A New Notion of Generalized Closed Sets in Topological Spaces

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Abstract — *In this paper, a new class of sets in topological spaces namely, alpha generalized star preclosed (briefly αg*p-closed) set is introduced. This class falls strictly in between the classes of gp*-closed and g*p-closed sets. Also we studied the characteristics and the relationship of this class of sets with the already existing class of closed sets. Further the complement of αg*p-closed set that is αg*p-open set has been imtroduced and investigated.*

Keywords —*αg*p-closed set, αg*p-open set , αg*p-closure , αg*p-interior , αg*p-derived set.* **.**

I. Introduction

In 1970, Levine [7] introduced the concept of generalized closed set and discussed their properties, closed and open maps, compactness, normal and separation axioms. A.S.Mashor, M.E.Abd El-Monsef and E1-Deeb.S.N., [12] introduced preopen sets in topological spaces and investigated their properties. Later in 1998 H.Maki, T.Noiri [11] introduced a new type of generalized closed sets in topological spaces called gp-closed sets. The study on generalization of closed sets has lead to significant contribution to the theory of separation axioms, generalization of continuous and irresolute functions. H.Maki et.al. ([9],[10])generalized α-open sets in two ways and introduced generalized α-closed(briefly gα-closed) sets and α-generalized closed(briefly αg-closed) sets in 1993 and 1994 respectively.

M.K.R.S.Veera kumar [21] introduced the concepts of generalized star preclosed sets and generalized star preopen sets in a topological space. Recently, P. Jaya kumar[6] introduced and studied gp*-closed sets.In this paper we introduce and study a new type of closed set namely 'αg*p-closed sets' in topological spaces.The aim of this paper is to study of αg*p-closed sets thereby contributing new innovations and concepts in the field of topology through analytical as well as research works. The notion of αg*p-closed sets and its different characterizations are given in this paper.

Throughout this paper (X, τ) and (Y,σ) represents topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X, the closure of A and interior of A will be denoted by cl(A) and int(A) respectively. The union of all preopen sets of X contained in A is called pre-interior of A and it is denoted by pint(A). The intersection of all preclosed sets of X containing A is called pre-closure of A and it is denoted by pcl(A).

II. PRELIMINARIES

Definition 2.1:

A subset A of a topological space (X, τ) is called

(i) preopen [12] if A $\subseteq$ int (cl (A)) and preclosed if cl (int(A)) $\subseteq$ A.

(ii) semi-open [8] if A $\subseteq$ cl (int (A)) and semi-closed if int (cl (A)) $\subseteq$ A.

(iii) α-open [16] if A $\subseteq$ int (cl (int (A))) and α-closed if cl(int(cl(A))) $\subseteq$ A .

(iv) semi-preopen [1] (β-open) if A $\subseteq$ cl(int(cl(A))) and semi-preclosed (β-closed) if int (cl (int (A))) $\subseteq$ A .

(v) regular open [20] if A = int (cl(A)) and regular closed if A = cl (int (A)) .

Definition 2.2:

A subset A of a topological space (X, τ) is called (i) generalized closed (briefly, g-closed)[7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(ii) semi-generalized closed (briefly, sg-closed)[3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.

(iii) generalized semi-closed(briefly, gs-closed)[2] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(iv) generalized α-closed(briefly, gα-closed)[10] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X.

(v) α-generalized closed(briefly, αg-closed)[9] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(vi) generalized preclosed(briefly, gp-closed)[11] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X .

(vii)generalized semi-preclosed(briefly,gsp-closed)[4] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X .

(viii)generalized pre regular closed(briefly,gpr-closed) [5] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X .

(ix)weakly generalized closed (briefly, wg-closed) [14] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(x)regular weakly generalized closed (briefly, rwg-closed)[14] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.

(xi)strongly generalized closed (briefly ,g*-closed)[23] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.

(xii)mildly generealized closed (briefly, mg-closed) [18] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.

(xiii)generalized star preclosed (briefly, g*p-closed set) [21] if pcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.

(xiv)generalized pre star closed (briefly gp*-closed set) [6] if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is gp-open in X.

(xv)α-generalized star closed (briefly , αg*-closed set) if αcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is αg-open in X.

(xvi)presemi-closed set[22] if spcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is αg-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3:[15]

Let (X, τ) be a topological space , A $\subseteq$ X and x $\in$ X then x is said to be a pre-limit point of A iff every preopen set containing x contains a point of A different from x.

Definition 2.4:[15]

Let (X, τ) be a topological space and A $\subseteq$ X. The set of all pre-limit points of A is said to be the pre derived set of A and is denoted by D_p[A].

Definition 2.5:

Let (X, τ) be a topological space and let A , B be two non-void subsets of X . Then A and B are said to be pre-separated if A $\cap$ pcl(B) = pcl(A) $\cap$ B = ϕ.

III. αG*P - CLOSED SETS

In this section we introduce alpha generalized star preclosed sets and investigate some of their properties.

Definition 3.1: A subset A of a topological space (X, τ) is called alpha generalized star preclosed set (briefly αg*p-closed) if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is αg-open in X.

Theorem 3.2: Let A be αg*p-closed in (X , τ).Then pcl (A) \ A does not contain any non-empty αg-closed set.

Proof: Suppose that F is a αg-closed subset of pcl (A) \ A. Since X \ F is an αg-open set with A $\subseteq$ X \ F and A is αg*p- closed , pcl (A) $\subseteq$ X \ F. Thus implies F $\subseteq$ (X \ pcl(A)) $\cap$ (pcl(A) \ A) $\subseteq$ (X \ pcl(A)) $\cap$ pcl(A) = ϕ.Therefore F = ϕ.

Corollary 3.3: Let A be a αg*p-closed set in (X , τ). Then pcl (A) \ A does not contain any non-empty α-closed set.

Theorem 3.4: Every preclosed set is αg*p-closed.

Proof: Let A be any preclosed set in X . Let U be any αg-open set containing A. Since A is a preclosed set ,we have pcl(A) = A.Therefore pcl(A)$\subseteq$ U.Hence A is αg*p-closed in X.

The converse of above theorem need not be true as seen from the following example.

Example 3.5: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a},{b},{a,b},{a,c},{a,b,c}, X }.Then {a,b,d} is a αg*p-closed set but not preclosed in X .

Theorem 3.6: Every closed(resp. α-closed, regular closed) set is αg*p-closed.

Proof: The proof follows from the definitions and the fact that every closed(resp. α-closed, regular closed) set is preclosed.

Example 3.7: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a,b}, X}.Then {a,c} is a αg*p-closed set but not α-closed in X.

Example 3.8: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a},{b},{a,b}, X}.Then {d} is a αg*p-closed set but not closed in X.

Example 3.9: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a},{b,c},{a,b,c} , X}.Then {b} is a αg*p-closed set but not regular closed in X.

Theorem 3.10: Every gp*-closed(resp. αg*-closed) set is αg*p-closed.

Proof: Let A be any gp*-closed (resp. αg*-closed) set in X . Let U be αg-open set containing A. Since every αg-open set is gp-open, we have pcl(A)⊆U. Hence A is αg*p-closed.

Example 3.11: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a}, X }. The set {c} is αg*p-closed but not gp*-closed set.

Example 3.12: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{c,d}, X }. The set {c} is αg*p-closed but not αg*-closed.

Theorem 3.13: i)Every αg*p-closed set is gp-closed.
ii) Every αg*p-closed set is a gpr-closed set.
iii) Every αg*p-closed set is gsp-closed.
iv) Every αg*p-closed set is g*p-closed.
Proof: i)Let A be any αg*p-closed set in X . Let U be open set containing A. Since every open set is αg-open, we have pcl(A) ⊆ U. Hence A is gp-closed.
The proof of (ii) to (iv)follows from the definitions and the fact that every open(g-open) set is αg-open .

The converse of above theorem need not be true as seen from the following example.

Example 3.14: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a},{b},{a,b},{a,b,c}, X }.Then {a,d} is a gp-closed set but not αg*p-closed in X.

Example 3.15: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a},{b},{a,b},{a,b,c}, X }. Then {a,b} is a gpr-closed set but not αg*p-closed in X.

Example 3.16: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a,b}, X }. Then {a,b,d} is a gsp-closed set but not αg*p-closed in X.

Example 3.17: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a},{b},{a,b},{a,b,c}, X }. Then {a,d} is a g*p-closed set but not αg*p-closed in X.

Theorem 3.18: i)Every αg*p-closed set is mildly generalized (mg) closed.
ii) Every αg*p-closed set is weakly generalized(wg) closed.
iii)Every αg*p-closed set is regular weakly generalized(rwg) closed.
Proof: i)Let A be any αg*p-closed set in X . Let U be any g-open set containing A. Since every g-open set is αg-open and cl(int(A) ⊆ pcl(A) , we have cl(int(A)) ⊆ U. Hence A is mildly generalized closed.
(ii) and (iii)follows from the fact that every mg-closed set is wg-closed(rwg-closed).

The converse of above theorem need not be true as seen from the following example.

Example 3.19: Let X = {a,b,c,d} be given the topology τ = {ϕ,{a},{b},{a,b},{b,c},{a,b,c}, X}. Then {a,b,d} is mildly generalized closed set but not αg*p-closed in X.

Example 3.20: Let X = {a,b,c,d} be given the topology τ = { ϕ,{a},{b},{a,b}, X}.
Then {b,c} is a weakly generalized closed set but not αg*p-closed in X.

Example 3.21: Let X = {a,b,c,d} be given the topology τ ={ ϕ,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}, X}. Then {a,b} is rwg-closed but not αg*p-closed.

Theorem 3.2: Every αg*p-closed set is pre-semi closed .
Proof: Let A be any αg*p-closed set in X. Let U be any g-open set containing A. Since every g-open set is αg-open and every preclosed set is semi-preclosed, we have spcl(A) ⊆ pcl(A) ⊆ U. Hence A is pre-semi closed.

The converse of above theorem need not be true as seen from the following example.

Example 3.23: Let X = {a,b,c,d} be given the topology τ = {ϕ,{a},{b},{a,b},{a,b,c}, X}. Then {a} is a pre-semi closed set but not αg*p-closed in X.

Remark 3.24: αg*p-closed sets are independent of semi-closed sets and semi-preclosed sets as seen from the following example.

Example 3.25: Let X = {a,b,c,d} be given the topology τ = {ϕ,{a},{b},{a,b},{a,c},{a,b,c}, X}.
In this topological space (X, τ), a subset {b,c} is semi-closed and semi-preclosed but not αg*p-closed and {a,b,d} is a αg*p-closed set but not semi-closed , semi-preclosed.

Remark 3.26: The following examples show that αg*p-closed sets are independent of g-closed , g*-closed , sg-closed , gs-closed , αg-closed , gα-closed .

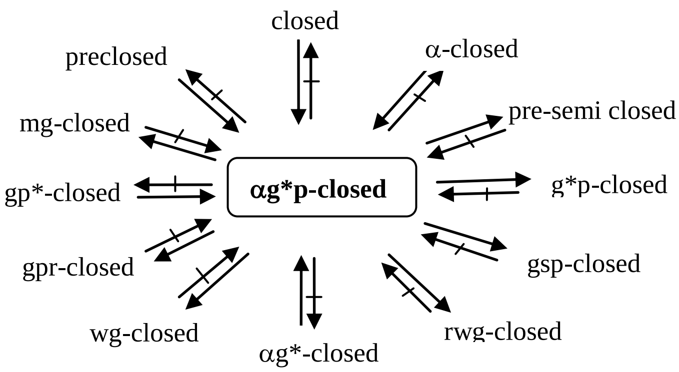
Example 3.27: Let X = {a,b,c,d} be given the topology τ = {ϕ ,{a},{b},{a,b},{a,b,c}, X}.
The set {b,d} is g-closed and g*-closed but not αg*p-closed . The set {c} is αg*p-closed but not g-closed and g*-closed .

Example 3.28: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a,b},X}. The sets {a},{b} are αg*p-closed but not sg-closed, gs-closed , αg-closed and gα-closed .

Example 3.29: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a},{b},{a,b},X}.The set {a,c} is sg-closed and gs-closed but not αg*p-closed .

Example 3.30: Let X = {a,b,c,d} be given the topology τ = { ϕ ,{a},{b},{a,b},{b,c},{a,b,c},X}.The set {b,d} is αg-closed but not αg*p-closed.

From the above discussions we have the following implications:



IV. PROPERTIES OF αG*P-CLOSED SETS AND αG*P-OPEN SETS

Theorem 4.1: Let A be a αg*p-closed set in (X , τ). Then A is preclosed iff pcl (A) \ A is αg-closed.

Proof: Suppose A is preclosed in X. Then pcl(A) = A and so pcl(A) \ A = ϕ which is αg-closed in X. Conversely , Suppose pcl(A) \ A is αg-closed in X. Since A is αg*p-closed , pcl(A) \ A does not contain any non-empty αg-closed set in X. That is pcl(A) \ A = ϕ. Hence A is preclosed.

Theorem 4.2: If A is αg*p-closed in (X, τ) and if A $\subseteq$ B $\subseteq$ pcl(A) then B is also αg*p-closed in (X, τ).

Proof: Let U be an αg-open set of (X , τ) such that B $\subseteq$ U. Since A $\subseteq$ U and A is αg*p-closed , pcl(A) $\subseteq$ U. Since B $\subseteq$ pcl(A) , we have pcl(B) $\subseteq$ pcl(pcl(A)) = pcl(A). Thus pcl(B) $\subseteq$ U. Hence B is a αg*p-closed set of (X , τ).

Remark 4.3: Union of two αg*p-closed sets need not be αg*p-closed .

Remark 4.4: The intersection of two αg*p-closed set is need not be a αg*p-closed set .

Example 4.5: Let X = {a,b,c,d} with τ = {ϕ ,{a,b}, X }. The sets {a} and {b} are αg*p-closed sets but their union {a,b} is not a αg*p-closed set.

Theorem 4.6: For an element x$\in$X , then the set X \ {x} is a αg*p-closed set (or) αg-open set.

Proof: Let x $\in$ X .Suppose that X \ {x} is not αg-open. Then X is the only αg-open set containing X \ {x}. This implies pcl(X \ {x}) $\subseteq$ X. Hence X \ {x} is αg*p-closed in X.

Theorem 4.7: If A is both αg-open and αg*p-closed in X then A is preclosed.

Proof: Suppose A is αg-open and αg*p-closed in X. Since A $\subseteq$ A, pcl(A) $\subseteq$ A. But Always A $\subseteq$ pcl (A). Therefore A = pcl (A). Hence A is preclosed.

Corollary 4.8: Let A be a αg-open set and αg*p-closed set in X. Suppose that F is preclosed in X. Then A $\cap$ F is αg*p-closed in X.

Proof: By theorem 4.7, A is preclosed. So A $\cap$ F is preclosed and hence A $\cap$ F is an αg*p-closed in X.

Theorem 4.9: If A is both open and αg-closed in X, then A is αg*p-closed in X.

Proof: Let A $\subseteq$ U and U be αg-open in X. Now A $\subseteq$ A. By hypothesis αcl(A) $\subseteq$ A.Since every α-closed set is preclosed , pcl(A) $\subseteq$ αcl(A). Thus pcl(A) $\subseteq$ A $\subseteq$ U.Hence A is αg*p-closed in X.

Definition 4.10: Let B $\subseteq$ A $\subseteq$ X. Then we say that B is αg*p-closed relative to A if $pcl_A(B)$ $\subseteq$ U where B $\subseteq$ U and U is αg-open in A.

Theorem 4.11: Let B $\subseteq$ A $\subseteq$ X and Suppose that B is αg*p-closed in X. Then B is αg*p-closed relative to A .

Proof: Given that B $\subseteq$ A $\subseteq$ X and B is αg*p-closed in X. Let us assume that B $\subseteq$ A$\cap$V, where V is αg-open in X. Since B is αg*p-closed set, B $\subseteq$ V implies pcl (B) $\subseteq$ V. It follows that $pcl_A(B)$ = pcl (B) $\cap$ A $\subseteq$ V $\cap$ A. Therefore B is αg*p-closed relative to A.

Theorem 4.12: Let A and B be αg*p-closed sets such that D(A) $\subseteq$ $D_p(A)$ and D(B) $\subseteq$ $D_p(B)$ then A $\cup$ B is αg*p-closed .

Proof: Let U be an αg-open set such that A $\cup$ B $\subseteq$ U. Then pcl(A) $\subseteq$ U and pcl(B) $\subseteq$ U.However, for any set E, $D_p(E)$ $\subseteq$ D(E).Therefore cl(A) = pcl(A) and cl(B) = pcl(B) and this shows cl(A $\cup$ B) = cl(A) $\cup$ cl(B) = pcl(A) $\cup$ pcl(B) .That is pcl(A $\cup$ B) $\subseteq$ U. Hence A $\cup$ B is αg*p-closed.

Definition 4.13: A subset A of a topological space (X, τ) is called αg*p-open set if and only if X \ A is αg*p-closed in X. We denote the family of all αg*p-open sets in X by αg*p-O(X).

Theorem 4.14: If pint (A) $\subseteq$ B $\subseteq$ A and if A is αg*p-open in X then B is αg*p-open in X.

Proof: Suppose that pint (A) $\subseteq$ B $\subseteq$ A and A is αg*p-open in X. Then X \ A $\subseteq$ X \ B $\subseteq$ pcl (X \ A). Since X \ A is αg*p-closed in X, we have X \ B is αg*p-closed in X. Hence B is αg*p-open in X.

Remark 4.15: The intersection of two αg*p-open sets in X is generally not αg*p-open in X.

Example 4.16: Let X = {a,b,c,d} with τ = {ϕ,{a,b}, X }.The sets {a,c} and {b,c} are αg*p-open sets but their intersection {c} is not αg*p-open set.

Theorem 4.17: A set A is αg*p-open iff F $\subseteq$ pint(A) whenever F is αg-closed and F $\subseteq$ A.

Proof: ***Necessity.*** Let A be αg*p-open set and suppose F $\subseteq$ A where F is αg-closed. By definition , X \ A is αg*p-closed. Also X \ A is contained in the αg-open set X \ F. This implies pcl(X \ A) $\subseteq$ X \ F. Now pcl(X \ A) = X \ pint(A). Hence X \ pint(A) $\subseteq$ X \ F.That is F $\subseteq$ pint(A)

Suffiency. If F is αg-closed set with F $\subseteq$ pint(A) where F $\subseteq$ A, it follows that X \ A $\subseteq$ X \ F and X \ pint(A) $\subseteq$ X \ F. That is pcl(X \ A) $\subseteq$ X \ F. Hence X \ A is αg*p-closed and A becomes αg*p-open.

Theorem 4.18: If A and B are pre-separated αg*p-open sets then A $\cup$ B is αg*p-open.

Proof: By definition, pcl(A $\cap$ B) =A $\cap$ pcl(B) = ϕ. Let F be a αg-closed set such that F $\subseteq$ A $\cup$ B then F $\cap$ pcl(A) $\subseteq$ pcl(A) $\cap$ (A$\cup$B) $\subseteq$ A $\cup$ ϕ =A. Similarly , F$\cap$ pcl(B) $\subseteq$ B. Hence by 4.17, F $\cap$ pcl(A) $\subseteq$ pint(A) and F $\cap$ pcl(B) $\subseteq$ pint(B).

Now, F = F $\cap$ (A $\cup$ B) = (F$\cap$A) $\cup$ (F$\cap$B) $\subseteq$ (F $\cap$ pcl(A)) $\cup$ (F $\cap$ pcl(B)) $\subseteq$ pint(A) $\cup$ pint(B) $\subseteq$ pint(A$\cup$B). Hence A $\cup$ B is αg*p-open.

V. αG*P-CLOSURE

In this section we introduce αg*p-closure in topological spaces by using the notions of αg*p-closed sets and study some of their properties.

Definition 5.1: For a subset A of (X, τ) ,the intersection of all αg*p-closed sets containing A is called the αg*p-closure of A and is denoted by αg*p-cl(A). That is , αg*p-cl(A) = $\cap${F : F is αg*p-closed in X , A $\subseteq$ F }.

Theorem 5.2: If A is a αg*p-closed subset of (X, τ) then αg*p-cl(A) = A .

Theorem 5.3: For a subset A of (X, τ) and x$\in$X, αg*p-cl(A) contains x iff V $\cap$ A $\neq$ ϕ for every αg*p-open set V containing x.

Proof: Let A $\subseteq$ X and x$\in$X ,where (X, τ) is a topological space. Suppose that there exists an αg*p-open set V containing x such that V $\cap$ A= ϕ . Since A $\subset$ X \ V , αg*p-cl(A) $\subset$ X \ V and then x $\notin$ αg*p-cl(A) , which is a contradiction.

Conversely , assume that x $\notin$ αg*p-cl(A). Then there exists an αg*p-closed set F containing A such that x $\notin$F. Since x$\in$ X \ F and X \ F is αg*p-open , (X \ F) $\cap$ A = ϕ which is a contradiction. Hence x $\in$ αg*p-cl(A) iff V $\cap$ A $\neq$ ϕ for every αg*p-open set V containing x .

Theorem 5.4: If A and B are subsets of (X, τ) then

(1) αg*p-cl(ϕ) = ϕ and αg*p-cl(X) = X

(2) A $\subseteq$ αg*p-cl(A).

(3)A $\subseteq$ B $\Rightarrow$ αg*p-cl(A) $\subseteq$ αg*p-cl(B)

(4) αg*p-cl(αg*p-cl(A)) = αg*p-cl(A)

(5) αg*p-cl(A $\cup$ B) = αg*p-cl(A) $\cup$ αg*p-cl(B)

(6) αg*p-cl(A $\cap$ B) $\subseteq$ αg*p-cl(A) $\cap$ αg*p-cl(B) .

Theorem 5.5: Let A and B subsets of X. If A is αg*p-closed, then αg*p-cl(A $\cap$ B) $\subseteq$ A $\cap$ αg*p-cl(B).

Proof: Let A be a αg*p-closed set, then αg*p-cl(A) = A and so αg*p-cl(A$\cap$B) $\subseteq$ αg*p-cl(A) $\cap$ αg*p-cl (B) = A $\cap$ αg*p-cl (B) which is the desired result.

VI. αG*P-INTERIOR

In this section we introduce αg*p-interior in topological spaces by using the notions of αg*p-open sets and study some of their properties.

Definition 6.1: Let A be a subset of a topological space X . A point x$\in$ X is called a αg*p-interior point of A if there exists a αg*p-open set G such that x$\in$G $\subseteq$ A. The set of all αg*p-interior points of A is called the αg*p-interior of A and is denoted by Int-αg*p(A).

Definition 6.2: Let A be a subset of a topological space (X, τ). A point x$\in$X is said to be αg*p-limit point of A if every αg*p-open set containing x contains a point of A different from x.

The set of all αg*p-limit points of A is called the αg*p-derived set of A and is denoted by $D_{\alpha g*p}(A)$.

Remark 6.3: Note that for a subset A of X, a point x $\in$ X is not a αg*p-limit point of A if and only if there exists a αg*p-open set G in X such that x$\in$G and G $\cap$ (A \ {x}) = ϕ or, equivalently, x $\in$ G and G $\cap$ A = ϕ or G $\cap$ A = {x}.

Example 6.4:

Let X ={a,b,c,d} with topology τ = {ϕ,{a},{b},{a,b}, X}, τ^c = {ϕ,{b,c,d},{a,c,d},{c,d}, X}.

Then i) $\tau_{\alpha g*p}$ = {ϕ,{a},{b},{a,b}{a,b,c},{a,b,d}, X}.

ii) If A = {a,c,d} then αg*p-cl(A) = {a,c,d},

Int-αg*p(A) ={a} and $D_{\alpha g*p}(A)$ = {c,d}.

Theorem 6.5: Let A be a subset of X and x$\in$X. Then the following are equivalent.

(i)For every G$\in\tau_{\alpha g*p}$, x$\in$G then A $\cap$ G $\neq$ ϕ.

(ii)x$\in$$\alpha$g*p-cl(A).

Proof: (i) $\Rightarrow$(ii) If x$\notin$$\alpha$g*p-cl(A), then there exists a αg*p-closed set F such that A $\subseteq$ F and x $\notin$F. Hence X \ F is a αg*p-open set containing x and A $\cap$ (X \ F) $\subseteq$ A $\cap$ (X \A) = ϕ. This is a contradiction, and hence (ii) is valid.

(ii) $\Rightarrow$ (i) Straightforward.

Corollary 6.6: For any subset A of X, we have $D_{\alpha g^*p}(A) \subseteq \alpha g^*p\text{-cl}(A)$.
Proof: Straightforward.

Theorem 6.7: For any subset A of X , $\alpha g^*p\text{-cl}(A)=A\cup D_{\alpha g^*p}(A)$.
Proof: Let $x\in\alpha g^*p\text{-cl}(A)$. Assume that $x\notin A$ and let $G\in \tau_{\alpha g^*p}$ with $x \in G$. Then $(G \cap A) \setminus \{x\} \neq \phi$,and so $x\in D_{\alpha g^*p}(A)$. Hence $\alpha g^*p\text{-cl}(A) \subseteq A \cup D_{\alpha g^*p}(A)$. The reverse inclusion is by $A \subseteq \alpha g^*p\text{-cl}(A)$and Corollary 6.6.

Proposition 6.8: For subsets A and B of X, the following assertions are valid.
(1) Int-$\alpha g^*p(A)$ is the union of all αg^*p-open subsets of A.
(2) A is αg^*p-open if and only if A = Int-$\alpha g^*p(A)$.
(3) Int-αg^*p(Int-$\alpha g^*p(A)$) = Int-$\alpha g^*p(A)$.
(4) Int-$\alpha g^*p(A) = A \setminus D_{\alpha g^*p}(X \setminus A)$.
(5) $X \setminus$ Int-$\alpha g^*p(A) = \alpha g^*p\text{-cl}(X \setminus A)$.
(6) $X \setminus \alpha g^*p\text{-cl}(A)$ = Int-$\alpha g^*p(X \setminus A)$.
(7) $A \subseteq B \Rightarrow$ Int-$\alpha g^*p(A) \subseteq$ Int-$\alpha g^*p(B)$.
(8) Int-$\alpha g^*p(A) \cup$ Int-$\alpha g^*p(B) \subseteq$ Int-$\alpha g^*p(A \cup B)$.
(9) Int-$\alpha g^*p(A \cap B)$ = Int-$\alpha g^*p(A) \cap$ Int-$\alpha g^*p(B)$.
Proof:
(1) and (2)follows from the definition.
(3) It follows from (1) and (2).
(4) If $x\in A \setminus D_{\alpha g^*p}(X \setminus A)$, then $x\notin D_{\alpha g^*p}(X \setminus A)$ and so there exists a αg^*p-open set G containing x such that $G \cap (X \setminus A) = \phi$. Thus $x\in G \subseteq A$ and hence $x\in$Int -$\alpha g^*p(A)$.
This shows that $A \setminus D_{\alpha g^*p}(X \setminus A) \subseteq$ Int -αg^*p (A).
Let $x\in$Int-$\alpha g^*p(A)$.
Since Int-$\alpha g^*p(A) \in\tau_{\alpha g^*p}$ and Int-$\alpha g^*p(A) \cap (X \setminus A) = \phi$, we have $x\notin D_{\alpha g^*p}(X \setminus A)$.
Therefore Int-$\alpha g^*p(A) = A \setminus D_{\alpha g^*p}(X \setminus A)$.
(5) Using (4) and Theorem 6.7, we have

$$\begin{aligned} X \setminus \text{Int}\alpha g^*p(A) &= X \setminus (A \setminus D_{\alpha g^*p}(X \setminus A)) \\ &= (X \setminus A) \cup D_{\alpha g^*p}(X \setminus A) \\ &= \alpha g^*p\text{-cl}(X \setminus A). \end{aligned}$$

(6) Using (4) and Theorem 6.7, we get Int-$\alpha g^*p(X \setminus A) = (X \setminus A) \setminus D_{\alpha g^*p}(A) = X \setminus (A \cup D_{\alpha g^*p}(A)) = X \setminus \alpha g^*p\text{-cl}(A)$.
(7) Straightforward. (8) and (9) They are by (7).

VII. CONCLUSION

The αg^*p-closed set can be used to derive continuity, irresolute function, closed map, open map , homeomorphism and new separation axioms.

REFERENCES

[1] D.Andrijevic, Semi-preopen sets, *Mat. Vesnik*, 38(1)(1986), 24-32.
[2] S.P. Arya and T. Nour, Characterizations of s-normal spaces, *Indian J. Pure. Appl. Math*., 21(8)(1990),717-719.
[3] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, *Indian J. Math*., 29(3)(1987), 375-382.
[4] J. Dontchev, On generalizing semi-preopen sets, *Mem. Fac. Sci. Kochi Univ. Ser.A. Math.*, 16(1995), 35-48.
[5] Y. Gnanambal, On generalized preregular closed sets in topological spaces, *Indian J. Pure. Appl. Math*., 28(3)(1997), 351-360.
[6] P. Jayakumar , K.Mariappa and S.Sekar , On generalized gp*-closed set in topological spaces, *Int. Journal of Math. Analysis*, 33(7)(2013),1635-1645.
[7] N. Levine, Generalized closed sets in topology, *Rend. Circ. Math.Palermo*, 19(2)(1970), 89-96.
[8] N.Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.
[9] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, *Mem. Fac. Sci. Kochi Univ. Ser.A.Math*., 15(1994), 51-63.
[10] H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology, *Bull. Fukuoka Univ. Ed. Part III*, 42(1993), 13-21.
[11] H. Maki, J. Umehara and T. Noiri, Every topological space is pre-$T_{1/2}$ space, *Mem. Fac. Sci. Kochi Univ. Ser.A. Math*., 17(1996), 33-42.
[12] A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, On pre-continuous and weak pre-continuous mapings, *Proc. Math. And Phys. Soc. Egypt*, 53(1982), 47-53.
[13] A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb, α-continuous and α- open mappings., *Acta Math. Hung*., 41(3-4)(1983), 213-218.
[14] N.Nagaveni , Studies on Generalizations of Homeomorphisms Topological spaces, *Ph.D .thesis, Bharathiar University, Coimbatore*,(1999).
[15] G.B.Navalagi, Pre-neighbourhoods, *The Mathematics Education*, Vol.XXXII,No.4,Dec.(1998),201-206.
[16] O.Njastad, On some classes of nearly open sets, *Pacific J.Math*.,15(1965), 961-970.
[17] N.Palaniappan and K.C.Rao, Regular generalized closed sets, *Kyungpook Math.J*., 33(2)(1993), 211-219.
[18] J.K.Park and J.H.Park , Mildly generalized closed sets ,almost normal and mildly normal spaces,*Chaos ,Solutions and Fractals* , 20(2004), 1103-1111.
[19] A.Pushpalatha, Studies on Generalizations of Mappings in Topological spaces, *Ph.D .thesis, Bharathiar University, Coimbatore,*(2000).
[20] M.Stone Application of the theory of Boolean rings to general topology .*Trans.Amer.Math.Soc*., 41(1937)374-481.
[21] M.K.R.S. Veera kumar, g*-preclosed sets, *Indian J.Math*., 44(2)(2002), 51-60.
[22] M.K.R.S. Veera kumar, Pre-semi closed sets, *Acta ciencia Indica (Maths) Meerut*, XXVIII(M)(1) (2002), 165-181.
[23] M.K.R.S. Veera kumar, Between closed sets and g-closed sets, *Mem. Fac. Sci. Kochi Univ.(Math).*,21(2000), 1-19.
[24] S.Willard, General Topology.*Addison Wesley*, (1970).