More Results on Subdivision of Heronian Mean Labeling of Graphs

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ABSTRACT

In this paper, we contribute some new results for Heronian Mean labeling of graphs. We have already proved that subdivisions of Heronian Mean Graphs are again Heronian Mean Graphs. We use some more standard graphs to derive the results of Heronian Mean labeling for subdivision of graphs.

Keywords: *Graph, Heronian Mean Graph, Crown, $P_n \odot K_{1,2}$, Quadrilateral Snake.*

AMS Subject Classification: 05C78

1. Introduction

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1]. For all other standard terminology and notations we follow Harary[2].The concept of Mean labeling has been introduced by S. Somasundaram and R. Ponraj [3] in 2004. S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. Motivated by the above works we introduced a new type of labeling called **Heronian Mean Labeling in [5].**

In this paper we investigate the Subdivision of Heronian Mean Labeling of graphs for some more new graphs. We will provide brief summary of definitions and other information which are necessary for our present investigation.

The corona $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2. The graph $C_n \odot K_1$ is called crown. A **Quadrilateral Snake** $\mathbf{Q_n}$ is obtained from a path $\mathbf{u_1, u_2, \ldots\ldots, u_n}$ by joining $\mathbf{u_i}$ and $\mathbf{u_{i+1}}$ to two new vertices $\mathbf{v_i}$ and $\mathbf{w_i}$ respectively and then joining $\mathbf{v_i}$ and $\mathbf{w_i}$. That is every edge of a path is replaced by a cycle $\mathbf{C_4}$.

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices **x∈V** with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e = uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil$$

$$(\mathbf{OR}) \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **Heronian Mean labeling** of G.

Definition 1.2:

If **e=uv** is an edge of G and w is not a vertex of G then e is said to be subdivided when it is replaced by the edges **uw** and **wv**. The graph obtained by subdividing each edge of a graph G is called the subdivision of G and is denoted by **S(G).**

Theorem 1.3: Crown, $\boldsymbol{C_n \odot K_1}$ is a Heronian mean graph for all $\boldsymbol{n \geq 3}$.

Theorem 1.4: $P_n \odot K_{1,2}$ is a Heronian mean graph.

Theorem 1.5: Any Quadrilateral Snake Q_n is a Heronian mean graph.

2. Main Results

Theorem: 2.1

Subdivision of any Crown $C_n \odot K_1$ is a Heronian mean graph.

Proof:

Let $\mathbf{C_n \odot K_1}$ be a graph obtained from the cycle $u_1u_2u_3\ldots\ldots u_nu_1$ by joining the vertex u_i to pendant vertices v_i.

Let $G = S(C_n \odot K_1)$ be a graph obtained by subdividing all the edges of $C_n \odot K_1$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge of cycle $u_1u_2u_3\ldots\ldots u_nu_1$ of $C_n \odot K_1$.

Let w_i , $1 \le i \le n$ be the vertices which subdivide the edges of cycle $u_1u_2u_3\ldots\ldots u_nu_1$.

Define a function $f: V(\mathbf{G}) \to \{1,2,3,\ldots\ldots,q+1\}$ by

$$f(u_i) = 3i - 1, 1 \le i \le n.$$

$$f(v_i) = 3i - 2, 1 \le i \le n$$

$$f(w_i) = 3i, 1 \le i \le n.$$

Then we get distinct edge labels. Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

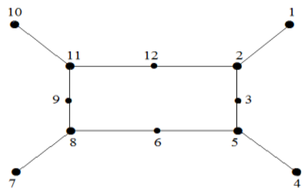

Figure:1

Case (ii):

Let G be a graph obtained by subdividing the edge u_iv_i of $C_n \odot K_1$. Let t_i , $1 \le i \le n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3,\ldots\ldots,q+1\}$ by

$$f(u_i) = \begin{cases} 3i, \text{if } i = 1,3,5,\ldots,n-1. \\ 3i-2, \text{if } i = 2,4,6,\ldots,n. \end{cases}$$

$$f(v_i) = \begin{cases} 3i-2, \text{if } i = 1,3,5,\ldots,n. \\ 3i, \text{if } i = 2,4,6,\ldots,n-1. \end{cases}$$

$$f(t_i) = 3i - 1, 1 \le i \le n.$$

Then we get distinct edge labels. Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

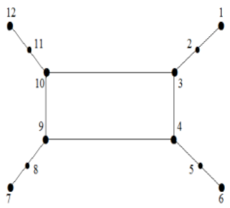

Figure:2

Case (iii):

Let G be a graph obtained by subdividing all the edges of $C_n \odot K_1$. Let w_i , $1 \le i \le n$ be the vertices which subdivide the edges of cycle $u_1u_2u_3\ldots\ldots u_nu_1$. Let t_i , $1 \le i \le n$ be the vertices which subdivide u_i and v_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3,\ldots\ldots,q+1\}$ by

$$f(u_i) = \begin{cases} 4i-1, \text{if } i = 1,3,5,\ldots,n-1. \\ 5i-5, \text{if } i = 2,4,6,\ldots,n. \end{cases}$$

$$f(v_i) = \begin{cases} 4i-3, \text{if } i = 1,3,5,\ldots,n-1. \\ 3i+1, \text{if } i = 2,4,6,\ldots,n. \end{cases}$$

$$f(w_i) = 4i, 1 \le i \le n.$$

$$f(t_i) = 4i - 2, 1 \le i \le n.$$

Then we get distinct edge labels. Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

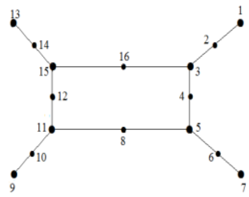

Figure:3

From all the above three cases, we conclude that $G = S(C_n \odot K_1)$ is a Heronian mean graph.

Theorem: 2.2

Subdivision of $P_n \odot K_{1,2}$ is a Heronian mean graph.

Proof:

Let $P_n \odot K_{1,2}$ be a graph obtained from a path $u_1u_2\ldots.u_n$ by joining the vertex u_i to two pendant vertices v_i and w_i.

Let $G = S(P_n \odot K_{1,2})$ be a graph obtained by subdividing all the edges of $P_n \odot K_{1,2}$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge u_iu_{i+1} of $P_n \odot K_{1,2}$. Let t_i, $1 \le i \le n-1$ be the vertices which subdivide the edges u_iu_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_i) = 4i - 2, 1 \le i \le n.$$

$$f(v_i) = 4i - 3, 1 \le i \le n.$$

$$f(w_i) = 4i - 1, 1 \le i \le n.$$

$$f(t_i) = 4i, 1 \le i \le n - 1.$$

Edges are labeled with, $f(u_iv_i) = 4i - 3, 1 \le i \le n$,

$$f(u_it_i) = 4i - 1, 1 \le i \le n - 1,$$

$$f(t_iu_{i+1}) = 4i, 1 \le i \le n - 1,$$

$$f(u_iw_i) = 4i - 2, 1 \le i \le n.$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

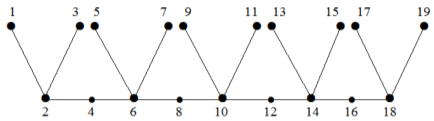

Figure:4

Case (ii):

Let G be a graph obtained by subdividing the edge u_iv_i and u_iw_i of $P_n \odot K_{1,2}$.

Let x_i , y_i, $1 \le i \le n - 1$ be the vertices which subdivide u_iv_i and u_iw_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_i) = 5i - 2, 1 \le i \le n.$$

$$f(v_i) = 5i - 4, 1 \le i \le n.$$

$$f(w_i) = 5i, 1 \le i \le n.$$

$$f(x_i) = 5i - 3, 1 \le i \le n.$$

$$f(y_i) = 5i - 1, 1 \le i \le n.$$

Edges are labeled with, $f(u_iu_{i+1}) = 5i, 1 \le i \le n - 1$,

$$f(u_ix_i) = 5i - 3, 1 \le i \le n,$$

$$f(x_iv_i) = 5i - 4, 1 \le i \le n,$$

$$f(u_iy_i) = 5i - 2, 1 \le i \le n,$$

$$f(y_iw_i) = 5i - 1, 1 \le i \le n.$$

Clearly f is a Heronian Mean labeling.

The labeling pattern is displayed below.

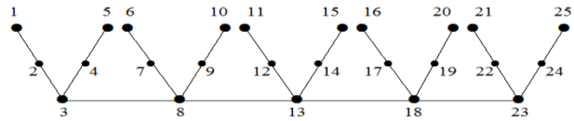

Figure:5

Case (iii):

Let G be a graph obtained by subdividing all the edges of $P_n \odot K_{1,2}$. Let x_i, y_i, $1 \le i \le n - 1$ be the vertices which subdivide u_iv_i and u_iw_i. Let t_i, $1 \le i \le n - 1$ be the vertices which subdivide the edges u_iu_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_i) = 6i - 5, 1 \le i \le n.$$

$$f(v_i) = 6i - 3, 1 \le i \le n.$$

$$f(w_i) = 6i - 1, 1 \le i \le n.$$

$$f(t_i) = 6i, 1 \le i \le n - 1.$$

$$f(x_i) = 6i - 4, 1 \le i \le n.$$

$$f(y_i) = 6i - 2, 1 \le i \le n.$$

Edges are labeled with, $f(u_it_i) = 6i - 2, 1 \le i \le n - 1$,

$$f(t_iu_{i+1}) = 6i, 1 \le i \le n - 1,$$

$$f(u_ix_i) = 6i - 5, 1 \le i \le n$$

$$f(x_iv_i) = 6i - 4, 1 \le i \le n.$$

$$f(u_iy_i) = 6i - 3, 1 \le i \le n,$$

$$f(y_iw_i) = 6i - 1, 1 \le i \le n.$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

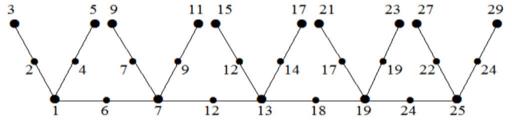

Figure:6

From all the above three cases, we conclude that $G = S(P_n \odot K_{1,2})$ is a Heronian mean graph.

Theorem: 2.3

Subdivision of any Quadrilateral Snake $\boldsymbol{Q_n}$ is a Heronian mean graph.

Proof:

Let $\mathbf{Q_n}$ be a Quadrilateral Snake obtained from a path $u_1 u_2 \ldots . u_n$ by joining u_i, u_{i+1} to new vertices w_i, z_i respectively and joining v_i and w_i for $1 \le i \le n-1$.

Let $G = S(\mathbf{Q_n})$ be a graph obtained by subdividing all the edges of $\mathbf{Q_n}$.

Here we consider the following cases.

Case (i):

Let G be a graph obtained by subdividing each edge $u_i u_{i+1}$ of $\mathbf{Q_n}$. Let x_i, $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_i) = 5i - 4, 1 \le i \le n.$$

$$f(v_i) = 5i - 3, 1 \le i \le n-1.$$

$$f(w_i) = 5i - 2, 1 \le i \le n-1.$$

$$f(x_i) = 5i, 1 \le i \le n-1.$$

Edges are labeled with, $f(u_i x_i) = 5i - 2, 1 \le i \le n-1$,

$$f(x_i u_{i+1}) = 5i, 1 \le i \le n-1.$$

$$f(u_i v_i) = 5i - 4, 1 \le i \le n-1,$$

$$f(u_{i+1} w_i) = 5i - 1, 1 \le i \le n-1,$$

$$f(v_i w_i) = 5i - 2, 1 \le i \le n-1,$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

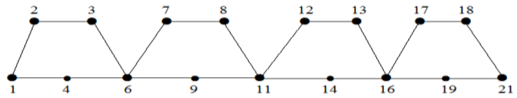

Figure:7

Case (ii):

Let G be a graph obtained by subdividing the edges $u_i v_i$ and $u_{i+1} w_i$ of $\mathbf{Q_n}$. Let s_i and t_i, $1 \le i \le n-1$ be the vertices which subdivide the edges $u_i v_i$ and $u_{i+1} w_i$.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f(u_i) = 6i - 5, 1 \le i \le n.$$

$$f(v_i) = 6i - 3, 1 \le i \le n-1.$$

$$f(w_i) = 6i - 2, 1 \le i \le n-1.$$

$$f(s_i) = 6i - 4, 1 \le i \le n-1.$$

$$f(t_i) = 6i - 1, 1 \le i \le n-1.$$

Edges are labeled with,

$$f(u_i u_{i+1}) = 6i - 2, 1 \le i \le n-1,$$

$$f(u_i s_i) = 6i - 5, 1 \le i \le n-1.$$

$$f(s_i v_i) = 6i - 4, 1 \le i \le n-1.$$

$$f(u_{i+1} t_i) = 6i, 1 \le i \le n-1.$$

$$f(t_i w_i) = 6i - 1, 1 \le i \le n-1.$$

$$f(v_i w_i) = 6i - 3, 1 \le i \le n-1.$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

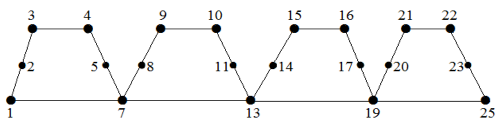

Figure:8

Case (iii):

Let G be a graph obtained by subdividing each edge $v_i w_i$ of $\mathbf{Q_n}$ .Let y_i , $1 \le i \le n-1$ be the vertices which subdivide v_i and w_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \dots\dots, q+1\}$ by

$$f(u_i) = 5i-4, 1 \le i \le n.$$

$$f(v_i) = 5i-3, 1 \le i \le n-1.$$

$$f(w_i) = 5i-1, 1 \le i \le n-1.$$

$$f(y_i) = 5i-2\,, 1 \le i \le n-1.$$

Edges are labeled with,

$$f(u_i u_{i+1}) = 5i-2\,, 1 \le i \le n-1.$$

$$f(u_i v_i) = 5i-4, 1 \le i \le n-1,$$

$$f(u_{i+1} w_i) = 5i\,, 1 \le i \le n-1,$$
$$f(v_i y_i) = 5i-3, 1 \le i \le n-1,$$

$$f(y_i w_i) = 5i-1, 1 \le i \le n-1.$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

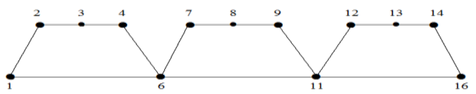

Figure:9

Case (iv):

Let G be a graph obtained by subdividing all the edges of $\mathbf{Q_n}$. Let x_i , $1 \le i \le n-1$ be the vertices which subdivide u_i and u_{i+1}. Let s_i and t_i , $1 \le i \le n-1$ be the vertices which subdivide the edges $u_i v_i$ and $u_{i+1} w_i$. Let y_i , $1 \le i \le n-1$ be the vertices which subdivide v_i and w_i.

Define a function $f: V(\mathbf{G}) \to \{1,2,3, \dots\dots, q+1\}$ by

$$f(u_i) = 8i-7, 1 \le i \le n.$$

$$f(v_i) = 8i-5, 1 \le i \le n-1.$$

$$f(w_i) = 8i-3, 1 \le i \le n-1.$$

$$f(x_i) = 8i-2\,, 1 \le i \le n-1.$$

$$f(y_i) = 8i-4\,, 1 \le i \le n-1.$$

$$f(s_i) = 8i-6, 1 \le i \le n-1.$$

$$f(t_i) = 8i-1, 1 \le i \le n-1.$$

Edges are labeled with,

$$f(u_i x_i) = 8i-5, 1 \le i \le n-1,$$

$$f(x_i u_{i+1}) = 8i-1\,, 1 \le i \le n-1.$$

$$f(u_i s_i) = 8i-7, 1 \le i \le n-1.$$

$$f(s_i v_i) = 8i-6, 1 \le i \le n-1.$$

$$f(u_{i+1} t_i) = 8i\,, 1 \le i \le n-1.$$

$$f(t_i w_i) = 8i-2, 1 \le i \le n-1.$$

$$f(v_i y_i) = 8i-4, 1 \le i \le n-1,$$

$$f(y_i w_i) = 8i-3, 1 \le i \le n-1.$$

Clearly f is a Heronian Mean labeling. The labeling pattern is displayed below.

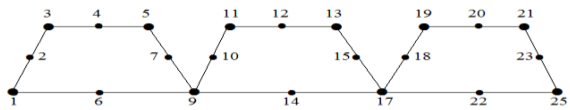

Figure:10

From all the above four cases, we conclude that $G = S(Q_n\,)$ is a Heronian mean graph.

3. Conclusion:

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigate subdivision of Heronian mean graphs which admit Heronian Mean Labeling. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

4. Acknowledgement:

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