Vertex Bimagic Total Labeling for Graphs

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Abstract. A vertex magic total labeling on a graph with v vertices and e edges is a one to one map taking the vertices and edges onto the integers $1,2,3,...v + e$ with the property that the sum of the label on the vertex and the labels of its incident edges is a constant, independent of the choice of the vertex. A graph with vertex magic total labeling with two constants k_1 or k_2 is called a vertex bimagic total labeling. The constants k_1 and k_2 are called magic constants. In this paper we have found that the star graphs $K_{1,n}$ for all n, cycle C_n when n is even, crown graphs C_nK_1, $(3,n)-$kite graph for odd $n > 3$, wheel graph W_n ,the fan graph F_n admits vertex bimagic total labeling. It has been proved that the star graphs $K_{1,n}$ has vertex bimagic total labeling for all n, but does not have vertex magic total labeling for $n \neq 2$. It has also been discussed that the vertex bimagic total labeling of Petersen graphs $P(n,1)$ and $P(n,2)$ when n is a multiple of 4, $m \leq \left\lceil \frac{n}{2} \right\rceil$. Also the extremities of bimagic constants in vertex bimagic total labeling graphs has also been discussed in this paper.

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Key words: vertex bimagic total labeling, bimagic constants.

I. INTRODUCTION

W. D. Wallis and others [9][10], introduced Edge-magic total labelings that generalize the idea of a magic square and can be referred for a discussion of magic labelings and a standardization of the terminology. J. Baskar Babujee & V.Vishnu Priya have introduced (1,1) edge bimagic labeing in their paper "Edge Bimagic labeling in certain types of graphs obtained by some standard graphs" [2]. Also V.Vishnu Priya, K.Manimegalai & J. Baskar Babujee [8] have proved edge bimagic labeling for some trees like $B_{m,n}$, $K_{1,n,n}$,Y_{n+1} , J. Baskar Babujee has himself introduced (1,1) vertex bimagic labeling in [1]which is named vertex bimagic total labeling.

K. Manimekalai and K. Thirusangu [4] have worked on Pair Sum Labeling of Some Special Graphs. M. I. Moussa and E. M. Badr [5] have proved that the crown graphs are odd graceful. N. Murugesan and R. Senthil Amutha[6] have discussed that the bistar $B_{n,n}$ are vertex bimagic total labeling for odd $n > 1$ and even $n > 2$. S. Karthikeyan, S. Navaneethakrishnan , and R. Sridevi[3] , proved that the star graph, Subdivisions of bistar graphs are total edge Fibonacci irregular graphs. In this paper we have discussed that the graphs star graphs $K_{1,n}$ bistar $B_{n,n}$ (odd $n > 1$ and even $n > 2$), cycle C_n , wheel W_n ,crown graphs C_nK_1 and $(3,n)$ kite graphs $(n > 3, n \text{ is odd})$, Fan graph F_n have vertex bimagic total labeling. The extremities of bimagic constants in vertex bimagic total labeling graphs has also been discussed in this paper. We further establish if a regular graph possesses vertex magic total labeling, then it also possesses vertex bimagic total labeling and any non regular graph with two different degrees at vertices, with vertex magic total labeling possesses vertex bimagic total labeling.

A graph with vertex magic total labeling with two constants k_1 or k_2 is called a vertex bimagic total labeling and denoted by VBMTL. The constants k_1 and k_2 are called bimagic constants.

A Star graph with n vertices is a tree with one vertex having degree $n-1$ and the remaining $(n-1)$ vertices with degree one. The graphs obtained by joining a single pendant edge to each vertex of C_n is called crown graph and is denoted by $C_n \square K_1$. The (m,n) kite graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge. The (m,n) kite graph is also called (m,n) tadpole graph [7].

Definition1.1: A graph with vertex magic total labeling with two constants k_1 *or* k_2 is called a vertex bimagic total labeling and denoted by VBMTL. The constants k_1 and k_2 are called bimagic constants.

In this section, due to symmetry instead of generic values, specific vertices are considered throughout, to find thee vertex weights.

II. VERTEX BIMAGIC TOTAL LABELING FOR STAR RELATED GRAPHS

Theorem 2.1. The star graphs $K_{1,n}$ has vertex bimagic total labeling for all n.

Proof:

Let the star graph $K_{1,n}$ be with $n+1$ vertices and n edges with $V(G) = \{v, v_1, v_2, \dots, v_n\}$ be the vertex set of G such that v be the central vertex of the star and $v_1, v_2, \dots, v_{n-1}$ be the pendant vertices of the star graph and $E(G) = \{e_1, e_2, \dots, e_n\}$ be the edge set of G.

We can define the bijection as $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows.

$f(v) = n+1.$

$f(v_i) = n+1+i, \quad 1 \le i \le n$

$f(vv_i) = n+1-i, \quad 1 \le i \le n$

Now,

$w(v) = f(v) + \sum_{i=1}^{n} f(vv_i)$

$= (n+1) + \sum_{i=1}^{n} (n+1-i)$

$= (n+1) + n(n+1) - \sum_{i=1}^{n} i$

$= (n+1) + n(n+1) - \frac{n(n+1)}{2}$

$= \frac{1}{2}[2(n+1) + 2n(n+1) - n(n+1)]$

$= \frac{1}{2}[(n+1)(2+2n-n)] = \frac{(n+1)(n+2)}{2}$

Also,

$w(v_1) = f(v_1) + f(vv_1)$

$= (n+1+1) + (n+1-1)$

$= (2n+2) = 2(n+1)$

We find that the weight $\frac{(n+1)(n+2)}{2}$ exists for one vertex and the weight $2(n+1)$ exists for n vertices. We may consider the bimagic constants k_1 and k_2 such that k_1 is at the pendant vertices and k_2 is at the central vertex. Hence the star graph $K_{1,n}$ is vertex bimagic total labeling for all n

A complete bipartite graph $K_{1,1,n}$ is also a star and it has $n+2$ vertices and $n+1$ edges.

Now we discuss that bistar $\boldsymbol{B_{n,n}}$ are vertex bimagic total labeling for odd $\boldsymbol{n} > 1$ and even $\boldsymbol{n} > 2$

III. VERTEX BIMAGIC TOTAL LABELING FOR CYCLE RELATED GRAPHS

Theorem 3.1: The cycle C_n, has vertex bimagic total labeling, for n is even

Proof:

Let the cycle C_n be with n vertices and n edges with $V(G) = \{v_1, v_2, \dots, v_n\}$ be the vertex set and $E(G) = \{e_1, e_2, \dots, e_n\}$ be the edge set such that $e_i = (v_i v_{i+1})$ where $i = 1$ to n and the indices taken modulo n.

We can define the bijection as

$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows.

$f(v_i) = n+i, \quad 1 \le i \le n$

$$f(v_iv_{i+1}) = \begin{cases} \frac{n+1-i}{2} & for\ \ i\ is\ odd \\ \frac{n+i}{2} & for\ i=2 \\ n+2-\frac{i}{2} & for\ \ i\ is\ even\ and\ i \geq 4 \end{cases}$$

Now the weights are given by,

For the vertex v_1,

$w(v_1) = f(v_1) + f(v_nv_1) + f(v_1v_2) = (n+1) + \left(n+2-\frac{n}{2}\right) + \left(\frac{n+1-1}{2}\right) = 2n+3$

For the vertex v_2,

$w(v_2) = f(v_2) + f(v_1v_2) + f(v_2v_3) = (n+2) + \frac{n}{2} + \frac{n+2}{2} = 2n+3$

For the vertex v_n,

$w(v_n) = f(v_n) + f(v_{n-1}v_n) + f(v_nv_1) = 2n + \frac{n+1-n+1}{2} + n+2-\frac{n}{2} = \frac{1}{2}(5n+6)$

We find that the bimagic constants are $2n+3$ and $\frac{1}{2}(5n+6)$. Let these bimagic constants be k_1 and k_2 respectively. Hence the cycle C_n has vertex bimagic total labeling for even .

Theorem 3.3: The crown graphs $C_n \odot K_1$ has vertex bimagic total labeling, for all n.

Proof:

Let the crown graph $C_n \odot K_1$ be with $2n$ vertices and $2n$ edges with

$V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ be the vertex set such that $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_n; $u_1, u_2, \dots, u_n$ be the pendant vertices attached to $v_1, v_2, \dots, v_n$ respectively and t $E(G) = \{e_1, e_2, \dots, e_{2n}\}$ be the edge set.

We can define the bijection as $f: V(G) \cup E(G) \rightarrow \{1,2,\dots,4n\}$ as follows.

The vertex labeling is defined as

$f(u_i) = i, \quad i = 1,2,\dots,n$

$$f(v_i) = \begin{cases} 3n-i, & i=1,2,\dots,n-1 \\ 3i, & i=n \end{cases}$$

The edge labeling is defined as

$f(u_iv_i) = 4n+1-i, \quad i = 1,2,\dots,n$

$f(v_iv_{i+1}) = n+1+i, \quad i = 1,2,\dots,n-1$

$f(v_1v_n) = n+1.$

Now the weights are,

For $i < n$,

$w(v_1) = f(v_1) + f(v_1v_2) + f(v_1v_n) + f(u_1v_1)$

$= (3n-1) + (n+2) + (n+1) + 4n$

$= 9n+2$

For $i = n$,

$w(v_n) = f(v_n) + f(v_{n-1}v_n) + f(v_1v_n) + f(u_nv_n)$

$= (3n) + (n+1) + (n-1) + n+1 + (4n+1-n) = 9n+2$

And for pendant vertices,

$w(u_1) = f(u_1) + f(u_1v_1)$

$= 1 + 4n + 1 - 1 = 4n+1$

Let the bimagic constants $4n+1$ and $9n+2$ be k_1 and k_2 respectively. Hence the crown graphs, $C_n \odot K_1$ has vertex bimagic total labeling.

Theorem3.5: The $(3,n)$ kite graph has vertex bimagic total labeling for odd $n > 3$.

Proof:

Let G be the $(3,n)$ kite graph of $n+3$ vertices and $n+3$edges with $V(G) = \{v_1, v_2, v_3, u_1, u_2, \dots, u_n\}$ be the vertex set of G such that v_1, v_2, v_3 be the vertices of the cycle ; $u_1, u_2, \dots, u_n$ be the vertices of the path and $E(G) = \{e_1, e_2, \dots, e_{n+3}\}$ be the edge set of G.

Define $f: V(G) \cup E(G) \rightarrow \{1,2,\dots,2n+6\}$ as follows.

$f(v_i) = 2n+2i, \quad i = 1,2.$

$f(v_3) = 2n+2.$

$$f(u_i) = \begin{cases} 2n-2i+2, & for\ \ 1 \leq i \leq \frac{n-3}{2} \\ 3n-2i+2, & for\ \ \frac{n-1}{2} \leq i \leq n-1 \end{cases}$$

$f(u_n) = 2n+6$

$f(v_1u_1) = 4$

$f(v_iv_{i+1}) = i+1, \quad i = 1,2.$

$f(v_iv_{i-2}) = 1, \quad i = 3.$

$f(u_iu_{i+1}) = i+4, \quad 1 \leq i \leq n-1.$

The vertex weights are as follows,

For i*=1*, $w(v_1) = f(v_1) + f(v_1u_1) + f(v_1v_2) + f(v_3v_1)$ = $(2n+2) + 4 + 2 + 1$ = $2n+9$

For i*=3*, $w(v_3) = f(v_3) + f(v_2v_3) + f(v_3v_1)$ = $(2n+5) + 3 + 1$ = $2n+9$

For $1 \le i \le \frac{n-3}{2}$, $w(u_1) = f(u_1) + f(u_1v_1) + f(u_1u_2)$ =$2n + 4 + 5$ =$2n+9$

For $\frac{n-1}{2} \le i \le n-1$, $w(u_{n-1}) = f(u_{n-1}) + f(u_{n-1}u_n) + f(u_{n-2}u_{n-1})$

= $(3n - 2(n-1) + 2) + (n - 1 + 4) + (n - 2 + 4)$ =$3n+9$

For $i = n$, $w(u_n) = f(u_n) + f(u_{n-1}u_n)$ = $2n + 6 + n - 1 + 4$ =$3n+9$

So, we get the bimagic constants $2n+9$ and $3n+9$ which may be named as k_1 and k_2 respectively. Hence the $(3,n)$ kite graph has vertex bimagic total labeling.

Theorem 3.7: The wheel W_n, has vertex bimagic total labeling for all n

Proof:

Let the wheel W_n, be with $n+1$ vertices and 2n edges with $V(G) = \{v, v_1, v_2, \dots, v_n\}$ be the vertex set and $E(G) = \{e_1, e_2, \dots, e_{2n}\}$ be the edge set such that $e_i = \{(vv_i) \cup (v_iv_{i+1})\}$

We can define the bijection as $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3n+1\}$ *as follows,*

$f(v) = 1$

$f(v_i) = 2(n+1) + i, \quad 1 \le i \le n-1$

$f(v_n) = 2(n+1)$

$f(vv_i) = i + 1, \quad 1 \le i \le n$

$f(v_iv_{i+1}) = 2n + 1 - i, \quad 1 \le i \le n-1$

$f(v_1v_n) = 2n+1$

Then the weights can be found as,

For the vertex at the centre,

$w(v)$ = $f(v) + \sum_{i=1}^{n} f(vv_i)$ = $1 + \sum_{i=1}^{n}(i+1)$ = $(n+1) + \sum_{i=1}^{n} i$

= $(n+1) + \frac{n(n+1)}{2}$

= $\frac{1}{2}[2(n+1) + n(n+1)]$ =$\frac{1}{2}[(n+1)(2+n)]$ = $\frac{1}{2}[(n+1)(n+2)]$

For the vertices on the cycle , $1 \le i \le n-1$

$w(v_1)$ = $f(v_1) + f(v_1v_n) + f(v_1v_2) + f(vv_1)$ = $((2(n+1)+1) + (2n+1) + (2n+1-1) + 2$ = $6(n+1)$

For $i = n$,

$w(v_n)$ = $f(v_n) + f(v_1v_n) + f(v_{n-1}v_n) + f(vv_n)$

= $(2(n+1) + (2n+1) + 2n + 1 - (n-1) + n + 1$ = $6(n+1)$

The bimagic constants are $\frac{1}{2}[(n+1)(n+2)]$ and $6(n+1)$ say k_1 and k_2 respectively

Hence wheel W_n, has vertex bimagic total labeling for all n .

IV. Vertex Bimagic Total Labeling of Special Graphs

Theorem 4.1: The Fan F_n graph has vertex bimagic total labeling for all n.

Proof:

Let G be the Fan graph with $n+1$ vertices and $2n-1$ edges with $V(G) = \{u, u_1, u_2, \dots, u_n\}$ be the vertex set of Gsuch that u be the vertex at the centre; $u_1, u_2, \dots, u_n$ be the vertices adjacent to u and $E(G) = \{uu_i \cup u_iu_{i+1} \; with \; i = 1 \; to \; n\}$ be the edge set of G.

Let us define $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, 3n\}$ as follows,

$f(u) = 2n$

$f(u_i) = 3n + 1 - i, \quad i = 1 to\ n$

$$f(uu_i) = \begin{cases} 2n - 1 - i, & i = 1 \; to \; n-1 \\ 2n - 1, & i = n \end{cases}$$

$f(u_iu_{i+1}) = i, \quad i = 1 \; to \; n-1$

The vertex weights are given by,

At the vertex adjacent to u,

$w(u_1) = f(u_1) + f(uu_1) + f(u_1u_2)$ = $(3n + 1 - 1) + (2n - 1 - 1) + 1 = 5n - 1$

At the centre vertex,

$$w(u) = f(u) + \sum_{i=1}^{n} f(uu_i)$$

$= 2n + \sum_{i=1}^{n-1}(2n - 1 - i) + 2n - 1$

$= 2n + (2n - 1)(n - 1) - \sum_{i=1}^{n-1} i + 2n - 1$

$= 2n + (2n - 1)(n) - \frac{n(n-1)}{2}$

$= \frac{1}{2}(3n^2 + 3n) = \frac{3n(n+1)}{2}$

At the vertices u_i's the magic constant is $5n - 1$ and at the centre it is $\frac{3n(n+1)}{2}$ say k_1 and k_2 be the bimagic constants. Hence the Fan F_n graph is vertex bimagic total labeling for all n

Theorem4.3: The disconnected graph nK_3 has vertex bimagic total labeling for all n

Proof:

Let u_i, v_i, w_i be the nodes of the graph K_3 and nK_3 contains $3n$ vertices and 3n edges.Now we define the function $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, 6n\}$ as follows,

$$f(u_i) = i, \qquad i = 1 to\ n$$
$$f(v_i) = n + i, \qquad i = 1 to\ n$$
$$f(w_i) = 2n + i, \qquad i = 1 to\ n$$
$$f(u_i v_i) = 4n + i, \qquad i = 1 to\ n$$
$$f(u_i w_i) = 3n + i, \qquad i = 1\ to\ n$$
$$f(v_i w_i) = 5n + i, \qquad i = 1\ to\ n$$

The weights are given by,

$w(u_1) = f(u_1) + f(u_1 v_1) + f(u_1 w_1) = 1 + 4n + 1 + 3n + 1 = 7n + 3$

$w(v_1) = f(v_1) + f(u_1 v_1) + f(v_1 w_1) = n + 1 + 4n + 1 + 5n + 1 = 10n + 3$

$w(w_1) = f(w_1) + f(u_1 w_1) + f(v_1 w_1) = 2n + 1 + 3n + 1 + 5n + 1 = 10n + 3$

From these results we see that the disconnected graph nK_3 has vertex bimagic total labeling for all .

Theorem4.4: The star graphs $K_{1,n}$ has vertex bimagic total labeling for all n, but does not have vertex magic total labeling for $n \neq 2$

Proof:

The star graph $K_{1,n}$ be with $n + 1$ vertices and n edges with the bijection as

$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2n + 1\}$ with

$f(v) = n + 1.$

$f(v_i) = n + 1 + i, \quad 1 \leq i \leq n$

$f(vv_i) = n + 1 - i, \quad 1 \leq i \leq n$

and $w(v) = \frac{(n+1)(n+2)}{2}, w(v_1) = 2(n + 1)$,

we find that the values

$$2(n + 1) < \frac{(n+1)(n+2)}{2} \quad \text{when } n \neq 2$$

But when $n = 2$,

$$2(n + 1) = \frac{(n+1)(n+2)}{2}$$

Hence the star graphs $K_{1,n}$ has vertex bimagic total labeling for all n, but does not have vertex magic total labeling for $n \neq 2$

Theorem4.5: If a graph G is vertex bimagic total labeling, its disconnected n copies are also vertex bimagic total labeling.

Proof:

If a graph G is vertex bimagic total labeling, all the corresponding vertices (edges) in the n copies with consecutive numbers from 1 to n in the same manner as in G, the graph nG becomes vertex bimagic total labeling.

Theorem4.6: If a r-regular graph possesses vertex magic total labeling with the function $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, p + q\}$ and magic constant k, then it also possesses vertex bimagic total labeling using the function $f': V(G) \cup E(G) \rightarrow \{1,2, \dots, p + q\}$ defined by

$$f'(u) = \begin{cases} p + q - f(u) & for\ f(u) < p + q \\ p + q & for\ f(u) = p + q \end{cases}$$

$$f'(uv) = \begin{cases} p + q - f(uv) & for\ f(uv) < p + q \\ p + q & for\ f(uv) = p + q \end{cases}$$

Proof:

Consider a r-regular graph which possesses vertex magic total labeling with the function $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, p + q\}$ and the weight is given by $w_f(u)$ or magic constant k where

$w_f(u) = f(u) + \sum f(uv)$

$k = f(u) + \sum f(uv)$ ------------*(a)*

$\sum f(uv) = k - (p + q)$ ------------*(b)* then the function of the vertex bimagic total labeling given by

$f' : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ defined by

$$f'(u) = \begin{cases} p+q-f(u) & for\ f(u) < p+q \\ p+q & for\ f(u) = p+q \end{cases}$$

$$f'(uv) = \begin{cases} p+q-f(uv) & for\ f(uv) < p+q \\ p+q & for\ f(uv) = p+q \end{cases}$$

has weight to be $w_{f'}(u)$. Then there exists the following cases for vertex bimagic total labeling.

Case(i): $f'(uv) \neq p+q;\ f'(u) = p+q$

$w_{f'}(u) = f'(u) + \sum f'(uv)$

$= (p+q) + \sum (p+q) - f(uv)$

$= (p+q) + r(p+q) - \sum f(uv)$

$= f(u) + r(p+q) + \sum f(uv) - 2\sum f(uv)$

$= f(u) + \sum f(uv) + r(p+q) - 2\sum f(uv)$

$= k + r(p+q) - 2[k-(p+q)]$

$= k + r(p+q) - 2k + 2(p+q)$

$= (r+2)(p+q) - k$

Case(ii): $f'(uv) \neq p+q;\ f'(u) \neq p+q$

$w_{f'}(u) = f'(u) + \sum f'(uv)$

$= (p+q) - f(u) + \sum (p+q) - f(uv)$

$= (r+1)(p+q) - [f(u) + \sum f(uv)]$

$= (r+1)(p+q) - k$

Case(iii): $f'(uv) = p+q;\ f'(u) \neq p+q$

$w_{f'}(u) = f'(u) + \sum f'(uv) = [(p+q) - f(u)] + [\sum_1^{r-1}(p+q) - f(uv) + (p+q)]$

$= (p+q) - f(u) + r(p+q) - \sum_1^{r-1} f(uv)$

$= (r+1)(p+q) - [f(u) + \sum_1^{r-1} f(uv)]$

$= (r+1)(p+q) - [f(u) + \sum_1^{r} f(uv) - f(uv)]$

$= (r+1)(p+q) - k + f(uv)$

$= (r+1)(p+q) - k + (p+q$

$= (r+2)(p+q) - k$

when the edge label is $p+q$, the bimagic constants of vertex bimagic total labeling are as in cases (i) and (ii), when the vertex label is $p+q$, the bimagic constants of vertex bimagic total labeling are as in cases (ii) and (iii). Hence the proof.

***Theorem 4.8*:** If $f : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ is the function of vertex magic total labeling, then any non regular graph with two different degrees at vertices, with vertex magic total labeling possesses vertex bimagic total labeling with the function $f' : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ *as*

$$f'(u) = p+q+1-f(u)$$

$$f'(uv) = p+q+1-f(uv)$$

Proof:

Consider a non regular graph with two different degrees at vertices, which possesses vertex magic total labeling with the function $f : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ and the weight is given by $w_f(u)$ or magic constant k where

$$w_f(u) = f(u) + \sum f(uv)$$

then the function of the vertex bimagic total labeling given by

$f' : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ is defined by

$$f'(u) = p+q+1-f(u)$$

$$f'(uv) = p+q+1-f(uv)$$

has weight to be $w_{f'}(u)$. Then there exists the following cases for vertex bimagic total labeling.

Case(i): *if* $deg(u) = n$

$w_{f'}(u) = f'(u) + \sum f'(uv) = (p+q+1) - f(u) + \sum (p+q+1) - f(uv)$

$= (p+q+1) - f(u) + n(p+q+1) - \sum f(uv)$

$= (n+1)(p+q+1) - \left[f(u) + \sum f(uv)\right]$

$= (n+1)(p+q+1) - k$

Case(ii): if $deg(u) = n+1$

$$w_{f'}(u) = (n+2)(p+q+1) - k$$

As there are bimagic constants as in cases (i) and (ii), the result is true.

V. Basic Counting for VBMTL

Let s_p denote the sum of vertex lables and s_q be the sum of the edge labels in a vertex bimagic total labeling f, since the labels are the numbers $1,2,\dots,p+q$, the sum of all labels be represented by $= \sigma_0{}^{p+q}$ which means that the sum starts after 0, in the subscript upto the values p + q in the superscript, is

$$s_p + s_q = \sigma_0{}^{p+q} = \binom{p+q+1}{2}$$

$$= \frac{(p+q)(p+q+1)}{2} \quad \text{---------------}(1)$$

At each vertex u_i ,we get

$$f(u_i) + \sum f(u_i v_i) = k_1 \; or \; k_2$$

Summing this over n_1 and $n_2$vertices , u_i is equivalent to adding each vertex label once and each edge label twice so that ,

$$s_p + 2s_q = n_1k_1 + n_2k_2 \quad \text{--------------}(2)$$

Where n_1 and n_2 are the number of vertices which receive the constant k_1 and k_2 respectively and

$p = n_1 + n_2$

Combining (1) and (2) we get

$$\binom{p+q+1}{2} + s_q = n_1k_1 + n_2k_2 \quad \text{--------}(3)$$

The edge labels are all distinct (as are all vertex labels).The edges could receive the q smallest labels or

at the extreme q largest labels or anything between.

$\therefore$ we have, $\sigma o^q \le s_q \le \sigma p^{p+q}$ ------------*(4)*

A similar result holds for s_p.

Now, combining (3) and (4) we get ,

$$\binom{p+q+1}{2} + \binom{q+1}{2} \le n_1k_1 + n_2k_2$$

$$\le 2\binom{p+q+1}{2} - \binom{p+1}{2} \quad \text{--------------}(5)$$

which gives the feasible range of $k_i{}'s$

From (5) we get ,

$$\binom{p+q+1}{2} + \binom{q+1}{2} - n_2k_2 \le n_1k_1$$

$$\le 2\binom{p+q+1}{2} - \binom{p+1}{2} - n_2k_2$$

$$\Rightarrow \frac{1}{n_1}\binom{p+q+1}{2} + \frac{1}{n_1}\binom{q+1}{2} - \frac{n_2}{n_1}k_2 \le k_1 \le \frac{2}{n_1}\binom{p+q+1}{2} - \frac{1}{n_1}\binom{p+1}{2} - \frac{n_2}{n_1}k_2 \quad \text{-------}(6)$$ and

$$\binom{p+q+1}{2} + \binom{q+1}{2} - n_1k_1 \le n_2k_2$$

$$\le 2\binom{p+q+1}{2} - \binom{p+1}{2} - n_1k_1$$

$$\Rightarrow \frac{1}{n_2}\binom{p+q+1}{2} + \frac{1}{n_2}\binom{q+1}{2} - \frac{n_1}{n_2}k_1 \le k_2 \le \frac{2}{n_2}\binom{p+q+1}{2} - \frac{1}{n_2}\binom{p+1}{2} - \frac{n_1}{n_2}k_1 \quad \text{-------}(7)$$

(7)-(6) gives,

$$\binom{p+q+1}{2}\left(\frac{1}{n_2} - \frac{1}{n_1}\right) + \binom{q+1}{2}\left(\frac{1}{n_2} - \frac{1}{n_1}\right) - \frac{n_1}{n_2}k_1 + \frac{n_2}{n_1}k_2 \le k_2 - k_1$$

$$\le \binom{p+q+1}{2}\left(\frac{2}{n_2} - \frac{2}{n_1}\right) - \binom{p+1}{2}\left(\frac{1}{n_2} - \frac{1}{n_1}\right) - \frac{n_1}{n_2}k_1 + \frac{n_2}{n_1}k_2 \quad \text{---------}(9)$$

This relation shows the range of the magic constants in VBMTL of graphs.

Let us verify the above relation in a corona $C_n \odot K_1$.

Consider $n = 3$, then

$p = 2n = 6, q = 2n = 6, n_1 = n = 3, n_2 = n = 3, k_1 = 13, k_2 = 29$.

On substituting the values in equation (9), we get

$$13c_2\left(\frac{1}{3} - \frac{1}{3}\right) + 7c_2\left(\frac{1}{3} - \frac{1}{3}\right) - \frac{3}{3}(13) + \frac{3}{3}(29) \le (29 - 13)$$

$$\le 2 * 13c_2\left(\frac{1}{3} - \frac{1}{3}\right) - 7c_2\left(\frac{1}{3} - \frac{1}{3}\right) - \frac{3}{3}(13) + \frac{3}{3}(29)$$

$\Rightarrow 16 \le 16 \le 16$

V. CONCLUSIONS

In this paper we have discussed that vertex bimagic total labeling exists for the graphs $K_{1,n}$ bistar $B_{n,n}$ (odd $n > 1$ and even $n > 2$), cycle C_n , wheel W_n ,crown graphs C_nK_1 and $(3,n)$ kite graphs ($n > 3, n$ is odd) , Fan graph F_n . Also the extremities of bimagic constants in vertex bimagic total labeling graphs has been discussed. We further establish the relation between vertex magic total labeling and vertex bimagic total labeling of a regular graph and non regular graph with two different degrees at vertices. Various such interesting facts can be worked on vertex bimagic total labeling .

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