Lucky Edge Labeling of Triangular Graphs

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Abstract - *Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper coloring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1, 2,….,k} is the lucky number of G denoted by η(G).*

A graph which admits lucky edge labeling is called Lucky Edge Graph.

In this paper, it is proved that Triangular Snake T_n, Book with triangular page B_3^n and Triangular Prism $P_n \times C_3$ are lucky edge graphs.

Keywords: *Lucky Edge Graph, Lucky Edge Labeling, Lucky Number.*
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I. Introduction

A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each e={uv} of vertices in E is called an edge or a line of G. For Graph Theoretical Terminology, [2].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots, v_n$. In this paper, it is proved that Triangular Snake T_n, Book with triangular page B_3^n and Triangular Prism $P_n \times C_3$ are lucky edge graphs.

II. Preliminaries

Definition:2.1

Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be **Lucky Edge Labeling** if the edge set E(G) is a proper coloring of G,that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1, 2,….,k} is the **Lucky Number** of G denoted by **η(G)**.

A graph which admits lucky edge labeling is called**Lucky Edge Graph**.

Definition:2.2

A **Triangular Snake** is obtained from a path by replacing every edge by a triangle C_3. It is denoted by $\boldsymbol{T_n}$.

Definition:2.3

One edge union of cycles of same length is called a Book. The common edge is called as the Base of the book. If we consider *n* copies if cycles of length 3, then the book is called **Book with Triangular page** and it is denoted by $\boldsymbol{B_3^n}$.

Definition:2.4

The product graph of path P_n and cycle C_3 is called **Triangular Prism** and it is denoted by $\mathbf{P_n \times C_3}$.

III. Main Results

Theorem:3.1

Triangular Snake T_n is a Lucky Edge Graph

Proof:

Let $G = T_n$ be the graph.

Let $V(G) = \{ u_i : 1 \leq i \leq n+1\}, \{v_i : 1 \leq i \leq n\}$

$E(G) = \{(u_i v_i) \cup (u_i u_{i+1}) \cup (v_i u_{i+1}) : 1 \leq i \leq n\}$.

Let $f : V[G] \rightarrow \{1, 2, 3, 4, 5\}$ defined by

$$f(u_i) = \begin{cases} 1 & i \equiv 1 \bmod 3 \\ 2 & i \equiv 2 \bmod 3 \\ 3 & i \equiv 0 \bmod 3 \end{cases}, \quad 1 \leq i \leq n+1.$$

$$f(v_i) = \begin{cases} 4 & i \equiv 1 \bmod 2 \\ 5 & i \equiv 0 \bmod 2 \end{cases}, \quad 1 \leq i \leq n.$$

Thus the induced edge labeling are

$$f^*(u_1u_i)=\begin{cases}5 & i\equiv 1 \bmod 6\\ 6 & i\equiv 4,5 \bmod 6\\ 7 & i\equiv 2,3 \bmod 6\\ 8 & i\equiv 0 \bmod 6\end{cases},\quad 1\le i\le n.$$

$$f^*(v_iu_{i+1})=\begin{cases}5 & i\equiv 3 \bmod 6\\ 6 & i\equiv 1,0 \bmod 6\\ 7 & i\equiv 4,3 \bmod 6\\ 8 & i\equiv 2 \bmod 6\end{cases},\quad 1\le i\le n.$$

$$f^*(u_iu_{i+1})=\begin{cases}3 & i\equiv 1 \bmod 3\\ 4 & i\equiv 0 \bmod 3\\ 5 & i\equiv 2 \bmod 3\end{cases},\quad 1\le i\le n.$$

and $\eta(T_5)$= 8

For example, lucky edge labeling of T_5 is given in figure 1 and $\eta(T_5)$= 8.

Thus T_n has Lucky Edge labeling and the labeling is {3, 4, 5, 6, 7, 8} and $\eta(T_n)$= 8.

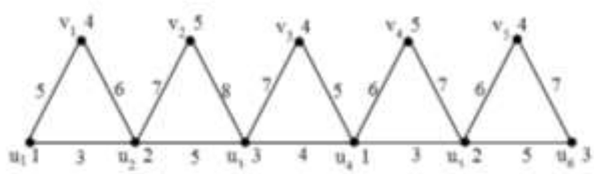

Figure 1

Thoerem:3.2

Book with triangular page B_3^n is Lucky Edge Graph.

Proof:

Let $G = B_3^n$ be the graph.

Let V(G) = $\{u_1, u_2, v_i : 1 \le i \le n\}$

E(G) = $\{(u_1u_2) \cup (u_1v_i) \cup (u_2v_i) : 1 \le i \le n\}$.

Let f : V[G] →{1, 2,…….., n+2} defined by

$f(u_i) = i$, i = 1, 2.

$f(v_i) = i+2$, $1 \le i \le n$.

Thus the induced edge labeling are

$f^*(u_1u_2)$ = 3

$f^*(u_1v_i)$ = i+3, $1 \le i \le n$

$f^*(u_2v_i)$ = i+4, $1 \le i \le n$

and $\eta(B_3^4)$= 8.

For example, lucky edge labeling of B_3^4 is given in figure 2 and $\eta(B_3^4)$= 8.

Thus B_3^n has Lucky Edge labeling and the labeling is {3, 4,…….,n+4} and $\eta(B_3^n)$= n+4.

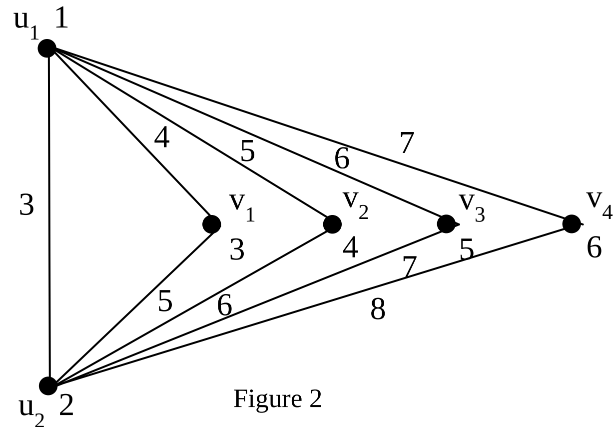

Figure 2

Theorem:3.3

Triangular Prism $P_n \times C_3$ is Lucky Edge Graph.

Proof:

Let $G = P_n \times C_3$ be the graph.

Let V(G) = $\{u_i, v_i, w_i : 1 \le i \le n\}$

E(G) = $\{(u_iv_i) \cup (u_iw_i) \cup (v_iw_i): 1 \le i \le n\} \cup \{(u_iu_{i+1}) \cup (v_iv_{i+1}) \cup (w_iw_{i+1}): 1 \le i \le n\text{-}1\}$

Let f : V[G] →{1, 2,…,6} defined by

$$f(u_i)=\begin{cases}1 & i\equiv 1,2 \bmod 4\\ 4 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

$$f(v_i)=\begin{cases}2 & i\equiv 1,2 \bmod 4\\ 5 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

$$f(w_i)=\begin{cases}3 & i\equiv 1,2 \bmod 4\\ 6 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

Thus the induced vertex coloring are

$$f^*(u_iu_{i+1})=\begin{cases}2 & i\equiv 1 \bmod 4\\ 5 & i\equiv 0,2 \bmod 4\\ 8 & i\equiv 3 \bmod 4\end{cases},\quad 1\le i\le n\text{-}1.$$

$$f^*(v_iv_{i+1})=\begin{cases}4 & i\equiv 1 \bmod 4\\ 7 & i\equiv 0,2 \bmod 4\\ 10 & i\equiv 3 \bmod 4\end{cases},\quad 1\le i\le n\text{-}1.$$

$$f^*(w_iw_{i+1})=\begin{cases}6 & i\equiv 1 \bmod 4\\ 9 & i\equiv 0,2 \bmod 4\\ 12 & i\equiv 3 \bmod 4\end{cases},\quad 1\le i\le n\text{-}1.$$

$$f^*(u_iv_i)=\begin{cases}3 & i\equiv 1,2 \bmod 4\\ 9 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

$$f^*(u_iw_i)=\begin{cases}4 & i\equiv 1,2 \bmod 4\\ 10 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

$$f^*(v_iw_i)=\begin{cases}5 & i\equiv 1,2 \bmod 4\\ 11 & i\equiv 0,3 \bmod 4\end{cases},\quad 1\le i\le n.$$

and $\eta(P_5 \times C_3)$= 12

For example, lucky edge labeling of $P_5 \times C_3$ is given in figure 2 and $\eta(P_5 \times C_3)$= 8.

Thus $P_n \times C_3$ has Lucky Edge labeling and the labeling is {2,3, 4,…….,12} and

$\eta(P_n \times C_3)$= 12.

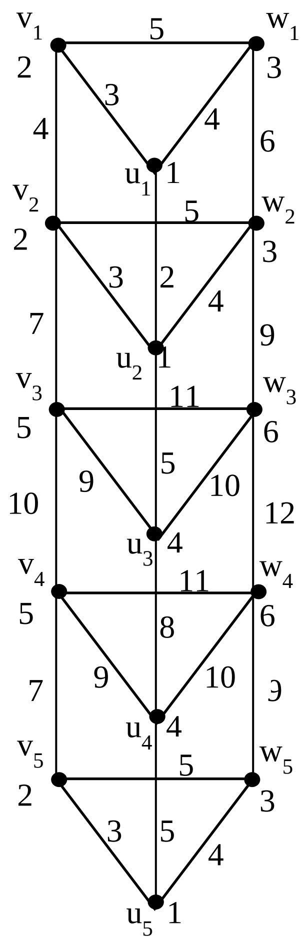

Figure 3

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