An Absolute Difference of Cubic and Square Sum Labeling of Certain Class of Graphs

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Abstract *- A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we introduce the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few graphs for cubic and square sum labeling.*

Keywords — Graph labeling, sum square graph, square sum graphs, cubic graphs.

Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q) graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from Frank Harary [2]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph[1]. In this paper we investigated some new results on ADCSS labeling of product related graphs.

Definition: 1.1 [1]

Let G = (V(G), E(G)) be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection

f : V(G) $\rightarrow$ {1,2,----------------,p} such that the induced function f^*_{adcss}: E(G) $\rightarrow$ multiples of 2 is given by $f^*_{adcss}(uv)$ = $|f(u)^3 + f(v)^3 - (f(u)^2 + f(v)^2)|$ is injective.

Definition: 1.2

A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.

Main Results

Definition 2.1

Let G be a graph with a fixed vertex v_0 and $v_{1,0}$, $v_{2,0}$, $v_{3,0}$,--------$v_{m,0}$ be the new vertices in m copies of G corresponding to the vertex v_0. The graph $[P_m;G]$ is graph obtained from m copies of G by joining $v_{i,0}$ and $v_{(i+1),0}$ by an edge, for each i , $1 \leq i \leq m-1$.

Definition 2.2

Let $C_n^{(2)}$ be a friendship graph. $[P_m; C_n^{(2)}]$ be the graph obtained from m copies of $C_n^{(2)}$ and the path P_m: u_1u_2………………u_m by joining u_i with the common vertex in the i^{th} copy of $C_n^{(2)}$, for $,1 \leq i \leq m$.

Theorem: 2.1

The graph $[P_m: C_n^{(2)}]$ is the absolute difference of the css-labeling.

Proof:

Let G = $[P_m: C_n^{(2)}]$ and let v_1,v_2,----------------,$v_{m(2n-1)}$ are the vertices of G.

Here $|V(G)|$ = m(2n-1) and $|E(G)|$ = 2nm + m-1

Define a function f : V $\rightarrow$ {1,2,3,--------,m(2n-1)} by

$f(v_i) = i$, i = 1,2,------,m(2n-1).

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1})$ = $(i+1)^2 i + i^2(i-1)$

i = 1,2,---------------,n-2

i = n,n+1,-----------------,2n-3

i = 2n-1, 2n,----------------,3n-4

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i = (2m-1)n-(2n-4), -----------, 2mn – (2n-1)

$f^*_{adcss}(v_{(n-1)i+1}\, v_{2m(n-1)+i+1})$ =

$((n-1)i+1)^2(n-1)i$ +

$(2m(n-1)+i+1)^2(2m(n-1)+i)$

i = 0,1,2,------------------,m-1.

$f^*_{adcss}(v_{(n-1)i}\, v_{2m(n-1)+i})$ =

$\{(n-1)i\}^2\{(n-1)i-1\}$ +

$(2m(n-1)+i)^2(2m(n-1)+i-1)$.

i = 1,2,3,-----------------,m

$f^*_{adcss}\,(v_{(n-1+i)m+1}\ v_{2m(n-1)+i+1})$ =
{(n-1+i)m+1}^{2}{(n-1+i)m} +
{2m(n-1)+i+1}^{2}{2m(n-1)+i}
i = 0,1,2,-------m-1

$f^*_{adcss}\,(v_{(n+i)m}\ v_{2m(n-1)+i+1})$ =
{(n+i)m}^{2}{(n+i)m-1} +
{2m(n-1)+i+1}^{2}{2m(n-1)+i}
i= 0,1,2,-------------m-1.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence [P_m: $C_n^{(2)}$] admits absolute difference of css-labeling.

Theorem: 2.2

The graph $C_n \ominus P_m$ is the absolute difference of the css-labeling.

Proof :

Let G = $C_n \ominus P_m$ and let v_1, v_2,----------------,$v_{(m+1)n)}$ are the vertices of G.

Here |V(G)| =(m+1)n and |E(G)| = n(2m-1) + n

Define a function f : V → {1,2,3,--------,(m+1)n} by
f(v_i) = i , i = 1,2,------,(m+1)n.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_{(j-1)m+j}\ v_{(j-1)m+j+i})$ =
{(j-1)m+j}^{2}{(j-1)m+j-1} +
{(j-1)m+j+i}^{2}{(j-1)m+j+i-1}
j= 1,2,------------,n
i = 1,2,------------,m

$f^*_{adcss}(v_{(i-1)(m+1)+1}\ v_{(i-1)(m+1)+m+2})$ =
{(i-1)(m+1)+1}^{2}{(i-1)(m+1)}+
{(i-1)(m+1)+m+2}^{2}{(i-1)(m+1)+m+1}
i = 1,2,------------,n-1

$$f^*_{adcss}(v_1\ v_{(n-2)(m+1)+m+2}) = \{(n-2)(m+1)+m+2\}^2\{(n-2)(m+1)+m+1\}$$

$f^*_{adcss}(v_i\ v_{i+1})$ = (i+1)^{2}i+i^2(i-1)
i = 2,3,4,------------------------m
= m+3, m+4,------------------,2m+1
= 2m+4,2m+5, ----------------, 3m+2
= 3m+5.3m+6,-----------------,4m+3
= ---------------------------------------
=(n-1)m+n+1,----------------------, nm+n-1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence G admits absolute difference of css-labeling.

Definition 2.3

Let G be a finite undirected graph with no loops and multiple edges. The central graph C(G) of a graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G

Theorem 2.3

The central graph of the path P_n, C(P_n) is the absolute difference of the css-labeling.

Proof:

Let G = C(P_n) and let v_1,v_2,----------------,v_{2n-1} are the vertices of G.

Here |V(G)| =2n-1and |E(G)| = 2n-2 +$\frac{(n-2)(n-1)}{2}$

Define a function f : V → {1,2,3,--------,2n-1 } by
f(v_i) = i , i = 1,2,------,2n-1.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\ v_{i+1})$ = i^2(i-1) + (i+1)^{2}i ,
i = 1,2,-------------,2n - 2

$f^*_{adcss}(v_1\ v_{2i+3})$ = (2i+3)^2 (2i+2),
i = 1,2,-------------,n - 2

$f^*_{adcss}(v_3\ v_{2i+5})$ = (2i+5)^2 (2i+4) + 18 ,
i = 1,2,-------------,n – 3

$f^*_{adcss}(v_5\ v_{2i+7})$ = (2i+7)^2 (2i+6) + 100,
i = 1,2,-------------,n – 4

$f^*_{adcss}(v_{2n-5}\ v_{2i+2n-3})$ = (2n - 5)^2 (2n -6) + (2i+2n-3)^2 (2i+2n-4), i = 1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence C(P_n) admits absolute difference of css-labeling.

Definition 2.4

The armed crown is a graph obtained from a cycle C_n by attaching a path P_m at each vertex of C_n and it is denoted by $C_n \ominus P_m$.

Theorem 2.4

The armed crown is the absolute difference of the css-labeling.

Proof:

Let G be the graph and let v_1,v_2,----------------,v_{mn} are the vertices of G.

Here |V(G)| =mn and |E(G)| = (m-1)n + n

Define a function f : V → {1,2,3,--------,mn } by
f(v_i) = i , i = 1,2,------,mn

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\ v_{i+1})$ = i^2(i-1) + (i+1)^{2}i ,
i = 1,2,------------------,n-1.

$f^*_{adcss}(v_1\ v_n)$ = n^2(n-1) .

$f^*_{adcss}(v_{i+jn}\ v_{i+(j+1)n})$ = (i+jn)^2 (i+jn-1) +
{i+(j+1)n}^{2}{i+(j+1)n-1}

i = 1,2,----------------,n
j = 0,1,2,-----------,m-2

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence G admits absolute difference of css-labeling.

Definition 2.5

Bi- armed crown $C_n \odot 2P_m$ is a graph obtained from a cycle C_n by identifying the pendant vertices of two vertex disjoint paths of same length m-1 at each vertex of the cycle.

Theorem 2.5

The Bi- armed crown is the absolute difference of the css-labeling.

Proof:

Let G be the graph and let v_1, v_2,----------------,$v_{(2m-1)n}$ are the vertices of G.

Here $|V(G)|$ =(2m-1)n and $|E(G)|$ = (2m-1)n

Define a function $f : V \rightarrow \{1,2,3,\text{--------},(2m-1)n\}$ by

$f(v_i) = i$, i = 1,2,------,(2m-1)n.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = i^2(i-1) + (i+1)^2 i$,
i = 1,2,-------------------,n-1.

$f^*_{adcss}(v_1\, v_n) = n^2(n-1)$

$f^*_{adcss}(v_{i+jn}\, v_{i+(j+1)n}) = (i+jn)^2 (i+jn-1) + \{i+(j+1)n\}^2\{i+(j+1)n-1\}$
i = 1,2,----------------,n
j = 0,1,2,---------------,m-2

$f^*_{adcss}(v_i\, v_{i+mn}) = i^2(i-1) + (i+mn)^2 (i+mn-1)$,
i = 1,2,-------------------,n.

$f^*_{adcss}(v_{i+(m+)jn}\, v_{i+(m+j+1)n}) = (i+(m+j)n)^2 (i+(m+j)n-1) + \{i+(m+j+1)n\}^2\{i+(m+j+1)n-1\}$
i = 1,2,----------------,n
j = 0,1,2,---------------,m-3

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence G admits absolute difference of css-labeling.

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