A new multidimensional integral transform concerning the multivariable Aleph-function

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ABSTRACT
In the present document, we use the multidimensional integral transform introduced by Chandel et al [2] concerning the multivariable Aleph-function defined by Ayant [1]. Some interesting special cases are also discussed.

KEYWORDS : Aleph-function of several variables, multidimensional integral transform, Multivariable I-function, Aleph-function of two variables, I-function of two variables.

1.Introduction

Chandel et al [2] introduce a new multidimensional integral transform defined by :

$$R^{(a,b)}_{\alpha_1,\cdots,\alpha_r}\{\}=\frac{\Gamma(\alpha_1+\cdots\alpha_r)\Gamma(1/2+a-b+\alpha_1+\cdots\alpha_r)2^{2a+2\alpha_1+\cdots+2\alpha_r-1}}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)\Gamma(2a+2\alpha_1+\cdots+2\alpha_r)\Gamma(1/2-(a+b+\alpha_1+\cdots+\alpha_r))}$$

$$\int_0^\infty\cdots\int_0^\infty(x_1+\cdots+\alpha_r)^a(1+x_1+\cdots+x_r)^{-1/2}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{2b}$$

$$x_1^{\alpha_1-1}\cdots x_r^{\alpha_r-1}\{\}\ \mathrm{d}x_1\cdots\mathrm{d}x_r \tag{1.1}$$

where $0<Re(a+\alpha_1+\cdots+\alpha_r)<1/2-Re(b), Re(\alpha_i)>0, i=1,\cdots,r$

and give two dimentional integral transforms concerning the multivariable H-function defined by Srivastava et al [6]. Here in the present document, we extend this work with the multivariable Aleph-function. The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [4] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $$\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$$

$$\begin{array}{l}[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:\\ \dots\dots\dots\dots\dots\dots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:\end{array}$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ ;[(\mathrm{c}_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;;[(\mathrm{d}_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

with $\omega=\sqrt{-1}$

For more details, see Ayant [1]. The reals numbers τ_i are positives for $i=1,\cdots,R$, $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1,\cdots,R^{(k)}$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|argz_k| < \frac{1}{2}A_i^{(k)}\pi \text{ , where}$$

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} - \tau_i\sum_{j=\mathfrak{n}+1}^{p_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=n_k+1}^{p_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{m_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=m_k+1}^{q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k=1\cdots,r, i=1,\cdots,R \text{ , } i^{(k)}=1,\cdots,R^{(k)} \tag{1.3}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,), max(\,|z_1|,\cdots,|z_r|\,) \rightarrow 0$$

$$\aleph(z_1,\cdots,z_r) = 0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,), min(\,|z_1|,\cdots,|z_r|\,) \rightarrow \infty$$

where, with $k=1,\cdots,r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R \text{ ; } V = m_1, n_1; \cdots; m_r, n_r \tag{1.4}$$

$$\text{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.5}$$

$$A = \{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.6}$$

$$B = \{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.7}$$

$$C = \{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \cdots, \{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.8}$$

$$D = \{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \cdots, \{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.9}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_r) = \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} z_1 & \text{A : C} \\ \cdot & \\ \cdot & \cdots \\ \cdot & \\ z_r & \text{B : D} \end{array}\right) \tag{1.10}$$

2. Required formulas

$$R_{\alpha_1,\cdots,\alpha_r}^{(a,b)}\{1\} = 1 \tag{2.1}$$

$$R_{\alpha_1,\cdots,\alpha_r}^{(a,b)}\left\{(x_1+\cdots+x_r)^{\zeta_1+\cdots+\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2(\eta_1+\cdots+\eta_r)}\right\}$$

$$= \frac{\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{4^{\zeta_1+\cdots+\zeta_r}\Gamma(2a+2\alpha_1+\cdots+2\alpha_r)\Gamma(1/2-(a+b+\alpha_1+\cdots+\alpha_r))} \times \tag{2.2}$$

$$\frac{\Gamma(2(a+\alpha_1+\cdots+\alpha_r+\zeta_1+\cdots+\zeta_r))\Gamma(1/2-(a+b+\alpha_1+\cdots+\alpha_r)+\eta_1-\zeta_1+\cdots+\eta_r-\zeta_r)}{\Gamma(1/2+a-b+\alpha_1+\zeta_1+\eta_1+\cdots+\alpha_r+\zeta_r+\eta_r)}$$

$$R^{(a,b)}_{\alpha_1,\cdots,\alpha_r}\Big\{x_1^{\lambda_1}\cdots x_r^{\lambda_r}(x_1+\cdots+x_r)^{\zeta_1+\cdots+\zeta_r}$$

$$\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2(\eta_1+\cdots+\eta_r)}\Big\}$$

$$= \frac{\Gamma(\alpha_1+\cdots+\alpha_r)\prod_{i=1}^r\Gamma(\alpha_i+\lambda_i)\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)4^{-(\zeta_1+\lambda+1+\cdots+\zeta_r+\lambda_r)}}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)\Gamma(\alpha_1+\lambda_1+\cdots+\alpha_r+\lambda_r)\Gamma(2(a+\alpha_1+\cdots+\alpha_r))\Gamma(1/2-a-\alpha_1-\cdots-\alpha_r)}$$

$$\Gamma(2(a+\alpha_1+\zeta_1+\lambda_1+\cdots+\alpha_r+\zeta_r+\lambda_r))$$

$$\frac{\Gamma(1/2-ab+\alpha_1-\lambda_1-\alpha_r-\lambda_r+\eta_1-\zeta_1+\cdots+\eta_r-\zeta_r)}{\Gamma(1/2+a-b+\alpha_1+\lambda_1+\zeta_1+\eta_1+\cdots+\alpha_r+\lambda_r+\zeta_r+\eta_r)} \tag{2.3}$$

valid if $0 < Re(a+\zeta_1+\alpha_1+\lambda_1+\cdots+\zeta_r+\alpha_r+\lambda_r) < Re(1/2-b+\eta_1+\cdots+\eta_r)$

$Re(\alpha_i) > 0, Re(\lambda_i) > 0, i = 1,\cdots,r$

3. Main integrals

In this section, making an appeal to (2.2) and (2.3), we derive the following results involving the multivariable Aleph-function defined by Ayant [1]

$$R^{(a,b)}_{\alpha_1,\cdots,\alpha_r}\left\{ \aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} z_1(x_1+\cdots+x_r)^{\zeta_1}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r(x_1+\cdots+x_r)^{\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_r} \end{array}\right|\right.$$

$$\left.\begin{array}{c} A:C \\ \cdots \\ \cdots \\ B:D \end{array}\right\} = \frac{\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{\Gamma(2a+2\alpha_1+\cdots+2\alpha_r)\Gamma(1/2-a-b-\alpha_1-\cdots-\alpha_r)}\aleph^{0,\mathfrak{n}+2:V}_{U_{21}:W}\left(\begin{array}{c} 4^{-\zeta_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_r}z_r \end{array}\right|$$

$$\left.\begin{array}{c} (1\text{-}2a\text{-}2\alpha_1-\cdots-2\alpha_r;2\zeta_1,\cdots,2\zeta_r),(1/2+a+b+\alpha_1+\cdots+\alpha_r;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),A:C \\ \cdots \\ \cdots \\ (1/2\text{-}a\text{+}b\text{-}\alpha_1-\cdots-\alpha_r;\zeta_1+\eta_1,\cdots,\zeta_r+\eta_r),B:D \end{array}\right) \tag{3.1}$$

where $U_{21} = p_i + 2; q_i + 1; \tau_i; R$

Provided that

a) $0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, i = 1, \cdots, r$

b) $\left|\frac{arg z_k}{4^{\zeta_k}}\right| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.3)

$$R_{\alpha_1,\cdots,\alpha_r}^{(a,b)}\left\{ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} z_1 x_1^{\lambda_1}(x_1+\cdots+x_r)^{\zeta_1}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x_r^{\lambda_r}(x_1+\cdots+x_r)^{\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_r} \end{array} \right| \right.$$

$$\left. \begin{array}{c} \mathrm{A} : \mathrm{C} \\ \cdots \\ \cdots \\ \mathrm{B} : \mathrm{D} \end{array} \right\}$$

$$= \frac{\Gamma(\alpha_1+\cdots+\alpha_r)\prod_{i=1}^{r}\Gamma(\alpha_i+\lambda_i)\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)\Gamma(\alpha_1+\lambda_1+\cdots+\alpha_r+\lambda_r)\Gamma(2(a+\alpha_1+\cdots+\alpha_r))\Gamma(1/2-a-b-\alpha_1-\cdots-\alpha_r)}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 4^{-\zeta_1-\lambda_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_r-\lambda_r}z_r \end{array} \left| \begin{array}{c} (\text{1-2a-2}\alpha_1-\cdots-2\alpha_r; 2(\zeta_1+\lambda_1),\cdots,2(\zeta_r+\lambda_r)), \\ \cdots \\ \cdots \\ (\text{1/2-a+b-}\alpha_1-\cdots-\alpha_r; \zeta_1+\eta_1+\lambda_1,\cdots,\zeta_r+\eta_r+\lambda_r) \end{array} \right. \right.$$

$$\left. \begin{array}{c} (\text{1/2+a+b+}\alpha_1+\cdots+\alpha_r; \eta_1-\zeta_1-\lambda_1,\cdots,\eta_r-\zeta_r-\lambda_r), A : C \\ \cdots \\ \cdots \\ \mathrm{B} : \mathrm{D} \end{array} \right) \quad (3.2)$$

where $U_{21} = p_i + 2; q_i + 1; \tau_i; R$

Provided that

a) $0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, Re(\lambda_i) > 0, i = 1, \cdots, r$

b) $\left|\frac{arg z_k}{4^{\zeta_k+\lambda_k}}\right| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.3)

4. Special cases

a) For $\lambda_i = 0, i = 1, \cdots, r$, (3.2) reduces to (3.1)

b) For $\eta_i = \zeta_i, i = 1, \cdots, r$, (3.1) reduces

$$R^{(a,b)}_{\alpha_1,\cdots,\alpha_r}\left\{ \aleph^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} z_1(x_1+\cdots+x_r)^{\zeta_1}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\zeta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r(x_1+\cdots+x_r)^{\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\zeta_r} \end{array} \right| \right.$$

$$\left. \begin{array}{c} \mathrm{A : C} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{B : D} \end{array} \right\} = \frac{\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{\Gamma(2a+2\alpha_1+\cdots+2\alpha_r)\Gamma(1/2-a-b-\alpha_1-\cdots-\alpha_r)} \aleph^{0,\mathfrak{n}+1:V}_{U_{11}:W}\left(\begin{array}{c} 4^{-\zeta_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_r}z_r \end{array} \right|$$

$$\left. \begin{array}{c} (1\text{-}2a\text{-}2\alpha_1-\cdots-2\alpha_r;2\zeta_1,\cdots,2\zeta_r), A:C \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ (1/2\text{-}a\text{+}b\text{-}\alpha_1-\cdots-\alpha_r;2\zeta_1,\cdots,2\zeta_r), B:D \end{array} \right) \tag{4.1}$$

where $U_{11} = p_i+1; q_i+1; \tau_i; R$

Provided that

a) $0 < Re(a+\alpha_1+\cdots+\alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, i = 1, \cdots, r$

b) $\left|\frac{argz_k}{4^{\zeta_k}}\right| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is given in (1.3)

5.Multivariable I-function

If $\tau_i, \tau_{i^{(1)}}, \cdots, \tau_{i^{(r)}} \rightarrow 1$, the Aleph-function of several variables degenere to the I-function of several variables. The two formulas have been derived in this section for multivariable I-functions defined by Sharma et al [3].

$$R^{(a,b)}_{\alpha_1,\cdots,\alpha_r}\left\{ I^{0,\mathfrak{n}:V}_{U:W}\left(\begin{array}{c} z_1(x_1+\cdots+x_r)^{\zeta_1}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r(x_1+\cdots+x_r)^{\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_r} \end{array} \right| \right.$$

$$\left.\begin{matrix} \text{A : C} \\ \cdots \\ \cdots \\ \text{B : D} \end{matrix}\right\} = \frac{\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{\Gamma(2a+2\alpha_1+\cdots+2\alpha_r)\Gamma(1/2-a-b-\alpha_1-\cdots-\alpha_r)} I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix} 4^{-\zeta_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_r}z_r \end{matrix}\right|$$

$$\left.\begin{matrix} (\text{1-2a-2}\alpha_1-\cdots-2\alpha_r;2\zeta_1,\cdots,2\zeta_r),(1/2+a+b+\alpha_1+\cdots+\alpha_r;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),A:C \\ \cdots \\ \cdots \\ (\text{1/2-a+b-}\alpha_1-\cdots-\alpha_r;\zeta_1+\eta_1,\cdots,\zeta_r+\eta_r),B:D \end{matrix}\right) \quad (5.1)$$

under the same notations and conditions that (3.1)

$$R_{\alpha_1,\cdots,\alpha_r}^{(a,b)}\left\{ I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{matrix} \mathrm{z}_1 x_1^{\lambda_1}(x_1+\cdots+x_r)^{\zeta_1}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x_r^{\lambda_r}(x_1+\cdots+x_r)^{\zeta_r}\left[(x_1+\cdots+x_r)^{1/2}+(1+x_1+\cdots+x_r)^{1/2}\right]^{-2\eta_r} \end{matrix}\right|\right.$$

$$\left.\begin{matrix} \text{A : C} \\ \cdots \\ \cdots \\ \text{B : D} \end{matrix}\right\}$$

$$= \frac{\Gamma(\alpha_1+\cdots+\alpha_r)\prod_{i=1}^r\Gamma(\alpha_i+\lambda_i)\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_r)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)\Gamma(\alpha_1+\lambda_1+\cdots+\alpha_r+\lambda_r)\Gamma(2(a+\alpha_1+\cdots+\alpha_r))\Gamma(1/2-a-b-\alpha_1-\cdots-\alpha_r)}$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{matrix} 4^{-\zeta_1-\lambda_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_r-\lambda_r}z_r \end{matrix}\middle| \begin{matrix} (\text{1-2a-2}\alpha_1-\cdots-2\alpha_r;2(\zeta_1+\lambda_1),\cdots,2(\zeta_r+\lambda_r)), \\ \cdots \\ \cdots \\ (\text{1/2-a+b-}\alpha_1-\cdots-\alpha_r;\zeta_1+\eta_1+\lambda_1,\cdots,\zeta_r+\eta_r+\lambda_r) \end{matrix}\right.$$

$$\left.\begin{matrix} (\text{1/2+a+b+}\alpha_1+\cdots+\alpha_r;\eta_1-\zeta_1-\lambda_1,\cdots,\eta_r-\zeta_r-\lambda_r),A:C \\ \cdots \\ \cdots \\ \text{B : D} \end{matrix}\right) \quad (5.2)$$

under the same conditions and notations that (3.2) with $\tau_i, \tau_{i^{(1)}}, \cdots, \tau_{i^{(r)}} \rightarrow 1$

6. Aleph-function of two variables

If $r = 2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following two relations.

$$R_{\alpha_1,\alpha_2}^{(a,b)}\left\{ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(x_1+x_2)^{\zeta_1}\left[(x_1+_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_2(x_1+x_2)^{\zeta_2}\left[(x_1+x_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_2} \end{array} \middle| \begin{array}{c} \mathrm{A}:\mathrm{C} \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array} \right) \right\}$$

$$= \frac{\Gamma(1/2+a-b+\alpha_1+\alpha_2)}{\Gamma(2a+2\alpha_1+2\alpha_2)\Gamma(1/2-a-b-\alpha_1-\alpha_2)} \aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 4^{-\zeta_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_2}z_2 \end{array} \middle| \right.$$

$$\left. \begin{array}{c} (\text{1-2a-2}\alpha_1-2\alpha_2;2\zeta_1,2\zeta_2),(1/2+a+b+\alpha_1+\alpha_2;\eta_1-\zeta_1,\eta_2-\zeta_2),A:C \\ \cdots \\ \cdots \\ (\text{1/2-a+b-}\alpha_1-\alpha_2;\zeta_1+\eta_1,\zeta_2+\eta_2),B:D \end{array} \right) \quad (6.1)$$

under the same notations and conditions that (3.1) with $r = 2$

$$R_{\alpha_1,\alpha_2}^{(a,b)}\left\{ \aleph_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1 x_1^{\lambda_1}(x_1+x_2)^{\zeta_1}\left[(x_1+_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_2 x_2^{\lambda_2}(x_1+x_2)^{\zeta_2}\left[(x_1+x_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_2} \end{array} \middle| \begin{array}{c} \mathrm{A}:\mathrm{C} \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array} \right) \right\}$$

$$= \frac{\Gamma(\alpha_1+\alpha_2)\prod_{i=1}^{2}\Gamma(\alpha_i+\lambda_i)\Gamma(1/2+a-b+\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_1+\lambda_1+\alpha_2+\lambda_2)\Gamma(2(a+\alpha_1+\alpha_2))\Gamma(1/2-a-b-\alpha_1-\alpha_2)}$$

$$\aleph_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 4^{-\zeta_1-\lambda_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_2-\lambda_2}z_2 \end{array} \middle| \begin{array}{c} (\text{1-2a-2}\alpha_1-2\alpha_2;2(\zeta_1+\lambda_1),2(\zeta_2+\lambda_2)), \\ \cdots \\ \cdots \\ (\text{1/2-a+b-}\alpha_1-\alpha_2;\zeta_1+\eta_1+\lambda_1,\zeta_2+\eta_2+\lambda_2) \end{array} \right.$$

$$\left.\begin{array}{c} (1/2+a+b+\alpha_1+\alpha_2;\eta_1-\zeta_1-\lambda_1,\eta_2-\zeta_2-\lambda_2),A:C \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right) \tag{6.2}$$

under the same conditions and notations that (3.2) with $r=2$

7. I-function of two variables

If $\tau_i,\tau_i',\tau_i''\rightarrow 1$, then the Aleph-function of two variables degenere in the I-function of two variables defined by sharma et al [4] and we obtain the same formulas with the I-function of two variables.

$$R_{\alpha_1,\alpha_2}^{(a,b)}\left\{ I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1(x_1+x_2)^{\zeta_1}\left[(x_1+_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_2(x_1+x_2)^{\zeta_2}\left[(x_1+x_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_2} \end{array}\left|\begin{array}{c} \mathrm{A}:\mathrm{C} \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right.\right)\right\}$$

$$=\frac{\Gamma(1/2+a-b+\alpha_1+\alpha_2)}{\Gamma(2a+2\alpha_1+2\alpha_2)\Gamma(1/2-a-b-\alpha_1-\alpha_2)} I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c} 4^{-\zeta_1}z_1 \\ \cdot \\ \cdot \\ \cdot \\ 4^{-\zeta_2}z_2 \end{array}\right|$$

$$\left.\begin{array}{c} (1\text{-}2a\text{-}2\alpha_1-2\alpha_2;2\zeta_1,2\zeta_2),(1/2+a+b+\alpha_1+\alpha_2;\eta_1-\zeta_1,\eta_2-\zeta_2),A:C \\ \cdots \\ \cdots \\ (1/2\text{-}a\text{+}b\text{-}\alpha_1-\alpha_2;\zeta_1+\eta_1,\zeta_2+\eta_2),B:D \end{array}\right) \tag{7.1}$$

under the same notations and conditions that (3.1) with $r=2$

$$R_{\alpha_1,\alpha_2}^{(a,b)}\left\{ I_{U:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c} \mathrm{z}_1x_1^{\lambda_1}(x_1+x_2)^{\zeta_1}\left[(x_1+_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_2x_2^{\lambda_2}(x_1+x_2)^{\zeta_2}\left[(x_1+x_2)^{1/2}+(1+x_1+x_2)^{1/2}\right]^{-2\eta_2} \end{array}\left|\begin{array}{c} \mathrm{A}:\mathrm{C} \\ \cdots \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right.\right)\right\}$$

$$= \frac{\Gamma(\alpha_1+\alpha_2)\prod_{i=1}^{2}\Gamma(\alpha_i+\lambda_i)\Gamma(1/2+a-b+\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_1+\lambda_1+\alpha_2+\lambda_2)\Gamma(2(a+\alpha_1+\alpha_2))\Gamma(1/2-a-b-\alpha_1-\alpha_2)}$$

$$I_{U_{21}:W}^{0,\mathfrak{n}+2:V}\left(\begin{array}{c|c} 4^{-\zeta_1-\lambda_1}z_1 & (1\text{-}2a\text{-}2\alpha_1-2\alpha_2;2(\zeta_1+\lambda_1),2(\zeta_2+\lambda_2)), \\ \cdot & \cdots \\ \cdot & \cdots \\ 4^{-\zeta_2-\lambda_2}z_2 & (1/2\text{-}a\text{+}b\text{-}\alpha_1-\alpha_2;\zeta_1+\eta_1+\lambda_1,\zeta_2+\eta_2+\lambda_2) \end{array}\right.$$

$$\left.\begin{array}{c} (1/2\text{+}a\text{+}b\text{+}\alpha_1+\alpha_2;\eta_1-\zeta_1-\lambda_1,\eta_2-\zeta_2-\lambda_2),A:C \\ \cdots \\ \cdots \\ \mathrm{B:D} \end{array}\right) \tag{7.2}$$

under the same conditions and notations that (3.2) with $r=2$

8. Conclusion

In this paper we have evaluated two multidimensional integral transform concerning the multivariable Aleph-function. The formulas established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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