Study on heat and mass transfer of MHD peristaltic flow of blood with chemical reaction

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Abstract

The purpose of this paper is to study the effect of chemical reaction on the peristaltic flow of MHD electrically conducting fluid in an asymmetric porous channel. The blood is considered as an incompressible electrically conducting fluid. The assumption of low Reynolds number and long wave length approximations are used. Expressions for axial velocity, temperature and concentration distribution are obtained analytically by solving the governing equations of the flow. The obtained results are displayed and discussed in detail with the help of graphs for the variation of different emerging flow parameters.

Keywords: Peristaltic flow, Electrically conducting fluid, Asymmtric channel, Magnetic field, Chemical reaction, Heat and mass transfer.

1 Introduction

Peristaltic pumping of physiological fluids takes a special status because of the propagation of progressive transverse waves along the walls of the channel (or) tube. It has drawn serious attention of the investigators working in the the area of physiological fluid dynamics. The mechanism of peristalsis is seen in many biological systems such as transport of urine from the kidney to the bladder, swallowing of food through oesophagus, movement of chyme in the gastro-intestinal tract, flow of bile, transport of spermatozoa in the ducts efferents of the male reproduction tract, movement of ovum in the fallopian tube, cilia movement, circulation of blood in small blood vessels. The mechanism of peristaltic transport has found ample industrial applications like sanitary fluid transport, transport of corrosive fluids, transport of noxious fluid in the nuclear industries, heart lung machines, dialysis and blood pump machines. A number of researchers have discussed the peristaltic flows including Newtonian and non-Newtonian fluids under different conditions[1]-[5].

Peristaltic transport through porous medium has got considerable attention in the last few decades due to its enormous applications in biological and engineering fields. Mekheimer and Al-Arabi[6] have studied non-linear peristaltic transport of MHD flow through a porous medium. Elshehawey et al.[7] investigated the peristaltic transport in an asymmetric channel through a porous medium. Haroun[8] examined the non-linear peristaltic flow of a fourth grade fluid in an asymmytric channel. Nadeem and Akram[9] have studied the effect of magnetic field on the peristaltic flow of a Williamson fluid in an asymmetric channel. Peristaltic flow of a couple-stress fluid in uniform and non-uniform channels with slip velocity has been investigated by Sobh[10]. The study of peristaltic flow with heat and mass transfer of an electrically conducting fluid through porous medium under the influence of magnetic field has attracted the interest of many researchers due to its manifold applications in diffrent branches of science and engineering [11]. The peristaltic flow through vertical porous tube with heat transfer is investigated by Vajravelu et al.[12]. The peristaltic transport of Newtonian fluid through vertical channel with porous medium and heat transfer is investigated by [13],[14]. The influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls has been presented by Srinivas and Kothandapani[15]. The effects of chemical reaction and space porosity on MHD mixed convective flow in a vertical channel with peristalsis is examined by Srinivas and Muthuraj[16]. Wang et al.[17] have studied magnetohydrodynamic peristaltic motion of sisko fluid in a symmetric (or) asymmetric channel. Beg and Ghosh[18] have discussed analytical study of MHD radiation-convection with surfce temperature oscillation and secondary flow effects. The effects of chemical reaction on vertical oscillating plate with variable temperature has been studied by Muthucumaraswamy[19]. More recently, Mishra et al.[20]investigated heat and mass transfer effect on MHD flow of visco-elastic fluid through porous medium with oscillaory suction and heat source. The effects of Hall current and chemical reaction on oscillatory mixed convection-radiation of a micropolar fluid in a rotating system has been examined by Pal et al[21]. Muthucumaraswamy et al.[22] have discussed radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Anjali Devi and Kandasamy[23] also analyzed the effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction and injection.

In view of the above, a mathematical model is presented to study the effect of chemical reaction on heat and mass transfer of the peristaltic flow of a blood under the influence of an external magnetic field. The highly non-linear differential equations are solved by simply using the low Reynolds number and high wavelength approximation approach. The expressions of velocity, temperature and concentration distribution are obtained which are shown and discussed with the help of graphs for the variations of different flow parameters.

2 Mathematical Formulation

We consider the flow of blood through an asymmetric porous channel in presence of magnetic field of strength B_0. The induced magnetic field is neglected in comparison with the applied magnetic field. The temperature on upper and lower walls of the channel are T_0 and T_1 respectively. The geometry of peristaltic flow through porous channel in dimensionless form is shown in Fig(1) and is given by

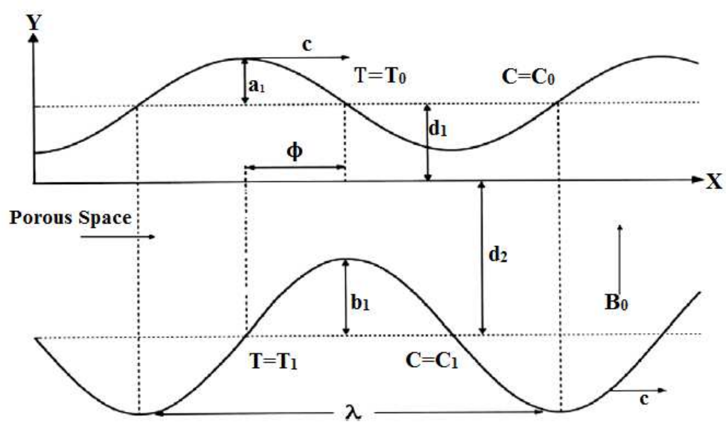

Figure 1: The Geometry of the problem

$$Y = H_1(X,Y,t) = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X-ct)\right] \tag{1}$$

$$Y = H_2(X,Y,t) = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(X-ct)+\phi\right] \tag{2}$$

where a_1, b_1 are the wave amplitudes, λ is the wave length, d_1+d_2 is the width of the channel, c is the wave speed, and $\phi(0 \le \phi \le \pi)$ is the phase difference respectively. Moreover, a_1, b_1, d_1, d_2 and ϕ satisfy the following relation

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1+d_2)^2 \tag{3}$$

The governing equations of motion in the laboratory frame (X, Y) are given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4}$$

$$\rho\left[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right] = -\frac{\partial P}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\mu}{K}U - \sigma B_0^2 U + \rho g\beta(T-T_0) + \rho g\beta^*(C-C_0) \tag{5}$$

$$\rho\left[\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right]=-\frac{\partial P}{\partial Y}+\mu\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\mu}{K}V \tag{6}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right) \tag{7}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=D_m\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right)-\overline{k_r}(C-C_0) \tag{8}$$

where $\rho, U, V, P, \mu, \sigma, K, B_0, g, \beta, \beta^*, T, t, k, c_p, D_m,$ are the fluid density, axial velocity, transverse velocity, pressure, viscosity, electrical conductivity of the fluid, permeability parameter, applied magnetic field, acceleration due to gravity, volumetric expansion co-efficient, co-efficient of expansion with concentation, temperature, time, thermal conductivity, specific heat at constant pressure, co-efficient of mass diffusivity respectively.

Defining in wave frame (x, y), the velocity components (u, v) and pressure p are given by

$$x=X-ct;\quad y=Y;\quad u=U-c;\quad v=V;\quad p(x)=P(X,t)$$

Introducing the dimensionless variables and parameters as follows:

$$\begin{aligned}\overline{x}=\frac{x}{\lambda},\overline{y}=\frac{y}{d_1},\overline{u}=\frac{u}{c},\overline{v}=\frac{v}{\delta c},\overline{p}=\frac{d_1^2 p}{\lambda\mu c},\bar{t}=\frac{tc}{\lambda},h_1=\frac{H_1}{d_1},h_2=\frac{H_2}{d_1},a=\frac{a_1}{d_1},b=\frac{a_2}{d_1},\\ d=\frac{d_2}{d_1}\delta=\frac{d_1}{\lambda},K_1=\frac{K}{d_1^2},Re=\frac{\rho c d_1}{\mu},Pr=\frac{\mu c_p}{k},\theta=\frac{T-T_0}{T_1-T_0},\Phi=\frac{C-C_0}{C_1-C_0},\\ G_r=\frac{\rho g\beta(T_1-T_0)d_1^2}{c\mu},G_c=\frac{\rho g\beta^*(C_1-C_0)d_1^2}{c\mu},\quad M^2=\frac{\sigma B_0^2 d_1^2}{\mu},k_r=\frac{\overline{k_r}\mu}{\rho d_1^2},\\ S_c=\frac{\mu}{\rho D_m},S_r=\frac{\rho D_m k_T(T_1-T_0)}{T_m\mu(C_1-C_0)}\end{aligned} \tag{9}$$

where δ, ϕ, R_e, Pr, G_r, G_c, S_r, S_c, M, and k_r are wave number, phase difference, Reynolds number, Prandtl number, Grashof number, modified Grashof number, Soret number, Schmidt number, Hartmann number and chemical reaction parameter respectively. By using equation (9) in equations (5-8) and applying the low Reynolds number and high wavelength approximation approach we arrive at

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\left(\frac{1}{K_1}+M^2\right)u+G_r\theta+G_c\Phi \tag{10}$$

$$\frac{\partial p}{\partial y}=0$$

$$\frac{\partial^2\theta}{\partial y^2}=0 \tag{11}$$

$$\frac{\partial^2\Phi}{\partial y^2}-S_c k_r\Phi=0 \tag{12}$$

The corresponding dimensionless boundary conditions are

$$u = -1, \quad \theta = 0, \quad \Phi = 0 \quad \text{at} \quad y = h_1 = 1 + a\cos[2\pi x] \tag{13}$$

$$u = -1, \quad \theta = 1, \quad \Phi = 1 \quad \text{at} \quad y = h_2 = -d - b\cos[2\pi x + \phi] \tag{14}$$

Method of Solution

Integrating equation (11) and applying the boundary condition (14), the value of θ becomes

$$\theta = \frac{y - h_1}{h_2 - h_1} \tag{15}$$

Solving equation (12) by using the associated boundary condition (15), we obtain

$$\Phi = A_2 Sinh[(y - h_1)A_1] \tag{16}$$

where

$$A_1 = \sqrt{S_c k_r} \tag{17}$$

and

$$A_2 = -Csch[(h_1 - h_2)A_1]$$

Solving equation(10) by substituting the equations (16) and (17), the velocity expression becomes

$$u = A_3\Big(- A_6(M^2 - \frac{dp}{dx})(Sinh[M(y-h_1)] - Sinh[M(y-h_2)]) + \frac{dp}{dx}A_6A_4 - Sinh[A_1(y-h_1)]A_7 + Sinh[M(y-h_1)]A_5 + A_6G_r(A_4(y-h_1) + Sinh[M(y-h_1)](-h_1+h_2))\Big) \tag{18}$$

where

$$A_3 = -\frac{1}{(e^{2Mh_1} - e^{2Mh_2})M^2(M^2 - A_1^2)(h_1 - h_2)}e^{M(h_1+h_2)}$$

$$A_4 = Sinh[M(h_1 - h_2)]$$

$$A_5 = 2M^2 Sinh[A_1(h_1 - h_2)]A_2G_c(h_1 - h_2)$$

$$A_6 = 2(M^2 - A_1^2)$$

$$A_7 = 2M^2 Sinh[M(h_1 - h_2)]A_2G_c(h_1 - h_2)$$

3 Results and Discussion

The aim of this section is to discuss the numerical and computational results with the help of graphical illustrations. The influence of different emerging physical parameters have been discussed with fussy prominence. Fig.2 illustrates the variation of axial velocity for different values of the Magnetic field parameter (M), Grashof number (G_r), modified Grashof number (G_c), Schmidt number (S_c) and chemical reaction parameter (k_r). From Fig. 2(a), it is observed that the axial velocity decreases by increasing the magnetic field strength. From Fig 2(b)-(c), it is depicted that the velocity increases by increasing Grashof number (G_r) and modified grashof number (G_c). Fig 2(d)-(e) illustrates that the Schmidt number and chemical reaction have similar effect on the velocity as the magnetic field (M). That is, the axial velocity decreases by increasing the value of Schmidt number (S_c) and chemical reaction parameter (k_r).

Fig.3. illustrates that the variation of temperture profile (θ) for different values of phase difference (ϕ). It is observed that the temperture increases by increasing the values of phase difference.

Fig.4. illustrates that the variation of concentration profile (Φ) for different values of chemical reaction parameter (k_r),phase difference (ϕ) and Schmidt number (S_c). From Fig 4(a)-(c), It is observed that the concentration profile decreases by increasing the values of k_r, ϕ, and S_c.

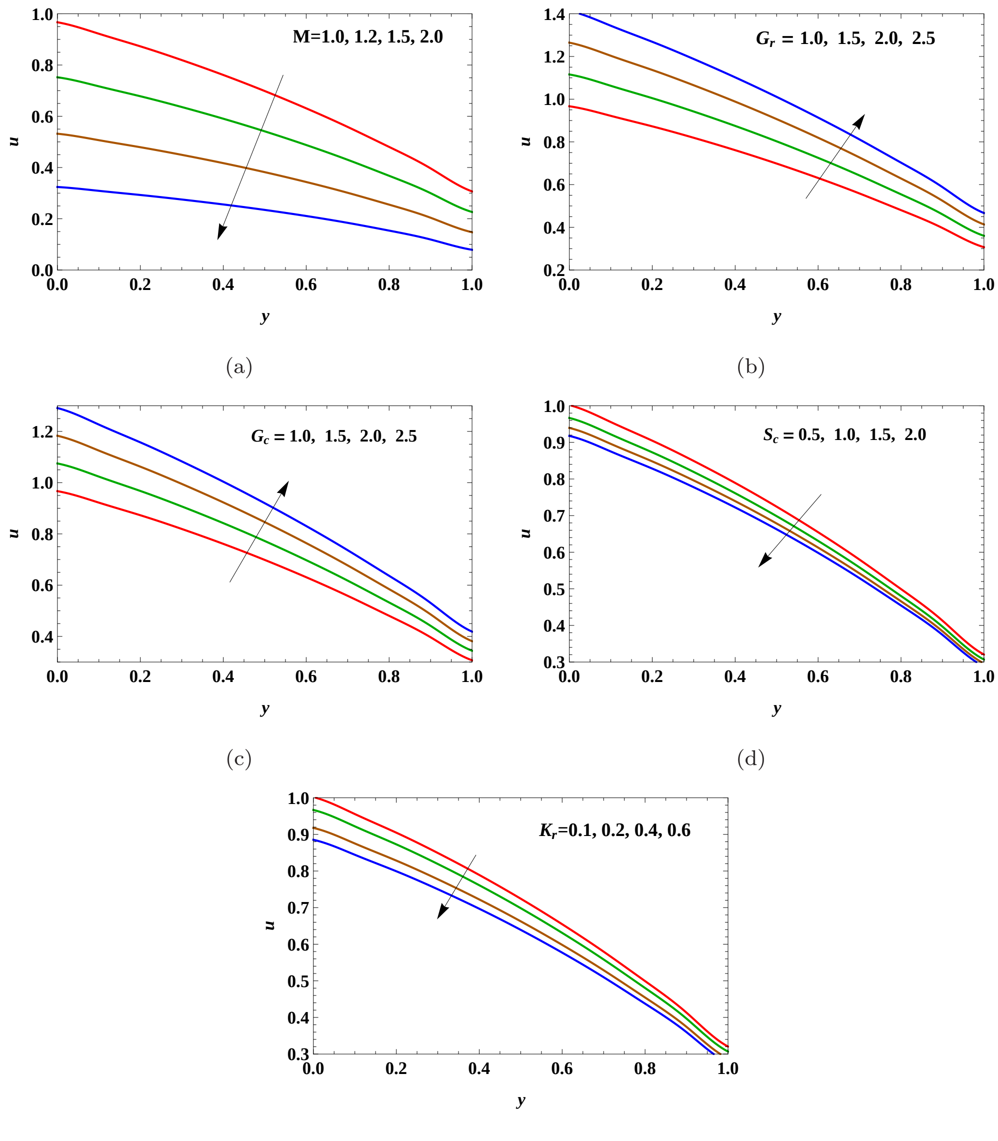

Figure 2: Variation of M, G_r, G_c, S_c, and k_r over the axial velocity (u) with respect to y with $a = 0.5$, $b = 0.5$, $d = 2$, $\frac{dp}{dx} = -3$, $x = 0$

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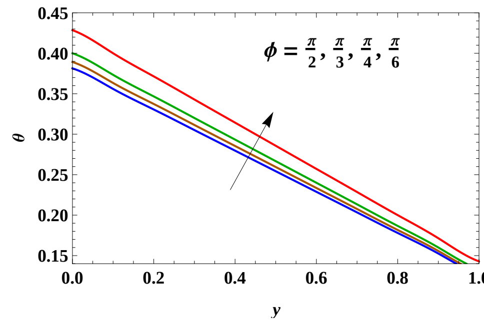

(a)

Figure 3: Variation of ϕ over the temperature profile (θ) with respect to y with $a = 0.5$, $b = 0.5$, $d = 2$, $x = 0$

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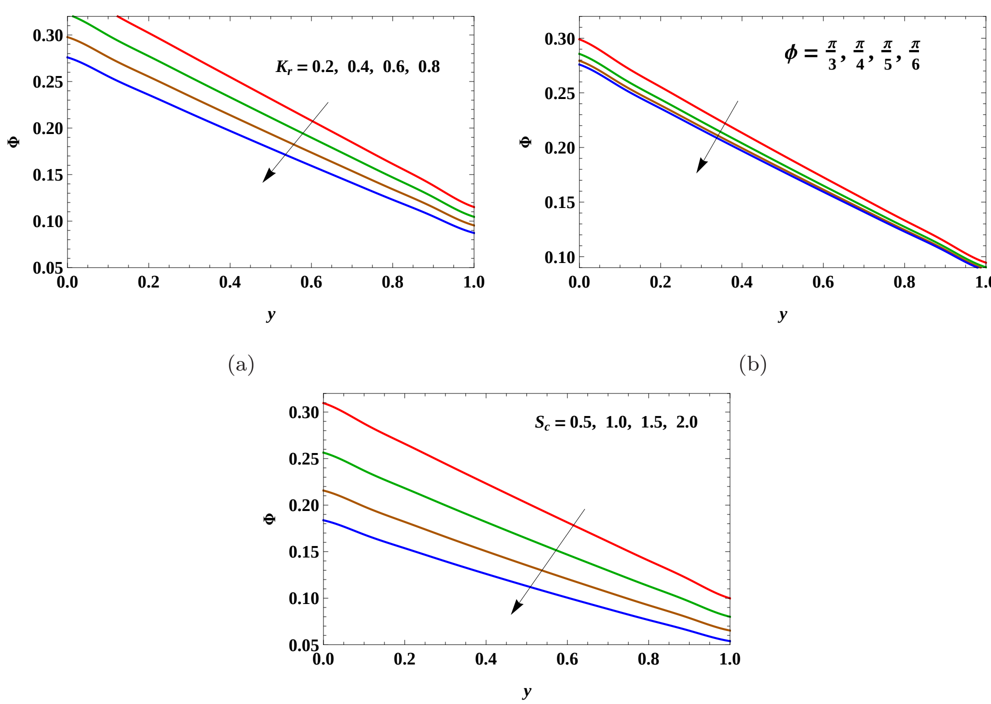

Figure 4: Variation of K_r, ϕ and S_c the concentration profile Φ with respect to y with $a = 0.5, b = 0.5, d = 2, x = 0$

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4 Conclusion

In this article we have studied the effect of chemical reaction on heat and mass transfer of the peristaltic flow of an electrically conducting fluid in the presence of an external magnetic field. The main findings are enlisted below:

- The velocity of a fluid decreases by increasing the magnetic field parameter.
- The behavior of Schmidt number and chemical reaction parameter on the axial velocity is similar to the magnetic field parameter.
- The Grashof number and modified Grashof number on velocity show opposite behavior when compared with the magnetic field.
- The behavior of amplitude ratio on temperature is totally opposite when compared with its effect on concentration.
- The chemical reaction parameter and Schmidt number show similar behavior on the concentration profile.

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