Numerical Solution of linear second order Linear Systems with Singular - A using He's Variational Iteration Method

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Abstract — *In this article, He's Variational Iteration Method (HVIM) is implemented for solving the second order linear systems with singular – A. This method is based on Lagrange multipliers for identification of optimal values of parameters in a functional. Using this method creates a sequence which tends to the exact solution of the problem. The results are compared with those obtained by the methods taken from the literature [4]. The works confirms that the He's Variational Iteration Method is superior and very faster to the other methods.*

Keywords — *Singular systems, Second order linear systems with singular-A, Leapfrog method, He's Variational Iteration Method.*

I. Introduction

One of the most important tasks in a study of dynamical systems is the numerical calculation of the trajectories. Thus far we have considered the integration method to be a black box into which we pop the system, initial conditions, method and time range and out pops a plot of the trajectories. Although this approach is common in courses on dynamical systems it obscures many of the pitfalls of numerical integration.

It is not possible at the present state of the art to choose a 'best' algorithm for the calculation of trajectories. There are several types of numerical algorithm, each with their own advantages and disadvantages. We shall consider a class of methods known as discrete variable methods. These methods approximate the continuous time dynamical system by a discrete time system. This means that we are not really simulating the continuous system but a discrete system which may have different dynamical properties. This is an extremely important point. [1-3, 5]

He's Variational Iteration Method can have a significant impact on what is considered a practical approach and on the types of problems that can be solved. S. Sekar and B. Venkatachalam alias Ravikumar [6,7] introduced the He's Variational Iteration Method to study the first and second order linear singular systems of time-invariant and time-varying cases. In this paper, we consider the second order linear systems with singular -A to solve by using the He's Variational Iteration Method. The results are compared with Leapfrog method [4] and with exact solution of the problem.

Hence, we use this He's Variational Iteration Method in the present paper to study second order linear systems with singular - A with initial conditions. The organized paper is as follows: In Section 2 presents general form of second order linear systems with singular – A. In Section 3, the He's Variational Iteration Method for solving second order linear systems with singular - A is introduced. In Section 4, the He's Variational Iteration Method and Leapfrog [4] method for solving second order linear systems with singular – A is solved.

II. Second Order Linear Systems with Singular - A

In this section, a second order linear systems with singular –A is of the form

$$\ddot{x}(t) = A\dot{x}(t) + Bx(t) + Cu(t) \qquad (1)$$

with initial condition $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$ is considered, where A is an $n \times n$ singular matrix, B are $n \times n$ and $n \times p$ constant matrices respectively. $x(t)$ is an n-state vector and $u(t)$ is the p-input control vector and C is an $n \times p$ matrix.

III. He's Variational Iteration Method

In this section, we briefly review the main points of the powerful method, known as the He's variational iteration method [6,7]. This method is a modification of a general Lagrange multiplier method proposed by [6,7]. In the variational iteration method, the differential equation

$$L[u(t)] + N[u(t)] = g(t) \qquad (2)$$

is considered, where L and N are linear and nonlinear operators, respectively and *g(t)* is an inhomogeneous term. Using the method, the correction functional

$$u_{n+1}(t) = u_n(t) + \int \lambda [L[u_n(s)] + N[\tilde{u}_n(s)] - g(s)] ds \quad (3)$$

is considered, where λ is a general Lagrange multiplier, u_n is the nth approximate solution and

$\tilde{u}_n$ is a restricted variation which means $\delta\tilde{u}_n = 0$ [6,7].

In this method, first we determine the Lagrange multiplier λ that can be identified via variational theory, i.e. the multiplier should be chosen such that the correction functional is stationary,

i.e. $\delta u_{n+1}(u_n(t), t) = 0$. Then the successive approximation $u_n, n \geq 0$ of the solution u will be obtained by using any selective initial function u_0 and the calculated Lagrange multiplier λ . Consequently $u = \lim_{n\to\infty} u_n$. It means that, by the correction functional (3) several approximations will be obtained and therefore, the exact solution emerges at the limit of the resulting successive approximations. In the next section, this method is successfully applied for solving the second order linear systems with singular-A.

IV. EXAMPLE FOR SECOND ORDER LINEAR SYSTEMS WITH SINGULAR - A

When the second order linear system with singular-*A* and *B=C=0* is considered, it becomes

$$\ddot{x}(t) = A\dot{x}(t) \qquad (1)$$

with initial conditions

$x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$

By taking $A = \begin{bmatrix} 0 & 2 \\ 0 & -6 \end{bmatrix}$ along with the initial conditions $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\dot{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ equation (1) becomes

$$\ddot{x}_1(t) = 2\dot{x}_2(t)$$
$$\ddot{x}_2(t) = -6\dot{x}_2(t)$$

Therefore the exact solution is

$$x_1(t) = \frac{1}{18}e^{-6t} + \left(\frac{4}{3}\right)t + \frac{17}{18},$$

$$x_2(t) = \frac{7}{6} - \frac{1}{6}e^{-6t},$$

$$\dot{x}_1(t) = -\frac{1}{3}e^{-6t} + \frac{4}{3} \text{ and } \dot{x}_2(t) = e^{-6t}$$

The approximate and exact solutions are calculated for the above problem mention in this section using Leapfrog method and He's Variational Iteration method for x_1 and x_2 for different time intervals and the error between them are shown in the Tables V - VI are presented to highlight the effectiveness of the He's Variational Iteration method.

V. TABLE
SOLUTIONS FOR THE PROBLEM IN IV AT VARIOUS VALUES OF $x_1(t)$

t	$x_1(t)$	
	Leapfrog	**HVIM**
0	0	0
0.25	1E-08	1E-09
0.5	1E-08	1E-09
0.75	2E-08	2E-09
1	2E-08	2E-09
1.25	2E-08	2E-09
1.5	3E-08	3E-09
1.75	3E-08	3E-09
2	3E-08	3E-09

VI. TABLE
SOLUTIONS FOR THE PROBLEM IN IV AT VARIOUS VALUES OF $x_2(t)$

t	$x_2(t)$	
	Leapfrog	**HVIM**
0	0	0
0.25	1E-08	1E-09
0.5	1E-08	1E-09
0.75	0	0
1	2E-08	2E-09
1.25	2E-08	2E-09
1.5	3E-08	3E-09
1.75	3E-08	3E-09
2	3E-08	3E-09

VII. SECOND ORDER MULTIVARIABLE LINEAR SYSTEM WITH SINGULAR – A INVOLVING THREE VARIABLES

When a second order linear multivariable system with singular-*A* of the form (1) is considered.

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

$C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$, and $u = [1 \quad 0 \quad 0]^T$

with $x(0) = [0 \quad 0 \quad 0]^T$ and $\dot{x}(0) = [0 \quad 2 \quad 3]^T$.

Hence, the equation (1) becomes

$$\ddot{x}_1 = \dot{x}_3 - 3x_1 + 1$$
$$\ddot{x}_2 = -2x_2$$
$$\ddot{x}_3 = -3x_3$$

The exact solution is

$$x_1 = -\left(\frac{\sqrt{3}}{2}\right) t \sin\sqrt{3}t - \frac{\cos\sqrt{3}t}{3} + \frac{1}{3}$$

$$x_2 = \sqrt{2}\sin\sqrt{2}t$$

$$x_3 = \sqrt{3}\sin\sqrt{3}t$$

Using Leapfrog method and He's Variational Iteration method to solve the above problem mention in this section, the approximate solutions have been determined and are presented in Tables VIII - X.

VIII. TABLE
SOLUTIONS FOR THE PROBLEM IN VII AT VARIOUS VALUES OF $x_1(t)$

t	$x_1(t)$	
	Leapfrog	**HVIM**
0	0	0
0.25	0	0
0.5	1E-07	1E-08
0.75	1E-07	1E-08
1	2E-08	2E-09
1.25	2E-08	2E-09
1.5	2E-09	2E-11
1.75	3E-09	3E-11
2	3E-08	3E-09

IX.TABLE
SOLUTIONS FOR THE PROBLEM IN VII AT VARIOUS VALUES OF $x_2(t)$

t	$x_2(t)$	
	Leapfrog	**HVIM**
0	1E-06	1E-07
0.25	1E-07	1E-08
0.5	1E-07	1E-08
0.75	1E-07	1E-08
1	2E-08	2E-09
1.25	2E-08	2E-09
1.5	2E-09	2E-11
1.75	3E-09	3E-11
2	3E-08	3E-09

X. TABLE
SOLUTIONS FOR THE PROBLEM IN VII AT VARIOUS VALUES OF $x_3(t)$

t	$x_3(t)$	
	Leapfrog	**HVIM**
0	1E-09	1E-11
0.25	1E-09	1E-11
0.5	1E-09	1E-11
0.75	1E-09	1E-11
1	2E-09	2E-11
1.25	2E-09	2E-11
1.5	2E-09	2E-11
1.75	3E-09	3E-11
2	3E-09	3E-11

XI. CONCLUSIONS

In this paper, He's Variational Iteration method has been successfully applied to find the solutions of second order linear systems with singular – A. All the examples show that the results of the present method are in excellent agreement with those of exact solutions and the obtained solutions are shown graphically. In our work, we use the C++ language to calculate the functions obtained from the He's Variational Iteration Method. The results show that this method provides excellent approximations to the solution of these second order linear systems with singular – A with high accuracy. This new method accelerated the convergence to the solutions. Finally, it has been attempted to show the capabilities and wide-range applications of the variational iteration method.

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