New Domination Parameters in Fuzzy Graphs

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Abstract – *In this paper, regular domination, regular split domination in fuzzy graph, regular non split domination in fuzzy graph, regular connected domination in fuzzy graph and totally regular domination in fuzzy graph are introduced and discuss its properties. Also introduced inverse regular connected domination number. Furthermore this new domination parameter is compare with other known domination parameters*

Keywords – *Regular domination, regular split domination number, regular non split domination number, regular connected domination, inverse regular connected domination*

AMS Classification: 05C72, 05C75

1. Introduction

The study of domination set in graphs was begun by Ore and Berge. Kulli V.R. et.al introduced the concept of split domination and non-split domination in graphs. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness. Nagoor Gani and Radha discussed on regular fuzzy graphs. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs. They defined domination using effective edges in fuzzy graph. In this paper, the concept of regular domination, regular split domination in fuzzy graph, regular non split domination in fuzzy graph, regular connected domination in fuzzy graph ,inverse regular connected domination and totally regular domination in fuzzy graph are discussed and establish the relationship with other parameter which is also investigated.

2. Preliminaries

Definition 2.1[9]

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=(σ, μ) is a set with two functions σ :V→[0, 1] and μ: E→[0, 1] such that $\mu(uv) \le \sigma(u)\wedge\sigma(v)$ for all u, v ∈ V.

Definition 2.2[9]

Let G=(σ, μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$ for all $u\in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(u\,v) = \mu(u\,v)$ for all $u,v \in V_1$, then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $\langle V_1\rangle$.

Definition 2.3[9]

The order p and size q of a fuzzy graph G= (σ, μ) are defined to be $p=\sum_{u\in V}\sigma(u)$ and $q=\sum_{uv\in E}\mu(uv)$.

Definition 2.4 [9]

Let G= (σ, μ) be a fuzzy graph on V and D⊆V then the fuzzy cardinality of D is defined to be $\sum_{u\in D}\sigma(u)$.

Definition 2.5[9]

An edge e= uv of a fuzzy graph is called an effective edge if $\mu(uv) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V / \mu(uv) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v\in N(u)}\sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u)|u\in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u)|u\in V(G)\}$.

Definition 2.6[9]

The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G}=(\sigma,\bar{\mu})$ where $\bar{\mu}(uv)=\sigma(u)\wedge\sigma(v)-\mu(uv)$.

Definition 2.7[9]

Let σ: V →[0,1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(uv) = \sigma(u) \wedge\sigma(v)$ for all uv∈E and is denoted by K_σ.

Definition 2.8[9]

A fuzzy graph G= (σ,μ) is said to be connected if any two vertices in G are connected.

Definition 2.9[9]

A fuzzy graph G=(σ, μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further if $\mu(u, v)=\sigma(u) \wedge \sigma(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

Definition 2.10[9]

Let $G=(\sigma,\mu)$ be a fuzzy graph on V. Let $u,v\in V$. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every $v\notin D$, there exists $u\in D$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$ or γ.

Definition 2.11[5]

A Dominating set D of a graph $G=(\sigma, \mu)$ is a split dominating set if the induced subgraph <V-D> is disconnected.

Definition 2.12[9]

A dominating set D of a fuzzy graph $G=(\sigma, \mu)$ is connected dominating set if the induced fuzzy sub graph $H=(<D>,\sigma',\mu')$ is connected. The minimum fuzzy cardinality of a connected dominating set of G is called the connected dominating number of G and is denoted by γ_c.

Definition 2.13[11]

A dominating set D of a fuzzy graph $G=(\sigma, \mu)$ is a non split dominating set if the induced fuzzy subgraph $H=(<V-D>,\sigma',\mu')$ is connected. The non split domination number $\gamma_{ns}(G)$ of G is the minimum fuzzy cardinality of a non split dominating set.

Definition 2.14[6]

Let $G = (\sigma, \mu)$ be a regular fuzzy graph on $G^*= (V, E)$. If $d_G(v) = k$ for all $v\in V$, (i.e) if each vertex has same degree k , then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph.

Remark 2.15 G is k-regular graph iff $\delta=\Delta=k$.

Definition 2.16[6]

Let $G = (\sigma, \mu)$ be a fuzzy graph on G^*.The total degree of a vertex $u\in V$ is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv\in E}\mu(uv) + \sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k- totally regular fuzzy graph.

Definition 2.17

The domination set D of the fuzzy graph $G = (\sigma, \mu)$ is said to be regular dominating set if the following conditions are satisfied (i) Every vertex in D is of same degree. The fuzzy regular domination number $\gamma_r(G)$ is the minimum fuzzy cardinality taken over all minimal regular dominating sets of G.

Definition 2.18

The domination set D of the fuzzy graph $G = (\sigma, \mu)$ is said to be regular split dominating set if the following conditions are satisfied (i) Every vertex in D is of same degree (ii) < V–D > is disconnected. The fuzzy regular split domination number $\gamma_{rs}(G)$ is the minimum fuzzy cardinality taken over all minimal regular split dominating sets of G.

Example 2.19

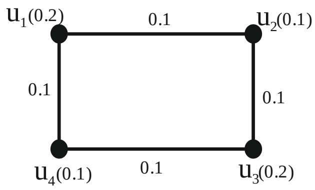

Fig.2.1

$D_{rs}(G) = \{u_2, u_4\}$, $\gamma_{rs}(G) = 0.2$

Definition 2.20

The dominating set D of the fuzzy graph $G = (\sigma, \mu)$ is said to be regular non split dominating set if the following conditions are satisfied (i) Every vertex in D is of same degree (ii) < V–D > is connected. The fuzzy regular non split domination number $\gamma_{rns}(G)$ is the minimum fuzzy cardinality taken over all minimal regular non split dominating sets of G.

Example 2.21

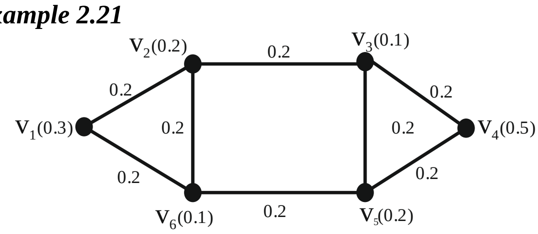

Fig.2.2

$D_{rns}(G) = \{v_1,v_4\}$, $\gamma_{rns}(G) = 0.8$

Definition 2.22

The dominating set D of a regular fuzzy graph $G = (\sigma, \mu)$ is said to be strong regular non split dominating set if all regular non split dominating D_{rns} having the same fuzzy cardinality. The fuzzy strong regular non split domination number $\gamma_{srns}(G)$ is the minimum fuzzy cardinality of a strong regular non split dominating set.

Example 2.23

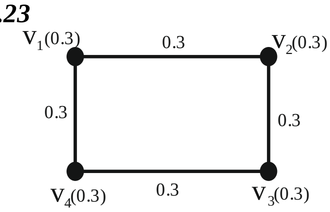

Fig.2.3

$D_{srns}(G) = \{v_1,v_2\}$, (or) $\{v_2,v_3\}$, (or) $\{v_3,v_4\}$, (or) $\{v_1,v_4\}$, $\gamma_{srns}(G) = 0.6$

Definition 2.24

The dominating set D of a regular fuzzy graph $G = (\sigma, \mu)$ is said to be regular connected dominating set if all <D> is connected. The fuzzy regular connected domination number $\gamma_{rc}(G)$ is the minimum fuzzy cardinality taken over all minimal regular connected dominating sets of G.

Example 2.25

From the figure 2.3

$D_{rc}(G) = \{v_1,v_2\}, \{v_2,v_3\}, \{v_1,v_4\}$

$<D_{rc}>$ is connected

$\gamma_{rc} = 0.6$

Definition 2.26

The dominating set D_{rc}' is an inverse regular connected dominating set if D_{rc}' is subset of $V - D_{rc}$ is a regular connected dominating set with respect to D_{rc}. The fuzzy inverse regular connected domination number $\gamma_{irc}(G)$ is the minimum fuzzy cardinality taken over all minimal inverse regular connected dominating sets of G.

Example 2.27

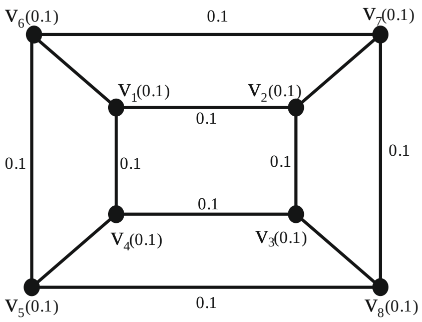

Fig. 2.4

$D_{irc} = \{v_1,v_2,v_3,v_4\}$, $<D_{irc}>$ is connected

$D_{irc}'(G) \subseteq V - D_{irc} = \{v_5,v_6,v_7,v_8\}$,

$< D_{irc}' >$ is connected

Remark 2.28 $\gamma_{irc}(G) + \gamma_{irc}'(G) = p$

Definition 2.29

The dominating set D of a fuzzy graphs $G = (\sigma, \mu)$ is said to be totally regular dominating set if all vertices in D are same total degree, the fuzzy totally regular dominating number γ_{tr} (G) is the minimum fuzzy cardinality taken over all minimal totally regular dominating set of G

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Remark *2.30*

deg(v) = sum of the fuzzy cardinality of all edges incident with V. It is denoted by $d_{(v)}$.

Remark *2.31* total degree (v) = d(v) + σ(v)

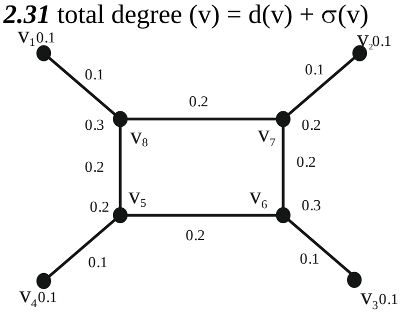

Fig.2.5

$D_{tr}(G) = \{v_5,v_7\}$

$\gamma_{tr} = 0.7$

3. Main Results

Theorem 3.1

For any regular fuzzy graph $G = (\sigma, \mu)$, $\gamma(G) \leq \gamma_{rs}(G)$.

Proof

By definition of $\gamma(G)$-set, $\gamma_{rs}(G)$-set the result is obvious.

Theorem 3.2

For any fuzzy graph $G = (\sigma, \mu)$, if D_{rs} is a regular split dominating set then $V - D_{rs}$ is a dominating set.

Theorem 3.3

For any regular fuzzy graph $G= (\sigma, \mu)$ has a γ_{srns}-set then (i) the number of vertices are even (ii) has a cycle(iii) Alternate vertices have same fuzzy cardinality.

Theorem 3.4

A regular split dominating set D of G is minimal if and only if for each vertex $v \in D$ one of the following conditions holds.

(i) There exists a vertex $u \in V - D$ such that $N(u) \cap D = \{v\}$.

(ii) v is an isolated vertex in <D>.

(iii) <V–D> is connected.

Proof

Suppose that D is minimal and there exists a vertex $v \in D$ such that v does not satisfy any of the above conditions. Then by conditions (i) and (ii) $D' = D - \{v\}$ is a dominating set of G, also by (iii) $<V-D'>$ is disconnected. This implies that D′ is a regular split dominating set of G, which is contradiction.

Theorem 3.5

For any regular fuzzy graph $G = (\sigma, \mu)$ is $\gamma_r(G)$-set all edges having same fuzzy cardinality.

Proof

Let $G = (\sigma,\mu)$ be a fuzzy graph then, the degree of all vertices in G are equal. By definition 2.18, all $\gamma_r(G)$ set having the same fuzzy cardinality. Every regular connected dominating set is a connected dominated set hence the theorem.

Theorem 3.6

If H is the spanning sub graph of the regular fuzzy graph $G = (\sigma, \mu)$ then $\gamma_{rns}(H) \geq \gamma_{rns}(G)$.

Proof

Let $(G) = (\sigma,\mu)$ be a fuzzy graph and $H = (\sigma',\mu')$ be the fuzzy spanning sub graph of G. $D_{rns}(G)$ is the minmum fuzzy cardinality regular non split dominating set of G. $D_{rns}(H)$ is the fuzzy regular non split dominating set of H but not minimum.

Therefore $\gamma_{rns}(H) \geq \gamma_{rns}(G)$.

Theorem 3.7

If $G=(\sigma, \mu)$ is a fuzzy graph then $\gamma_{rc}(G) + \gamma_{rc}'(G) = p$, where $\gamma_{rc}'(G)$ is the inverse regular connected dominating set of G.

Proof

Let $(G) = (\sigma, \mu)$ be a fuzzy graph and $D_{rc}(G)$ is the fuzzy regular connected dominating set of G and $D_{rc}'(G)$ then $\gamma_{rc}'(G) = \sum_{v \in V - D_{rc}} \sigma(v)$ then $\gamma_{rc}(G) + \gamma_{rc}'(G) = p$.

Theorem 3.8

For any regular fuzzy graph $G = (\sigma, \mu)$, $\gamma_c(G) \leq \gamma_{rc}(G)$.

Proof

Every regular connected dominating set is a connected dominated set hence the theorem

Theorem 3.9

For any regular fuzzy graph G = (σ, μ), $\gamma_{tr}(G) \le \gamma_r(G)$.

Theorem 3.10

If G=(σ,μ) is a regular fuzzy graph with all vertices having equal fuzzy cardinality then both γ_{tr} –set and γ_r – set exist also $\gamma_{tr} = \gamma_r$.

Theorem 3.11

For any regular fuzzy graph G = (σ, μ), $\gamma(G) \le \gamma_{tr}(G)$.

Observation3.12

For any regular fuzzy graph $\gamma_r(G) \le \gamma_{rs}(G) \le \gamma_{rc}(G) \le \gamma_{rns}(G) \le \gamma_{tr}(G)$.

ACKNOWLEDGEMENT

Thanks are due to the referees for their valuable comments and suggestions.

REFERENCES

[1]. Harary, E., 1969. *Graph Theory*. Addison Wesley, Reading, MA. McAlester, M.L.N., 1988. Fuzzy intersection graphs. Comp. Math. Appl. 15(10), 871-886.

[2]. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). *Domination in Graphs: Advanced Topics,* Marcel Dekker Inc. New York, U.S.A.

[3]. Haynes, T.W., Hedetniemi S.T. and Slater P.J. (1998). *Fundamentals of domination in graphs*, Marcel Dekker Inc. New York, U.S.A.

[4]. Kulli, V.R. and Janakiram B. (1997). *The split domination number of graph*. Graph Theory notes of New York. New York Academy of Sciences, XXXII, pp. 16-19.

[5]. Mahioub.Q.M. and Soner.N.D (2007), *"The split domination number of fuzzy graph* "Accepted for publication in Far East Journal of Applied Mathematics".

[6]. Nagoor Gani.A and Radha.K, (2008). *On Regular fuzzy Graphs*. Journal of Physical sciences, Vol 12,2008,33-40.

[7]. Hossein Rashmanlou and Madhumangal Pal., *Some Properties of Highly Irregular Interval Valued Fuzzy Graphs*, World Applied sciences journal 27 (12): 1756-1773,2013.

[8]. Rosenfeld, A., 1975. *Fuzzy graphs*. In : Zadeh, L.A., Fu, K.S., Shimura, M. (Eds.), Fuzzy Sets and Their Applications. Academic Press, New York.

[9]. Somasundaram, A., and Somasundaram, S., *Domination in fuzzy graphs-I*, Pattern Recognit. Lett. 19(9) 1998), 787-791.

[10]. Ponnappan C.Y, Surulinathan .P, Basheer Ahamed .S, *"The strong split domination number of fuzzy graphs"* International Journal of Computer & Organization Trends – Volume 8 Number 1 – May 2014.

[11]. Ponnappan C.Y, Surulinathan .P, Basheer Ahamed .S, *"The strong split domination number of fuzzy graphs"* International Journal of Computer & Organization Trends – Volume 8 Number 2 – May 2014.