERIVED SET

Definition 3.1 Let A be subset of a supra topological space X. An element $x \in X$ is said to be a supra N-limit point of A, if every supra N-open set U in X containing x, such that $U \cap (A - \{x\}) \neq \varphi$.

Definition 3.2 The set of all supra N-limit points of A is called supra N-derived set of A. It is denoted by $D_N^\mu(A)$.

Theorem 3.3 Let A, B be subsets of a supra topological space X, then

(i) $D_N^\mu(A) \subseteq D^\mu(A)$.

(ii) If $A \subseteq B$ then, $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(iii) $D_N^\mu(A \cap B) \subseteq D_N^\mu(A) \cap D_N^\mu(B)$.

(iv) $D_N^\mu(D_N^\mu(A)) - A \subseteq D_N^\mu(A)$.

(v) $D_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

Proof

(i) It is obvious, since every supra open set is supra N-open set.

(ii) Let $x \in D_N^\mu(A)$, then x is supra N-limit point of A. Then every neighbourhood of x contains a point of A different from x. Since $A \subseteq B$, every neighbourhood of x contains a point of B different from x. Hence $x \in D_N^\mu(B)$. Therefore $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(iii) Since $D_N^\mu(A \cap B) \subseteq D_N^\mu(A)$ and $D_N^\mu(A \cap B) \subseteq D_N^\mu(B)$, proof follows from (ii).

(iv) If $x \in D_N^\mu(D_N^\mu(A)) - A$ and U is a supra N-open set containing x, then $U \cap (D_N^\mu(A) - \{x\}) \neq \varphi$. Let $y \in U \cap (D_N^\mu(A) - \{x\})$. Then, since $y \in D_N^\mu(A)$ and $y \in U$, so $U \cap (A - \{y\}) \neq \varphi$. Let $z \in U \cap (A - \{y\})$. Then, $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A - \{x\}) \neq \varphi$. Therefore $x \in D_N^\mu(A)$.

(v) Let $x \in D_N^\mu[A \cup D_N^\mu(A)]$. If $x \in A$, the result is obvious. So let $x \in D_N^\mu[A \cup D_N^\mu(A)] - A$, then for supra N-open set U containing x, such that $U \cap [A \cup D_N^\mu(A) - \{x\}] \neq \varphi$. Thus $U \cap [A - \{x\}] \neq \varphi$ or $U \cap [D_N^\mu(A) - \{x\}] \neq \varphi$. Hence from (iv) $x \in D_N^\mu(A)$. Hence $x \in A \cup D_N^\mu(A)$. Therefore $D_N^\mu[A \cup D_N^\mu(A)] \subseteq A \cup D_N^\mu(A)$. The proof of the above theorem is shown by the following example.

Example 3.4 Let X={a, b, c} and τ = {X, φ, {a} , {a, b}}. τ^c = {X, φ, {b, c} , {c}}. supra N-open sets are {X, φ, {a},{b}, {a, b}, {a, c}}. supra N-closed sets are {X, φ, {b}, {c}, {b, c}, {a, c}}.

(i) Let A={a, b}. D_N^μ {a, b} = {c} and D^μ {a, b} = {b, c}. Hence $D_N^\mu(A) \subseteq D^\mu(A)$.

(ii) Let A={a} and B={a,b}. D_N^μ{a}={c} and D_N^μ{a,b}={c}. Hence $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(iii) Let A={a} and B={a,b}. D_N^μ{a}={c} and D_N^μ{a,b}={c}. Hence $D_N^\mu(A \cap B) \subseteq D_N^\mu(A) \cap D_N^\mu(B)$.

(iv) Let A={a, b}. D_N^μ{a,b}={c}. $D_N^\mu(D_N^\mu(\{a, b\}))$ − {a, b}= $D_N^\mu(\{c\})$ − {a, b}= φ. Hence $D_N^\mu(D_N^\mu(A)) - A \subseteq D_N^\mu(A)$.

(v) Let A={a}. $D_N^\mu(\{a\})$ = {c}. $A \cup D_N^\mu(\{a\})$ = {a, c} and $D_N^\mu(\{a, c\})$ = {c}. Hence $D_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

4. SUPRA N-BORDER

Definition 4.1 For a subset A of X, supra N-Border of A is defined as $Bd_N^\mu(A) = A - Nint^\mu(A)$.

Theorem 4.2 For a subset A of a supra topological space X, the following statement hold.

(i) $Bd_N^\mu(A) \subseteq Bd^\mu(A)$, where $Bd^\mu(A)$ denotes supra border of A.
(ii) $A = Nint^\mu(A) \cup Bd_N^\mu(A)$.
(iii) $Nint^\mu(A) \cap Bd_N^\mu(A) = \varphi$.
(iv) A is supra N-open iff $Bd_N^\mu(A) = \varphi$.
(v) $Bd_N^\mu(Nint^\mu(A)) = \varphi$.
(vi) $Nint^\mu(Bd_N^\mu(A)) = \varphi$.
(vii) $Bd_N^\mu(Bd_N^\mu(A)) = Bd_N^\mu(A)$.
(viii) $Bd_N^\mu(A) = A \cap Ncl^\mu(X - A)$.
(ix) $Bd_N^\mu(A) = D_N^\mu(X - A)$.

Proof

(i) Obvious, since every supra open set is supra N-open set.

(ii) $Nint^\mu(A) \cup Bd_N^\mu(A) = Nint^\mu(A) \cup (A - Nint^\mu(A)) = A$.

(iii) $Nint^\mu(A) \cap Bd_N^\mu(A) = Nint^\mu(A) \cap (A - Nint^\mu(A)) = \varphi$.

(iv) $Bd_N^\mu(A) = A - Nint^\mu(A)) = \varphi$ iff A is supra N-open.

(v) $Bd_N^\mu(Nint^\mu(A)) = Nint^\mu(A) - Nint^\mu(Nint^\mu(A)) = \varphi$.

(vi) If $x \in Nint^\mu(Bd_N^\mu(A))$, then $x \in Bd_N^\mu(A)$. On the other hand, since $Bd_N^\mu(A) \subseteq A$, $x \in Nint^\mu(Bd_N^\mu(A)) \subseteq Nint^\mu(A)$. Hence $x \in Nint^\mu(A) \cap Bd_N^\mu(A)$, which contradicts (iii). Hence $Nint^\mu(Bd_N^\mu(A)) = \varphi$.

(vii) $Bd_N^\mu(Bd_N^\mu(A)) = Bd_N^\mu(A - Nint^\mu(A)) = (A - Nint^\mu(A)) - Nint^\mu(A - Nint^\mu(A)) = A - Nint^\mu(A) = Bd_N^\mu(A)$.

(viii) $Bd_N^\mu(A) = A - Nint^\mu(A) = A-(X-Ncl^\mu(X - A)) = A \cap Ncl^\mu(X - A)$.

(ix) $Bd_N^\mu(A) = A - Nint^\mu(A) = A-(A-D_N^\mu(X - A)) = D_N^\mu(X - A)$.

The proof of the above theorem is shown by the following example.

Example 4.3 Let X={a, b, c} and τ = {X, φ, {a} , {a, b}}. τ^c = {X, φ, {b, c} , {c}}. supra N-open sets are {X, φ, {a} , {b} , {a, b} , {a, c}}. supra N-closed sets are {X, φ, {b} , {c} , {b, c} , {a, c}}.

(i) Let A={b, c}. Bd_N^μ {b, c} = {c} and Bd^μ {b, c} = {b, c}. Hence $Bd_N^\mu(A) \subset Bd^\mu(A)$.

(ii) Let A={b, c}. Bd_N^μ {b, c} = {c} and $Nint^\mu${b, c} = {b}. Hence $A = Nint^\mu \cup Bd_N^\mu(A)$.

(iii) Let A={b, c}. Bd_N^μ {b, c} = {c} and $Nint^\mu$ {b, c} = {b}. Hence $Nint^\mu \cap Bd_N^\mu (A) = \varphi$.
(iv) Let A={a, c} is supra N-open set. Bd_N^μ {a, c} = φ. Hence A is supra N-open iff $Bd_N^\mu(A) = \varphi$.
(v) Let A={a, c}. $Nint^\mu$ {a, c} = {a, c} and Bd_N^μ {a, c} = φ. Hence $Bd_N^\mu(Nint^\mu(A)) = \varphi$.
(vi) Let A={a, c}. $Nint^\mu(Bd_N^\mu \{a, c\}) = Nint^\mu(\varphi) = \varphi$. Hence $Nint^\mu(Bd_N^\mu (A)) = \varphi$.
(vii) Let A={c}. $Bd_N^\mu (Bd_N^\mu \{c\}) = Bd_N^\mu (\{c\}) = \{c\}$. Hence $Bd_N^\mu (Bd_N^\mu (A)) = Bd_N^\mu (A)$.
(viii) Let A={c}. Bd_N^μ {c} = {c}. $Ncl^\mu(X - \{c\}) = X$. $A \cap Ncl^\mu(X - A) = \{c\}$. Hence $Bd_N^\mu(A) = A \cap Ncl^\mu(X - A)$.
(ix) Let A={c}. Bd_N^μ {c} = {c} and $D_N^\mu (X - \{c\}) = \{c\}$. Hence $Bd_N^\mu(A) = D_N^\mu (X - A)$.

5. SUPRA N-FRONTIER

Definition 5.1 For a subset A of a supra topological space X, $Fr_N^\mu (A) = Ncl^\mu(A) - Nint^\mu(A)$ is said to be supra N-Frontier of A.

Theorem 5.2 For a subset A of a supra topological space X, the following statements hold:

(i) $Fr_N^\mu (A) \subseteq Fr^\mu(A)$, where $Fr^\mu(A)$ denotes supra frontier of A.
(ii) $Ncl^\mu(A) = Nint^\mu(A) \cup Fr_N^\mu (A)$.
(iii) $Nint^\mu(A) \cap Fr_N^\mu (A) = \varphi$.
(iv) $Bd^\mu_N (A) \subseteq Fr_N^\mu (A)$.
(v) $Fr_N^\mu (A) = Ncl^\mu(A) \cap Ncl^\mu(X - A)$.
(vi) $Fr_N^\mu (A) = Fr_N^\mu (X - A)$.
(vii) $Fr_N^\mu (A)$ is supra N-closed.
(viii) $Fr_N^\mu (Fr_N^\mu (A)) \subseteq Fr_N^\mu (A)$.
(ix) $Nint^\mu(A) = A - Fr_N^\mu (A)$.

Proof

(i) Let $x \in Fr_N^\mu (A)$ then $x \in Ncl^\mu(A) - Nint^\mu(A)$, implies $x \in cl^\mu(A) - int^\mu(A)$. Hence $x \in Fr^\mu(A)$. Therefore $Fr_N^\mu (A) \subseteq Fr^\mu(A)$.
(ii) $Nint^\mu(A) \cup Fr_N^\mu (A) = Nint^\mu(A) \cup (Ncl^\mu(A) - Nint^\mu(A)) = Ncl^\mu(A)$.
(iii) $Nint^\mu(A) \cap Fr_N^\mu (A) = Nint^\mu(A) \cap (Ncl^\mu(A) - Nint^\mu(A)) = \varphi$.
(iv) Let $x \in Bd^\mu_N (A)$, implies $x \in A - Nint^\mu(A)$, implies $x \in A$ and $x \notin Nint^\mu(A)$. Implies $x \in Ncl^\mu(A)$. Therefore $x \in Fr_N^\mu (A)$. Hence $Bd^\mu_N (A) \subseteq Fr_N^\mu (A)$.
(v) $Fr_N^\mu (A) = Ncl^\mu(A) - Nint^\mu(A) = Ncl^\mu(A) \cap Ncl^\mu(X - A)$.
(vi) $Fr_N^\mu (A) = Ncl^\mu(A) - Nint^\mu(A) = (X - Nint^\mu(A)) - (X - Ncl^\mu(A)) = Ncl^\mu(X - A) - Nint^\mu(X - A) = Fr_N^\mu (X - A)$.
(vii) $Ncl^\mu(Fr_N^\mu (A)) = Ncl^\mu(Ncl^\mu(A) \cap Ncl^\mu(X - A)) = Ncl^\mu(A) \cap Ncl^\mu(X - A) = Fr_N^\mu (A)$. Hence $Fr_N^\mu (A)$ is supra N-closed.
(viii) $Fr_N^\mu (Fr_N^\mu (A)) = Ncl^\mu(Fr_N^\mu (A)) \cap Ncl^\mu(X - Fr_N^\mu (A)) \subseteq Ncl^\mu(Fr_N^\mu (A)) = Fr_N^\mu (A)$. Hence $Fr_N^\mu (Fr_N^\mu (A)) \subseteq Fr_N^\mu (A)$.
(ix) $A - Fr_N^\mu (A) = A - (Ncl^\mu(A) - Nint^\mu(A)) = Nint^\mu(A)$. Hence $Nint^\mu(A) = A - Fr_N^\mu (A)$.

The proof of the above theorem is shown by the following example.

Example 5.3 Let X={a, b, c} and τ = {X, φ, {a, b}} and τ^c = {X, φ, {c}}. N- open set are {X, φ, {a, b} , {a} , {b}}. N-closed set are {X, φ, {b, c} , {a, c} , {c}}.

(i) Let A={a}. Fr^μ {a} = X and Fr_N^μ {a} = {c}. Therefore $Fr_N^\mu (A) \subseteq Fr^\mu(A)$.
(ii) Let A={a}. Ncl^μ {a} = {a, c}. $Nint^\mu$ {a} = {a} and Fr_N^μ {a} = {c}. Hence $Ncl^\mu(A) = Nint^\mu(A) \cup Fr_N^\mu (A)$.
(iii) Let A={a}. $Nint^\mu$ {a} = {a} and Fr_N^μ {a} = {c}. Hence $Nint^\mu(A) \cap Fr_N^\mu (A) = \varphi$.
(iv) Let A={b, c}. Bd^μ_N {b, c} = {c}. Fr_N^μ {b, c} = {c}. Hence $Bd^\mu_N (A) \subseteq Fr_N^\mu (A)$.
(v) Let A={a, b}. Ncl^μ {a, b} = X. $Ncl^\mu(X - \{a, b\}) = \{c\}$ and Fr_N^μ {a, b} = {c}. Hence $Fr_N^\mu (A) = Ncl^\mu(A) \cap Ncl^\mu(X - A)$.
(vi) Let A={a}. Fr_N^μ {a} = {c} and $Fr_N^\mu (X - \{a\}) = \{c\}$. Hence $Fr_N^\mu (A) = Fr_N^\mu (X - A)$.
(vii) Let A={b, c}. Fr_N^μ {b, c} = {c}, which is supra N-closed set.
(viii) Let A={b, c}. $Fr_N^\mu (Fr_N^\mu \{b, c\}) = Fr_N^\mu (\{c\}) = \{c\}$. Hence $Fr_N^\mu (Fr_N^\mu (A)) \subseteq Fr_N^\mu (A)$.
(ix) Let A={b, c}. $Fr_N^\mu (\{b, c\}) = \{c\}$. $Nint^\mu(\{b, c\}) = \{b\}$. $A - Fr_N^\mu (A) = \{b\}$. Hence $Nint^\mu(A) = A - Fr_N^\mu (A)$.

6. SUPRA N-EXTERIOR

Definition 6.1 For a subset A of a supra topological space X, $Ext_N^\mu (A) = Nint^\mu(X - A)$ is said to be supra N-exterior of A.

Theorem 6.2 For a subset A of a supra topological space X, the following statement holds.

(i) $Ext^\mu(A) \subseteq Ext_N^\mu (A)$, where $Ext^\mu(A)$ denotes supra exterior of A.
(ii) $Ext_N^\mu (A)$ is supra N-open.
(iii) $Ext_N^\mu (A) = Nint^\mu(X - A) = X - Ncl^\mu(A)$.
(iv) $Ext_N^\mu (Ext_N^\mu (A)) = Nint^\mu(Ncl^\mu(A))$.
(v) If $A \subseteq B$, then $Ext_N^\mu (A) \supseteq Ext_N^\mu (B)$.
(vi) $Ext_N^\mu (A \cup B) \subseteq Ext_N^\mu (A) \cup Ext_N^\mu (B)$.
(vii) $Ext_N^\mu (A \cap B) \supseteq Ext_N^\mu (A) \cap Ext_N^\mu (B)$.

(viii) $Ext_N^\mu(X) = \varphi$.
(ix) $Ext_N^\mu(\varphi) = X$.
(x) $Ext_N^\mu(A) = Ext_N^\mu(X - Ext_N^\mu(A))$.
(xi) $Nint^\mu(A) \subseteq Ext_N^\mu(Ext_N^\mu(A)$.
(xii) $X=Nint^\mu(A) \cup Ext_N^\mu(A) \cup Fr_N^\mu(A)$.

Proof

(i) $Ext^\mu(A) \subseteq Ext_N^\mu(A)$, Since every supra open set is supra N-open set.
(ii) $Nint^\mu(Ext_N^\mu(A))= Nint^\mu(Nint^\mu(X - A))= Nint^\mu(X - A)= Ext_N^\mu(A)$. Therefore $Ext_N^\mu(A)$ is supra N-open.
(iii) It is obvious from the definition.
(iv) $Ext_N^\mu(Ext_N^\mu(A))= Ext_N^\mu(Nint^\mu(X-A)) = Nint^\mu(X-Nint^\mu(X-A)) =Nint^\mu(Ncl^\mu(A))$.
(v) If $A \subseteq B$, then $Ncl^\mu(A) \subseteq Ncl^\mu(B)$, implies $X - Ncl^\mu(A) \supseteq X - Ncl^\mu(B)$. Hence $Ext_N^\mu(A) \supseteq Ext_N^\mu(B)$.
(vi) $Ext_N^\mu(A\cup B)=X-Ncl^\mu(A\cup B) \subseteq (X-Ncl^\mu(A))\cup(X-Ncl^\mu(B))= Ext_N^\mu(A)\cup Ext_N^\mu(B)$.
(vii) $Ext_N^\mu(A\cap B)=X-Ncl^\mu(A\cap B) \supseteq (X-Ncl^\mu(A))\cap(X-Ncl^\mu(B))= Ext_N^\mu(A)\cap Ext_N^\mu(B)$.
(viii) $Ext_N^\mu(X)=X-Ncl^\mu(X)=X-X=\varphi$.
(ix) $Ext_N^\mu(\varphi)=X-Ncl^\mu(\varphi)=X-\varphi=X$.
(x) $Ext_N^\mu(X - Ext_N^\mu(A))= Ext_N^\mu(X-Nint^\mu(X-A))= Nint^\mu(X-(X-Nint^\mu(X-A))) =Nint^\mu(Nint^\mu(X-A))=Nint^\mu(X - A)= Ext_N^\mu(A)$.
(xi) $Nint^\mu(A) \subseteq Nint^\mu(Ncl^\mu(A)) =Nint^\mu(X-Nint^\mu(X-A)) = Ext_N^\mu(X-Ext_N^\mu(A))= Ext_N^\mu(Ext_N^\mu(A))$, from (x).
(xii) $Nint^\mu(A) \cup Ext_N^\mu(A) \cup Fr_N^\mu(A)= Ext_N^\mu(A) \cup Ncl^\mu(A)=X$.

The proof of the above theorem is also seen from the following example.

Example 6.3 Let $X=\{a, b, c\}$ and $\tau = \{X, \varphi, \{a, b\}\}$ and $\tau^c = \{X, \varphi, \{c\}\}$. N- open set are $\{X, \varphi, \{a, b\}, \{a\}, \{b\}\}$. N-closed set are $\{X, \varphi, \{b, c\}, \{a, c\}, \{c\}\}$.

(i) Let $A=\{a\}$. $Ext_N^\mu(\{a\}) = \{b\}$, $Ext^\mu(\{a\}) = \varphi$. Hence $Ext^\mu(A) \subseteq Ext_N^\mu(A)$.
(ii) Let $A=\{a\}$. $Ext_N^\mu(\{a\}) = \{b\}$, which is supra N-open.
(iii) Let $A=\{a\}$. $Ext_N^\mu(\{a\}) =Nint^\mu(X - \{a\}) =X-Ncl^\mu(\{a\}) =\{b\}$.
(iv) Let $A=\{a\}$. $Ext_N^\mu(Ext_N^\mu(\{a\})) =Nint^\mu(Ncl^\mu(\{a\})) =\{a\}$.
(v) Let $A=\{a\}$ and $B=\{a, b\}$. $A \subseteq B$. $Ext_N^\mu(\{a\}) = \{b\}$. $Ext_N^\mu(\{a, b\}) = \varphi$. Hence $Ext_N^\mu(A) \supseteq Ext_N^\mu(B)$.
(vi) Let $A=\{a\}$ and $B=\{b\}$. $Ext_N^\mu(\{a\}) = \{b\}$. $Ext_N^\mu(\{b\}) = \{b\}$. $Ext_N^\mu(\{a, b\}) =\varphi$. Hence $Ext_N^\mu(A \cup B) \subseteq Ext_N^\mu(A) \cup Ext_N^\mu(B)$.
(vii) Let $A=\{a\}$ and $B=\{b\}$. $Ext_N^\mu(\{a\}) = \{b\}$. $Ext_N^\mu(\{b\}) = \{b\}$. $Ext_N^\mu(\{\varphi\}) =X$. Hence $Ext_N^\mu(A \cap B) \supseteq Ext_N^\mu(A) \cup Ext_N^\mu(B)$.
(viii) Obvious.
(ix) Obvious.
(x) Let $A=\{a\}$. $Ext_N^\mu(\{a\})= \{b\}$ and $Ext_N^\mu(X - Ext_N^\mu(\{a\})) = \{b\}$. Hence $Ext_N^\mu(A) = Ext_N^\mu(X - Ext_N^\mu(A)$.
(xi) Let $A=\{a, b\}$. $Nint^\mu(\{a, b\})=\varphi$. $Ext_N^\mu(Ext_N^\mu(\{a, b\})=X$. Hence $Nint^\mu(A) \subseteq Ext_N^\mu(Ext_N^\mu(A)$.
(xii) Let $A=\{a\}$. $Nint^\mu(\{a\}) = \{a\}$. $Ext_N^\mu(\{a\}) = \{b\}$. $Fr_N^\mu(\{a\}) = \{c\}$. Hence $X=Nint^\mu(A) \cup Ext_N^\mu(A) \cup Fr_N^\mu(A)$.

7. SOME SEPARATION AXIOMS.

Definition 7.1 A space (X, τ) is said to be supra N-T_0, if for $x,y \in X$ such that $x\neq y$, there exist a supra N-open set U of X containing x but not y (or) a supra N-open set V of X containing y but not x.

Definition 7.2 A space (X, τ) is said to be supra N-T_1, if for $x,y \in X$ such that $x\neq y$, there exist a supra N-open set U of X containing x but not y and a supra N-open set V of X containing y but not x.

Definition 7.3 A space (X, τ) is said to be supra N-T_2, if for $x,y \in X$ such that $x\neq y$, there exist disjoint supra N-open set U and V such that $x \in U$ and $y\in V$.

Remark 7.4 Every supra N-T_1 is supra N-T_0, converse need not be true. It is shown by the following example.

Example 7.5 Let $X=\{a, b, c\}$. $\tau = \{X, \varphi, \{a, b\}, \{a, c\}\}$. supra N-open sets are $\{X, \varphi, \{a, b\}, \{a, c\}, \{a\}\}$. Let x=a and y=b. Then X is supra N-T_0 space but not an supra N-T_1 space, since there is no supra N-open set containing one point but not the other.

Remark 7.6 Every supra N-T_2 is supra N-T_0, converse need not be true. It is shown by the following example.

Example 7.7 Let $X=\{a, b, c, d\}$. $\tau = \{X, \varphi, \{a, b\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}\}$. supra N-open sets are $\{X, \varphi, \{a, b\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{d\}, \{c\}\}$. Let x=c and y=d. Then X is supra N-T_0 space but not an supra N-T_2 space, since there is no disjoint supra N-open sets containing c and d of X.

Theorem 7.8 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a supra N-irresolute, injective map. If Y is supra N-T_1, then X is supra N-T_1.

Proof Let Y be supra N-T_1 space. Let $x,y \in X$, such that $x\neq y$. since f is injective map, then $f(x)\neq f(y)\in Y$. Since Y is supra N-T_1 space, there exist supra N-open set U and V in Y such that $f(x)\in U$, $f(x) \notin V$ and $f(y) \notin U$, $f(y)\in V$. Since f is supra N-irresolute, then $f^{-1}(U)$ and $f^{-1}(V)$ are supra

N-open sets in X. Then $x\in f^{-1}(U)$, $x\notin f^{-1}(V)$ and $y\in f^{-1}(V)$, $y\notin f^{-1}(U)$. Hence X is supra N-T_1 space.

Theorem 7.9 Let $f:(X, \tau) \to (Y, \sigma)$ be a supra N-irresolute, injective map. If Y is supra N-T_2, then X is supra N-T_2.

Proof Let Y be supra N-T_2 space. Let $x,y \in X$, such that $x\neq y$. since f is injective map, then $f(x)\neq f(y)\in Y$. Since Y is supra N-T_2 space, there exist supra N-open set U and V in Y such that $f(x)\in U$, $f(y)\in V$. Since f is supra N-irresolute, then $f^{-1}(U)$ and $f^{-1}(V)$ are supra N-open sets in X. Then $x\in f^{-1}(U)$, $y\in f^{-1}(V)$. Hence X is supra N-T_2 space.

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