Semi Regular Weakly Open Sets in Topological Spaces

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Abstract — *This paper considers a new class of sets called semi-regular weakly open (briefly srw-open) sets are introduced and studied in topological spaces. i.e. A subset G of topological space X is said to be semi-regular weakly open set, if $F \subseteq \sin t(A)$, whenever $F \subseteq A$ and F is rw-closed set in X. The new class strictly lies between semi-open sets, αrw-open sets and gs-open sets in topological spaces. Also, as applications, using some properties of srw-open sets and srw-closed sets we investigate srw-interior and srw-closure operators and their properties respectively.*

Keywords— *srw-closed sets, srw-open sets, srw-neighbourhoods, srw-interior, srw-closure.*

I. Introduction

Levine and Stone[6, 13] introduced generalized open sets, regular open sets in topological spaces respectively, then regular weakly open sets, generalized semi closed sets, generalized α-closed sets and α -generalized closed sets semi open sets, α regular w-closed sets, pgrw-closed sets and semi-regular weakly closed sets have been introduced and studied by Benchalli S. S. and Wali R. S.[2], Arya S.P. and Nour T.M.[1], Maki et al. [7], Levin [7], Wali R. S. and Mendalgeri [17], Wali R. S. and Chilakwad [18] and Wali R. S. and Mathad [16] respectively.

We introduce and study the semi-regular weakly open (briefly srw-open) sets, semi-regular weakly neighbourhood (briefly srw-nhd) and operators; srw-interior and srw-closure in topological space and obtain some of their properties.

II. PRELIMINARIES

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of topological space X, cl(A) and int(A) denote the closure of A and interior of A respectively. Let X\A denotes the complement of A in X. Now, we recall the following definitions.

Definition 2.1 A subset A of a topological space X is called

i) Regular open [14], if $A = \text{int}(cl(A))$ and regular closed if $cl(\text{int}(A)) = A$.
ii) Pre-open [10], if $A \subseteq \text{int}(cl(A))$ and pre-closed if $cl(\text{int}(A)) \subseteq A$.
iii) Semi open [7], if $A \subseteq cl(\text{int}(A))$ and semi-closed if $\text{int}(cl(A)) \subseteq A$
iv) α -open [11], if $A \subseteq \text{int}(cl(\text{int}(A)))$ and α -closed if $\text{int}(\text{int}(cl(A))) \subseteq A$
v) Semi pre open [11], if $A \subseteq cl(\text{int}(cl(A)))$ and semi pre closed if $\text{int}(cl(\text{int}(A))) \subseteq A$.
vi) π -open [19], if A is a finite union of regular open sets.

Definition 2.2 A subset A of a topological space X is called

i) Generalized closed (briefly g-closed) [7], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
ii) Semi-generalized closed (briefly sg-closed) [3], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
iii) Generalized semi-closed (briefly gs-closed) [1], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
iv) Generalized α -closed (briefly gα -closed) [4], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X.
v) α -generalized closed (briefly α g-closed) [9], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
vi) Generalized semi pre-closed (briefly gsp-closed) [5], if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
vii) Regular generalized closed (briefly rg-closed) [12], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
viii) Weakly closed (briefly w-closed) [13], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

ix) Regular weakly closed (briefly rw-closed) [2], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X.

x) α-regular weakly closed (briefly α rw-closed) [17], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is rw-open set in X.

The complements of above all closed sets are their respective open sets in the same topological space X.

The semi-pre-closure (resp. semi-closure, resp. pre-closure, resp. α-closure) of a subset A of X is the intersection of all semi-pre- closed (resp. semi- closed, resp. pre- closed, resp. α-closed) sets containing A and is denoted by spcl(A)(resp. scl(A), resp. pcl(A), resp. cl(A)).

Definition 2.3 A subset A of a space X is said to be semi regular weakly closed (briefly srw-closed) set [16], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is rw-open set in X.

We denote the family of all srw-closed sets, srw-open sets, α rw-open sets and semi-open sets of X by SRWC(X), SRWO(X), α RWO(X) and SO(X) respectively.

Lemma 2.4 i) For a subset A of X, αrw-closure of A [17] is denoted by $\alpha rw-cl(A)$ and defined as $\alpha rw-cl(A) = \cap F \subset X : A \subset F \in \alpha RWC(X)$.

ii) For a subset A of X, semi-closure of A [6] is denoted by $scl(A)$ and defined as $scl(A) = \cap F \subset X : A \subset F \in SC(X)$.

iii) For a subset A of X, gs-closure of A [1] is denoted by $gs-cl(A)$ and defined as $gs-cl(A) = \cap F \subset X : A \subset F \in GSC(X)$.

III. SEMI REGULAR WEAKLY OPEN (BRIEFLY SRW-OPEN) SETS

In this section, we introduce and study srw-open sets in topological space and obtain some of their basic properties.

Definition 3.1 A subset A of X is called Semi Regular Weakly open (briefly srw-open) set, if X\A is srw-closed set in X. The family of all semi regular weakly open sets in X is denoted as SRWO(X).

Theorem 3.2 If a subset A of space X is αrw-open, then it is srw-open in X but not conversely.

Proof: Let A be a αrw-open set in a space X. Then X\A is a αrw-closed set. By Theorem 3.2 of [16], X\A is srw-closed. Therefore A is a srw-open set in X.

The converse of the above Theorem need not be true as shown in example 3.3.

Example 3.3 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then {a, d} and {b, c, d} are srw-open sets in X but it is not αrw-open sets in X.

Theorem 3.4 If a subset A of space X is semi-open, then it is semi-open in X but converse is not true.

Proof: Let A be a semi-open set in a space X. Then X\A is a semi-closed set. By Theorem 3.6 of [16], X\A is srw-closed. Therefore A is a srw-open set in X.

The converse of the above Theorem need not be true as shown in example 3.5.

Example 3.5 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then {b} and {c} are srw-open sets in X but not semi-open sets in X.

Corollary 3.6 From Levine [7], it is evident that every open set is semi-open set but not conversely. By Theorem 3.4 every semi-open set is srw-open set in X but not conversely and hence every open set is srw-open set in X.

Corollary 3.7 From Wali and Prabhavati[17], it is evident that every α-open set is αrw-open set is srw-open set but not conversely and hence every α-open set is srw-open set but not conversely.

Corollary 3.8 From Stone [14], it is evident that every regular open set is open, but not conversely. By corollary 3.7, every open set is srw-open set but conversely and hence every regular open set is srw-open set in X.

Corollary 3.9 From Velicko [15], it is evident that every θ-open (δ-open) set is open but not conversely. By Corollary 3.7, every open set is srw-open set but not conversely and hence every θ-open (δ-open) set is srw-open set in X.

Theorem 3.10 If a subset A of a space X is srw-open, then it is a gs-open set in X.

Proof: Let A be a srw-open set in X, then X\A is a srw-closed set in X. By Theorem 3.4 of [16], every srw-closed set is gs-closed set in X i.e. X\A is a gs-closed set in X. Therefore A is a gs-open set in X.

The converse of the above Theorem need not be true as shown in example 3.11.

Example 3.11 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then {a, c} and {a, b} are gs-open sets in X but not srw-open sets in X.

Theorem 3.12 If a subset A of a space X is srw-open, then it is a gs-open set in X, but not conversely.

Proof: Let A be a srw-open set in X, then X\A is a srw-closed set in X. By Theorem 3.10 of [16], every srw-closed set is gsp-closed set in X i.e. X\A is a gsp-closed set in X. Therefore A is a gsp-open set in X.

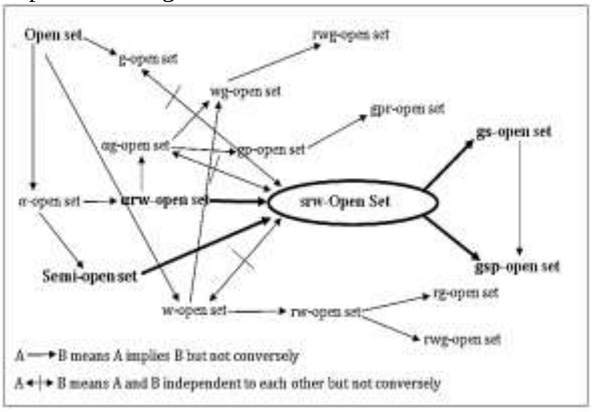
The converse of the above Theorem need not be true as shown in example 3.13.

Example 3.13 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then {a, b} and {c, d} are gsp-open sets in X but not srw-open sets in X.

The concepts of g-open, w-open, α g-open and wα -open sets are independent with the concept of srw-open set as shown in the following example 3.14.

Example 3.14 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then {a, d} is a srw-open, however it can be verified that it is not g-open, w-open, g-open and w -open set. Also, the set {a, b} and {a, c} are g-open, w-open, α g-open and wα -open set but not srw-open set in X.

Thus the above discussion leads to the following implication diagram:



Remark 3.15 Union and intersection of two srw-open sets need not be srw-open set as shown in the following example 3.16.

Example 3.16 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $SRWO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$.

Let A= {b}, B= {a, d} and C= {b, c, d}. Here A and B are srw-open sets but $A \cup B = \{a,b,d\}$ is not srw-open. Also B and C are srw-open sets but $B \cap C = \{d\}$ is not srw-open set in X.

Theorem 3.17 If $A \subseteq$ X is srw-closed, then scl(A)\A is srw-open set in X.

Proof: Let $A \subseteq X$ is srw-closed and let F be a rw-closed set such that $F \subseteq scl(A) \setminus A$. Then by Theorem 3.19 of [16], $F = \phi$ that implies $F \subseteq \sin t(scl(A) \setminus A)$ and Theorem 3.17 scl(A)\A is srw-open set in X.

Theorem 3.18 A subset A of a topological space X is srw-open if and only if $F \subseteq \sin t(A)$ whenever F is rw-closed and $F \subseteq A$.

Proof: Let $F \subseteq A$ is srw-closed and let F be a rw-closed set and $F \subseteq A$. Then $X \setminus A \subseteq X \setminus F$ where $X \setminus F$ is rw-open. Since $X \setminus A$ is srw-closed, $scl(X \setminus A) \subseteq X \setminus F$ and hence $X \setminus \sin t(A) \subseteq X \setminus F$ that implies $F \subseteq \sin t(A)$.

Conversely, suppose $F \subseteq \sin t(A)$ whenever $F \subseteq A$, F is rw-closed. To prove: A is srw-open. Suppose, $X \setminus U \subseteq A$ where U is rw-open. Then $X \setminus U \subseteq A$ where $X \setminus U$ is rw-closed. By assumption $X \setminus U \subseteq \sin t(A)$ that implies $scl(X \setminus A) \subseteq U$. This proves that $X \setminus A$ is srw-closed and hence A is srw-open set in X.

Theorem 3.19 Every singleton point set in a space X is either srw-open or rw-open in X.

Proof: Let $x \in X$ where X is a topological space. To prove: {x} is either srw-open or rw-open set in X i.e. to prove that $X \setminus \{x\}$ is either srw-closed or rw-open, which follows from Theorem 3.25 of [16].

The next Theorem shows that all the sets between sint(A) and A are srw-open whenever A is srw-open.

Theorem 3.20 If $\sin t(A) \subseteq B \subseteq A$ and A is a srw-open set in X, then B is srw-open set in X.

Proof: Let $\sin t(A) \subseteq B \subseteq A$ and A is a srw-open set. Then $X \setminus A \subseteq X \setminus B \subseteq X \setminus \sin t(A)$ that implies $X \setminus A \subseteq X \setminus B \subseteq \sin t(X \setminus A)$, since $X \setminus A$ is srw-closed set, by Theorem 3.23 of [16], $X \setminus B$ is srw-closed set. Therefore B is srw-open in X.

Theorem 3.21 If $A \subseteq X$ is srw-closed, then $scl(A) \setminus A$ is srw-open set in X.

Proof: Let $A \subseteq X$ is srw-closed set and F be a rw-closed set such that $F \subseteq \sin t(A) \setminus A$. By Theorem 3.19 of [16], $F = \phi$, so $F \subseteq \sin t(scl(A) \setminus A)$ By Theorem 3.18 $scl(A) \setminus A$ is srw-open set in X.

The converse of above Theorem does not hold shown by example 3.22.

Example 3.22 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then A={c, d} then scl(A)={b, c, d} and $scl(A) \setminus A$ ={b} is an srw-open set, but A is not an srw-closed set in X.

Theorem 3.23 If a subset A is srw-open in X and if G is rw-open in X with $\sin t(A) \cup (X \setminus A) \subseteq G$ then G=X.

Proof: Suppose that G is an rw-open set and $\sin t(A) \cup (X \setminus A) \subseteq G$. Now $(X \setminus A) \subseteq X \setminus scl(A) \cap X \setminus (X \setminus A)$ implies that $(X \setminus G) \subseteq scl(X \setminus A) \cap A$. Suppose A is srw-open. Since $X \setminus G$ is rw-closed and $X \setminus A$ is srw-closed, then by Theorem 3.19 of [16], $X \setminus G = \phi$ and hence G=X.

The converse of the above Theorem need not be true in general as shown in example 3.24.

Example 3.24 Let X= {a, b, c, d} with topology $\tau=\{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $SRWO(X)=\{X,\phi,\{a\},\{b\},\{c\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$.

and

$RWO(X)=\{X,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{c,d\},\{b,d\},\{a,c\},\{a,b,c\}\}$.

Let A= {a, b, d}{a, b, d} is not an srw-open set in X. However $\sin t(A)\cup(X\setminus A)=\{a,d\}\cup\{c\}=\{a,c,d\}$. So for some rw-open set G, such that $\sin t(A)\cup(X\setminus A)=\{a,c,d\}\subset G$ gives G=X but A is not srw-open set in X.

Theorem 3.25: Let X be a topological space and A, B⊆X. If B is srw-open and $\sin t(B)\subseteq A$, then $A\cap B$ is srw-open in X.

Proof**:** Since B is srw-open and $\sin t(B)\subseteq A$, then $\sin t(B)\subseteq A\cap B\subseteq B$, then by Theorem 3.20 of [16], $A\cap B$ is srw-open set in X.

IV. Semi Regular Weakly Neighbourhoods (briefly srw-nhd)

Definition 4.1 Let (X,τ) be a topological space and let $x\in X$. A subset N is said to be srw-nbd of x, if and only if there exists a srw-open set G such that $x\in G\subseteq N$.

Definition 4.2 **i)** A subset N of X is a srw-nhd of $A\subseteq X$ in topological space (X,τ), if there exists an srw-open set G such that $A\subseteq G\subseteq N$.

ii) The collection of all srw-nhd of $x\in X$ is called srw-nhd system at $x\in X$ and shall be denoted by srw-N(x).

Theorem 4.3 Every neighborhood N of $x\in X$ is a srw-nbd of x.

Proof**:** Let N be neighbourhood of point $x\in X$. To prove that N is a srw-nbd of x. By definition of neighbourhood, there exists an open set G such that $x\in G\subseteq N$. Hence N is srw-nhd of x.

Remark 4.4 In general, a srw-nbd N of x in X, as shown from example 4.5.

Example 4.5 Let X= {a, b, c, d} with topology $\tau=\{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $SRWO(X)=\{X,\phi,\{a\},\{b\},\{c\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$.

the set {a, b, d} is srw-nhd of the point b, since the srw-open set {b} is such that $b\in\{b\}\subset\{a,b,d\}$. However, the set {a, b, d} is not a neighbourhood of the point b, since no open set G exists such that $b\in G\subset\{a,b,d\}$.

Theorem 4.6 If a subset N of a space X is srw-open, and then N is a srw-nhd of each of its points.

Proof**:** Suppose N is srw-open. Let $x\in N$ we claim that N is a srw-nhd of x. For N is a srw-open set such that $b\in N\subset N$. Since x is an arbitrary point of N, it follows that N is a srw-nhd of each of its points.

The converse of the above theorem is not true in general as seen from the following example 4.7.

Example 4.7 Let X= {a, b, c, d} with topology $\tau=\{X,\phi,\{a\},\{b,c\},\{a,b,c\}\}$ .Then $SRWO(X)=\{X,\phi,\{a\},\{b\},\{c\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$.

. The set {a, c} is srw-nhd of the point a, since the srw-open set {a} is such that $a\in\{a\}\subset\{a,c\}$. Also the set {a, c} is a srw-nhd of the point c, since the srw-open set {c} is such that $c\in\{c\}\subset\{a,c\}$ i.e. {a, c} is a srw-nhd of each of its points. However the set {a, c} is not a srw-open set in X.

Theorem 4.8 Let X be a topological space. If F is a srw-closed subset of X and $x\in(X\setminus A)$, then there exists a srw-nhd N of x such that $N\cap F=\phi$.

Proof**:** Let F be srw-closed subset of X and $x\in(X\setminus F)$. Then $(X\setminus F)$ is an srw-open set of X. By Theorem 4.6, $(X\setminus F)$ contains a srw-nhd of each of its points. Hence there exists a srw-nhd N of x such that $N\cap F=\phi$.

Theorem 4.9 Let X is a topological space and for each $x\in X$, let srw-N(x) be the collection of all srw-nhds of x. Then we have the following results.

i) $\forall x\in X, srw-N(x)\neq\phi$.

ii) $X\in srw-N(x)\Rightarrow x\in N$.

iii) $N\in srw-N(x)$ and $N\subset M\Rightarrow M\in srw-N(x)$.

iv) $N\in srw-N(x)\Rightarrow\exists M\in srw-N(y)$ for every $y\in M$.

Proof**:** **i)** Since X is an srw-open set, it is a srw-nhd of every $x\in X$. Hence there exists at least one srw-nhd(X) for each $x\in X$. Hence $srw-N(x)\neq\phi$ for every $x\in X$.

ii) If $N\in srw-N(x)$, then N is a srw-nhd of x. So, by definition of srw-nhd $x\in X$.

iii) Let $N\in srw-N(x)$ and $N\subset M$, then there is a srw-open set G such that $x\in G\subset N$ .Since $N\subset M$, $x\in G\subset M$ and so M is a srw-nhd of x. Hence $M\in srw-N(x)$.

iv) If $N\in srw-N(x)$, then there exists an srw-open set M is an srw-open set, it is a srw-nhd of each of its points. Therefore $M\in srw-N(y)$ for $y\in M$.

V. SEMI-REGULAR WEAKLY INTERIOR OPERATOR

In this section, the notion of srw-interior is defined and some of its basic properties are studied.

Definition 5.1 Let A be a subset of X. A point $x \in A$ is said to be srw-interior point of A, if A is a srw-nhd of x. The set of all srw-interior of A and is denoted by srw-int(A).

Definition 5.2 For a subset A of X, srw-interior of A is defined as srw-int(A) to be the union of all srw-open sets contained in A. In symbolically, $srw-\text{int}(A) = \cup\{G \subset X : G \subseteq A$ and G is srw-open in X}.

Theorem 5.3 If A is a subset of X, then $srw-\text{int}(A) = \cup\{G \subset X : G \subseteq A$ and G is srw-open in X}

Proof: Let A be a subset of X. Let $x \in srw-\text{int}(A) \Leftrightarrow$ x is a srw-interior point of A $\Leftrightarrow$ A is a srw-nhd of point x $\Leftrightarrow$ there exists an srw-open set G such that $x \in G \subset A$ $\Leftrightarrow x \in \cup\{G \subset X : G$ is srw-open, $G \subset A\}$. Hence $srw-\text{int}(A) = \cup\{G \subset X : G \subseteq A$ and G is srw-open in X}.

Theorem 5.4 Let A and B are subsets of X. Then

i) srw-int(X)=X and srw-int(ϕ)=ϕ.

ii) srw-int (A) $\subset$ A.

iii) If B is any srw-open set contained in A, then B $\subset$ srw-int (A).

iv) If A $\subset$ B, then srw-int (A) $\subset$ srw-int (B).

v) srw-int (srw-int (A)) = srw-int (A).

Proof: **i)** Since X is only srw-open set contained in X. i.e. by definition 5.2, srw-int (A) = $\cup\{G \subset X : G$ is srw-open, G $\subset$ A} $= X \cup$ {all srw-open sets}=X. Hence srw-int(X)=X . Since ϕ is only srw-open set contained in ϕ .Hence srw-int (ϕ) = ϕ .

ii) Let $x \in srw-\text{int}(A) \Rightarrow$ x is an srw-interior of A $\Rightarrow$ A is an srw-nhd of x $\Rightarrow x \in A$. Hence $x \in srw-\text{int}(A) \Rightarrow x \in A$. Hence $srw-\text{int}(A) \subset A$.

iii) Let B be any srw-open set such that $B \subset A$. Let $x \in B$ then since B is srw-open set contained in A. x is srw-interior point of A i.e. $x \in srw-\text{int}(A)$. Hence $B \subset srw-\text{int}(A)$.

iv) Let A and B subsets of X such that $A \subset B$ let $x \in srw-\text{int}(A)$ Then x is srw-interior point of A and so A is srw-nhd of x. Since $A \subset B$, B is also srw-nhd of x $\Rightarrow x \in srw-\text{int}(B)$. Thus we have shown that $x \in srw-\text{int}(A) \Rightarrow x \in srw-\text{int}(B)$. Hence $x \in srw-\text{int}(A) \subset x \in srw-\text{int}(B)$.

v) Let A be any subset of X. By the definition of srw-interior $\text{srw-int}(A) = \{G : G \subset A \,\&\, G \in SRWO(X)\}$, if $G \subset A$ then applying srw-interior on both sides, $srw-\text{int}(G) \subset srw-\text{int}(A) \Rightarrow G \subset srw-\text{int}(A)$. Since G is srw-open set contained in srw-int(A), by iii. $G \subset srw-\text{int}(srw-\text{int}(A))$. Hence $\cup\{G : G \subset A \,\&\, G \in SRWO(X)\} \subset srw-\text{int}(srw-\text{int}(A)) \Rightarrow$ srw-int(A) $\subset srw-\text{int}(srw-\text{int}(A))$

i.e. $srw-\text{int}(srw-\text{int}(A))$ = srw-int(A).

Theorem 5.7 If a subset A of X is srw-open, then srw-int(A)=A.

Proof: Let A be srw-open subset of X. We know that srw-int (A) $\subset$ A. Also, A is srw-open set contained in A. From theorem 5.6(iii), A $\subset$ srw-int (A) Hence srw-int (A) =A.

The converse of Theorem 5.7 need not be true as seen in the following example 5.8.

Example 5.8 Let X= {a, b, c, d}with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Let $SRWO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$.

Let A= {a, b} then srw-int(A)=A but A is not a srw-open set in X.

Theorem 5.9 If A and B are subsets of X, then

i) $srw-\text{int}(A) \cup srw-\text{int}(B) \subset srw-\text{int}(A \cup B)$.

ii) $srw-\text{int}(A \cap B) \subset srw-\text{int}(A) \cap srw-\text{int}(B)$.

Proof: **i)** We know that $A \subset (A \cup B)$ and $B \subset (A \cup B)$. We have by Theorem 5.6 (iv), $srw-\text{int}(A) \subset srw-\text{int}(A \cup B)$ and $srw-\text{int}(B) \subset srw-\text{int}(A \cup B)$. This implies that srw-int(A) $\cup$ srw-int(B) $\subset srw-\text{int}(A \cup B)$

ii) We know that $(A \cap B) \subset A$ and $(A \cap B) \subset B$ We have by Theorem 5.6 (iv), $srw-\text{int}(A \cap B) \subset srw-\text{int}(A)$ and $srw-\text{int}(A \cap B) \subset srw-\text{int}(B)$.

I.e. $srw-\text{int}(A \cap B) \subset$ srw-int(A) $\cap$ srw-int(B).

Theorem 5.10 If AA is subset of X, then

i) $\alpha rw-\text{int}(A) \subset srw-\text{int}(A)$.

ii) $\sin t(A) \subset srw-\text{int}(A)$.

Proof: **i)** Let A is subset of X. Let $x \in \alpha rw-\text{int}(A) \Rightarrow x \in \cup\{G : G$ is αrw -open, $G \subset A\} \Rightarrow$ there exist an αrw-open set G such that $x \in G \subset A \Rightarrow$ here exist a srw-open set G such that $x \in G \subset A$, as every αrw -open set is srw-open set in X. $\Rightarrow x \in \cup\{G \subset X : G$ is srw-open, G $\subset$ A} $\Rightarrow x \in srw-\text{int}(A)$. Thus $x \in \alpha rw-\text{int}(A) \Rightarrow x \in srw-\text{int}(A)$. Hence $\alpha rw-\text{int}(A) \subset$ srw-int(A).

ii) Let A is subset of X. Let $x \in srw-\text{int}(A) \Rightarrow x \in \cup\{G \subset X : G$ is srw-open, G $\subset$ A} implies that there exists a semi-open set G such that $x \in G \subset A \Rightarrow$ here there exists a srw-open set G such that $x \in G \subset A$, As every semi open set is srw-open set in X. $\Rightarrow x \in \cup\{G \subset X : G$ is srw-open, G $\subset$A} implies that $x \in srw-\text{int}(A)$. Thus $x \in \sin t(A) \Rightarrow x \in srw-\text{int}(A)$. Hence $\sin t(A) \subset srw-\text{int}(A)$.

Remark 5.11 If A is subset of X, then

i) $\alpha-\text{int}(A) \subset srw-\text{int}(A)$.

ii) $\text{int}(A) \subset srw-\text{int}(A)$.

iii) $r-\text{int}(A) \subset srw-\text{int}(A)$.

Theorem 5.12 If A is subset of X, then $srw-\text{int}(A) \subset gs-\text{int}(A)$.

Proof: Let A be a subset of X, let $x \in srw-\text{int}(A) \Rightarrow x \in \cup\{G \subset X : G$ is srw-open, G $\subset$A}$\Rightarrow$ there exists srw-open set G such that $x \in G \subset A \Rightarrow$ there exists gs-open set G such that $x \in G \subset A$, as every srw-open set is gs-open set in X. $\Rightarrow x \in \cup\{G \subset X : G \subset A$, G is gs-open set in X} $\Rightarrow x \in gs-\text{int}(A)$. Thus $x \in srw-\text{int}(A) \Rightarrow x \in gs-\text{int}(A)$. Hence $srw-\text{int}(A) \subset gs-\text{int}(A)$.

Remark 5.13 Containment relations in the above theorem 5.10 may be proper as seen in the following example 5.12.

Example 5.14 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Let $SRWO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$.

Let A= {a, b} and B= {a, c}, then

i) $\alpha rw-\text{int}(A) \subset srw-\text{int}(A) \Rightarrow \{a\} \subset \{a,b\}$.

But $\alpha rw-\text{int}(A) \neq srw-\text{int}(A)$.

ii) $\sin t(A) \subset srw-\text{int}(A) \Rightarrow \{a\} \subset \{a,c\}$.

But $\sin t(A) \neq srw-\text{int}(A)$.

Remark 5.15 If A is subset of X, then $srw-\text{int}(A) \subset gsp-\text{int}(A)$.

VI. Semi-Regular Weakly Closure Operator

Now we introduce the concept of srw-closure in topological spaces by using the notations of srw-closed sets and obtain some of their properties. For any $A \subset X$, it is proved that the complement of srw-interior of srw-closure of the complement of A.

Definition 6.1 For a subset A of X, srw-closure of A is defined as srw-cl(A) to be the intersection of all srw-closed sets containing A. In symbolically, srw-cl(A)=$\cap\{F \subset X : A \subset F \;\&\; F \in SRWO(X)\}$.

Theorem 6.2 If A and B are subsets of a space X. Then

i) srw-cl(X)=X and srw-cl(ϕ)=ϕ.

ii) $A \subset srw-cl(A)$.

iii) If B is any srw-closed set containing A then $srw-cl(A) \subset B$.

iv) If $A \subset B$ then $srw-cl(A) \subset srw-cl(B)$.

v) $srw-cl(A) = srw-cl(srw-cl(A))$.

Proof: **i)** By definition 3.1, X is the only srw-closed set containing X. Therefore srw-cl(X) =$\cap$ all srw-closed sets containing X=$\cap$ {X}=X. That is srw-cl(X)=X. By the definition of srw-closure, srw-cl(ϕ)= Intersection of all srw-closed sets containing $\phi = \phi \cap$ {any srw-closed set containing ϕ }=ϕ.

ii) By definition 6.1, it is obvious that $A \subset srw-cl(A)$.

iii) Let B be any srw-closed set containing A. Since srw-cl(A) is the intersection of all srw-closed sets containing A, srw-cl(A) is contained in every srw-closed set containing A. Hence in particular srw-cl(A)$\subset$B.

iv) Let A and B be subsets of X such that $A \subset B$. By definition 6.1, If $B \subset F \in SRWC(X)$, then $srw-cl(B) \subset F$. Since $A \subset B$, $A \subset B \subset \quad F \in SRWC(X)$. We have $srw-cl(A) \subset F$. Therefore $srw-cl(B) \subset \cap\{F : B \subset F \in SRWC(X)$ =srw-cl(B).

v) Let A be any subset of X. By definition 6.1, If $A \subset F \in SRWC(X)$, then $srw-cl(A) \subset F$. Since F is srw-closed set containing srw-cl(A), by (iii), $srw-cl(srw-cl(A)) \subset F$. Hence $srw-cl(srw-cl(A)) \subset$

$\cap\{F : A \subset F \in SRWC(X)\}$ =srw-cl(A). i.e. $srw-cl(srw-cl(A)) = srw-cl(A)$.

Remark 6.3 **i)** srw-closure of a set A is not always srw-closed set.

ii) If $A \subset X$ is srw-closed, then srw-cl(A)=A.

***Proof*:** **ii)** Let A be srw-closed subset of X. We know that $A \subset srw-cl(A)$. Also $A \subset A$ and A is srw-closed. By the theorem 6.2 (iii), srw-cl(A) $\subset$ A. Hence srw-cl(A)=A. However if srw-cl(A)=A then it is not true that A is srw-closed as seen from following example.

Example 6.4 Let X= {a, b, c, d} with topology $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let A= {b} then srw-cl(A)={b} but A is not a srw-closed set.

Theorem 6.5 If A and B are subsets of a space X, then

i) $srw-cl(A \cap B) \subset srw-cl(A) \cap srw-cl(B)$.

ii) $srw-cl(A) \cup srw-cl(B) \subset srw-cl(A \cup B)$.

***Proof*:** Let A and B be subsets of X.

i) Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 6.2 (iv), srw-cl($A \cap B$) $\subset$ srw-cl(A) and srw-cl($A \cap B$) $\subset$ srw-cl(B). Hence srw-cl($A \cap B$) $\subset$ srw-cl(A) $\cap$ srw-cl(B).

ii) Clearly $A \subset (A \cup B)$ and $B \subset (A \cap B)$. By Theorem 6.2 (iv), $srw-cl(A) \subset srw-cl(A \cup B)$ and $srw-cl(B) \subset srw-cl(A \cup B)$. Hence $srw-cl(A) \cup srw-cl(B) \subset srw-cl(A \cup B)$.

Theorem 6.6 If A is subset of a space X, then

i) $srw-cl(A) \subset \alpha rw-cl(A)$.

ii) $srw-cl(A) \subset scl(A)$.

***Proof*:** Let A be subset of space X.

i) From lemma 2.4 (i), If $A \subset F \in \alpha RWC(X)$, then $A \subset F \in SRWC(X)$, because every αrw-closed set is srw-closed. That is $srw-cl(A) \subset F$. Therefore srw-cl(A) $\subset \cap\{F : A \subset F \in \alpha RWC(X)\} = \alpha rw-cl(A)$.

ii) From lemma 2.4 (ii), If $A \subset F$ and F is semi closed subset of X then $A \subset F \in SRWC(X)$ because of every semi closed set is srw-closed subset in X. that is $srw-cl(A) \subset F$. Therefore srw-cl(A) $\subseteq \cap\{F \subset X : A \subset F \,\&\, F$ is semi-closed set in X}=scl(A).

Remark 6.7 Containment relations in the above theorem 6.6 may be proper as seen in the following example 6.7.

Example 6.8 Let X= {a, b, c} with topological space τ= {X, ϕ, {a}, {b}, {b, c}, {a, b, c}}. SRWC(X)= {X, ϕ, {a}, {d}, {b, c}, {a, d}, {b, c, d}, {a, c, d}, {a, b, d}}. Let A= {a} and B= {a, b} then srw - cl(A) = {a}, $\alpha rw-cl(A) = \{a, d\}$, scl(B)=X, scl(B)= {a, b, d}.

i) $srw-cl(\{a\}) \subset \alpha rw-cl(\{a\}) \Rightarrow \{a\} \subset \{a, d\}$. But $srw-cl(A) \neq \alpha rw-cl(A)$.

ii) $srw-cl(\{a\}) \subset scl(\{a\}) \Rightarrow \{a\} \subset \{a, d\}$. But $srw-cl(A) \neq scl(A)$.

Remark 6.9 If A is subset of a space X, then

i) $srw-cl(A) \subset \alpha-cl(A)$.

ii) $srw-cl(A) \subset cl(A)$.

iii) $srw-cl(A) \subset r-cl(A)$.

Theorem 6.10 If A is subset of a space X, then $gs-cl(A) \subset srw-cl(A)$.

***Proof*:** Let A be subset of space X. From lemma 2.4 (iii), If $A \subset F \in SRWC(X)$, then $A \subset F \in GSC(X)$, because of every srw-closed set is gs-closed set. That is $gs-cl(A) \subset F$ .Therefore $gs-cl(A) \subset \cap \{F \subset X : A \subset F \in SRWC(X)\}$=srw-cl(A).

Remark 6.11 If A is subset of a space X, then $gsp-cl(A) \subset srw-cl(A)$.

Theorem 6.12 Let $x \in X$, then x is srw-cl(A) if and only if $V \cap A \neq \phi$ for every srw-open set V containing x.

***Proof*:** Let $x \in srw-cl(A)$. Suppose there exists a srw-open set V containing x such that $V \cap A = \phi$. Since $A \subseteq X \setminus V$ and by 6.2 (iv), $srw-cl(A) \subseteq X \setminus V$. This implies $x \in srw-cl(A)$ which is contradiction.

Conversely, we assume that $V \cap A = \phi$ for every srw-open set V containing x. Suppose $x \notin srw-cl(A)$, then by definition 6.2 (i), there exists a srw-closed subset F containing A such that $x \notin X$. Therefore $x \in X \setminus F$ and $X \setminus F$ is an srw-open. Since $A \subseteq F$, $(X \setminus F) \cap A = \phi$ which is impossible as $x \in X \setminus F$ and $x \in A$. Hence $x \in srw-cl(A)$.

Theorem 6.13 Let A is a subset of X. Then

(i) $X \setminus (srw-\text{int}(A)) = srw-cl(X \setminus A)$

(ii) $srw-\text{int}(A) = X \setminus (srw-cl(X \setminus A))$

(iii) $srw-cl(A) = X \setminus (srw-\text{int}(X \setminus A))$.

***Proof*:** **(i)** Let $x \in X \setminus (srw-\text{int}(A))$. Then $x \notin srw-\text{int}(A)$, i.e. every srw-open set U containing x is such that $U \not\subset A$. i.e. every srw-open set U containing x is such that $U \cap X \setminus A \neq \phi$. By Theorem 6.12, $x \in srw-cl(X \setminus A))$. Therefore $X \setminus (srw-\text{int}(A)) \subset srw-cl(X \setminus A)$.

Conversely, let $x \in srw-cl(X \setminus A))$. Then by Theorem 6.12, every srw-open set U containing x is such that $U \cap X \setminus A \neq \phi$. i.e. every srw-open set U containing x is such that $U \not\subset A$ implies that by definition of srw-int(A), $x \notin srw-\text{int}(A)$. i.e. $x \in X \setminus (srw-\text{int}(A))$ and $srw-cl(X \setminus A) \subset X \setminus (srw-\text{int}(A))$. Thus $X \setminus (srw-\text{int}(A)) = srw-cl(X \setminus A)$.

(ii) By taking complements to above (i).

(iii) Follows by replacing A by X\A in (i).

VII. Conclusion

In this article we have studied most of the basic properties. With the help of these properties we will investigate srw-continuous and irresolute functions in topological spaces and fuzzy topological spaces.

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