Some V_4 -cordial families with its balanced V_4-cordial labeling

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Abstract: *In this paper we discussed about balanced V_4-cordial labeling. We proved that $\boldsymbol{G^*}$, $\boldsymbol{P_n} \times \boldsymbol{G}$ and $\overline{\boldsymbol{G}}^*$ are balanced V_4-cordial graphs, when G is a balanced V_4-cordial graph.*

Keywords: *V_4-cordial graph, balanced V_4-cordial labeling, Star of a graph G and the complete star of a graph G.*

Mathematics Subject Classification-05C78

I. Introduction

Labeled graph have many diversified applications. The cordial labeling introduced by Cahit [1] is a weaker version of graceful and harmonious labeling.We follow Harary [2] for the basic notation and terminology of graph theory. Gallian[3] provide vast amount of literature on survey of different types of graph labeling.Also he proved that the complete graph $K_n$is cordial if and only if$n \leq 3$. After this, many researchers have studied cordial graph and similar type graph labeling.V_4-cordial labeling was introduced by Riskin [4] in 2013.

A cordial graph G with a cordial labelling f is called a balanced cordial graph if $|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 0$. Kaneria, Patadiya and Teraiya [5] proved that $P_n \times C_{4t}, C_n \times C_{4t}$is balanced cordial. Also Kaneria, Teraiya and Patadiya [6] proved that $P(t.C_{4t})$ is a balanced cordial if t is odd and it is vertex balanced cordial if t is even, where $n \in \mathbb{N}. C(t.C_{4t})$ is a balanced cordial if $t \equiv 0\ (mod\ 4)$ and it is vertex balanced cordial if $t \equiv 1, 3\ (mod\ 4)$ where $n \in \mathbb{N}$ and$C_{4n}{}^*$is a balanced cordial graph, $\forall\ n \in \mathbb{N}$.

Let V_4 = {e, a, b, c} be the Klein four group with the binary operation *. A V_4-cordialgraph G with a V_4-cordial labeling f is said to be a balanced V_4-cordial graph if $|v_f(p) - v_f(q)|, |e_f(p) -$ *efqϵ0,∀ p,q ϵV4.* Let G be a V_4-cordial graph with a V_4-cordial labeling f on G. Define $g: V(G) \to V_4$ by f(u) = g(u),when f(u) = 0 and $\{f(x), f(y), f(z)\} = \{g(x), g(y), g(z)\}$, *when* $f(x), f(y), f(z)\ \epsilon\ \{a, b, c\}$.Observed that g is also a V_4-cordial labeling on G. Also observed that g isa balanced V_4-cordial labeling, when f is a balanced V_4-cordial labeling for G.

Star of graph G is denoted by G^* andit obtain by $|V(G)| + 1$ copies of G say $G^{(0)}, G^{(1)}, G^{(2)}, \dots, G^{(p)}$, where $V(G) = \{v_1, v_2, \dots, v_p\}$. It is obtained by joining each vertex of $G^{(0)}$ with the corresponding vertex v_i of $G^{(i)}$, $\forall\ i = 1,2, \dots, p$. We call $G^{(0)}$ as central copy of G^* . It is obvious that $K_1{}^* = K_2$, $K_2{}^* = P_6$.

Complete star of a graph G, we mean a graph obtain by p + 1copies$G^{(0)}, G^{(1)}, G^{(2)}, \dots, G^{(p)}$ ofthe graph G and it is obtained by joining each vertex of$G^{(0)}$ with all the corresponding vertices of all copies $G^{(1)}, G^{(2)}, \dots, G^{(p)}$, where p = |V (G)|.We denote such graph by $\bar{G}^*$ .It is obvious that$\overline{K_1}^* = K_1{}^* = K_2$ and $\overline{K_2}^* = P_2 \times P_3$.

In this paper we have obtain a balanced V_4 -cordial labeling for G^* , $\bar{G}^*$ and$P_n \times G$,where G is balanced V_4-cordial graph.

II. Main Results

Theorem -2.1
If G is a balanced V_4-cordial graph, then so is G^*.

Proof: Let $V(G) = \{v_1, v_2, \dots, v_p\}$ and $q = |E(G)|$. Let $f: V(G) \to V_4$ be a balanced cordiallabeling for G. It is obvious that $p, q \equiv 0\ (mod\ 4)$and $V_f(s) = \frac{p}{4}$, $e_f(s) = \frac{q}{4}$,$\forall\ s\ \in V_4$. Let H $= G^*$ and $V(H) = \bigcup_{i=0}^{p}(V(G^{(i)}) = \{v_1{}^{(i)}, v_2{}^{(i)}, \dots, v_p{}^{(i)} / \forall i = 0,1, \dots, p\}$.

Note that $V(H) = p(p + 1) and\ |E(H)| = (p + 1)q + p$.

Define $g: V(H) \to V_4$as follows.
For any $v_j{}^{(i)} \in V(H)$

$$g(v_j^{(i)}) = \begin{cases} f(v_j) & when\ f(v_j) = 0 \\ a, & when\ f(v_j) = c \\ b, & when\ f(v_j) = a \\ c, & when\ f(v_j) = b, \end{cases}$$

$$\forall\, i = 0,1,\dots,p, \forall\, j = 1,2,\dots p.$$

Note that $v_g(0) = v_f(0) + p\, v_f(0) = \frac{(1+p)p}{4} = v_g(a) = v_g(b) = v_g(c)$, $e_g(0) = e_f(0) + p\, e_f(0) + v_f(0) = \frac{q}{4} + \frac{pq}{4} + \frac{p}{4} = \frac{1}{4}[pq + p + q] =$ *ega=egb=egc.*

Because for each $(v_j^{(0)}, v_j^{(j)}) \in E(H)$,

$$g(v_j^{(0)}) * g(v_j^{(j)}) = \begin{cases} 0, & if\ f(v_j^{(0)}) = 0 \\ a, & if\ f(v_j^{(0)}) = b \\ b, & if\ f(v_j^{(0)}) = c \\ c, & if\ f(v_j^{(0)}) = a \end{cases}$$

$$\forall\, j = 1,2,\dots,p.$$

Thus, G* is a graph which satisfies $v_g(0) = v_g(a) = v_g(b) = v_g(c) = \frac{(1+p)p}{4}$ and $e_g(0) = e_g(a) = e_g(b) = e_g(c) = \frac{(pq+p+q)}{4}$.

So, H = G* is balanced V_4-cordial graph.

Theorem -2.2

If G is a balanced V_4-cordial graph, then so is $P_n \times G$.

Proof: Let $V(G) = \{v_1, v_2, \dots, v_p\}$ and $q = |E(G)|$. Let $f: V(G) \to V_4$ be a balanced cordiallabeling for G. It is obvious that $p, q \equiv 0\ (mod\ 4)$ and $V_f(s) = \frac{p}{4}$, $e_f(s) = \frac{q}{4}$, $\forall\, s \in V_4$ in G.

Let $G^{(1)}, G^{(2)}, \dots, G^{(n)}$ be n copies of the graph G. Join vertex $v_j^{(i)}$ of $G^{(i)}$ and vertex $v_j^{(i+1)}$ of $G^{(i+1)}$ by an edge, $\forall\, i = 1,2,\dots,n-1$, $\forall\, j = 1,2,\dots,p.$ to form the graph $P_n \times G$. Thus, $|V(P_n \times G)| = np$ and $|E(P_n \times G)| = nq + (n-1)\,p$.

Define $g: V(H) \to V_4$ as follows.

$$g(v_i) = \begin{cases} 0, & when\ f(v_j) = 0 \\ a, & when\ f(v_j) = c \\ b, & when\ f(v_j) = a \\ c, & when\ f(v_j) = b \end{cases}$$

$$\forall\, i = 1,\dots,p.$$

Note that above defined labeling function g on G is also balanced V_4-cordial labeling.

Define $h : V(P_n \times G) \to V_4$ as follows.

For any $v_j^{(i)} \in V(P_n \times G)$

$$h(v_j^{(i)}) = \begin{cases} f(v_j) & when\ i\ is\ odd \\ g(v_j) & when\ i\ is\ even \end{cases}$$

$$\forall\, i = 1,2,\dots n, \forall\, j = 1,2,\dots,p.$$

It is observed that $v_h(0) = \frac{np}{4} = v_h(a) = v_h(b) = v_h(c)$ and $e_h(0) = \frac{nq}{4} + \frac{(n-1)p}{4} = e_h(a) = e_h(b) = e_h(c)$.

Because for each $(v_j^{(i)}, v_j^{(i+1)}) \in E(P_n \times G)$

$$h(v_j^{(i)}) * h(v_j^{(i+1)}) = |f(v_j) * g(v_j)|$$

$$= \begin{cases} 0, & if\ f(v_j) = 0 \\ a, & if\ f(v_j) = b \\ b, & if\ f(v_j) = c \\ c, & if\ f(v_j) = a, \end{cases}$$

$$\forall\, i = 1,2,\dots,n-1, \forall\, j = 1,2,\dots,p$$

Thus, $P_n \times G$ satisfies $|v_h(p) - v_h(q)|, |e_h(p) -$ *ehq∈0,∀ p,q ∈V4.*

So, it is balanced V_4-cordial.

Theorem -2.3

If G is a balanced V_4-cordial graph, then so is $\bar{G}^*$.

Proof: Let $G^{(0)}$ be the central copy of G in $\bar{G}^*$, $V(G^{(0)}) = \{v_1^{(0)}, v_2^{(0)}, \dots, v_p^{(0)}\}$ and $q = |E(G^{(0)})|$. Let $f: V(G^{(0)}) \to V_4$ be a balanced V_4-cordial labeling for $G = G^{(0)}$. Let H $= \bar{G}^*$,the complete star of the graph G. It is obvious that $P = |V(H)| = p(p+1)$ and $Q = |E(H)| = (p+1)\,q + p^2$. Take $V(H) = \cup_{i=0}^{p}(V(G^{(i)}) =$ *{v1i,v2i,...,vpi/∀ i=0,1,...,p }.*

Since G is a balancedV_4-cordial graph, $p, q \equiv 0\ (mod\ 4)$ and $v_f(0) = v_f(a) = v_f(b) = v_f(c) = \frac{p}{4}$, $e_f(0) = e_f(a) = e_f(b) = e_f(c) = \frac{q}{4}$ in G.

Define $g: V(H) \to V_4$ as follows.

For any $v_j^{(i)} \in V(H)$,

$$g(v_j^{(0)}) = f(v_j^{(0)}), \qquad \forall\, j = 1,2,\dots,p.$$

$$g\left({v_j}^{(i)}\right) = \begin{cases} 0, & when\ f(v_j) = 0 \\ a\ , & when\ f(v_j) = c \\ b, & when\ f(v_j) = a \\ c, & when\ f(v_j) = b \end{cases}$$

$$\forall\ i = 1,2, \dots, p, \forall\ j = 1,2, \dots, p.$$

It is observed that $v_g(0) = (p+1)v_f(0) = \frac{(p+1)p}{4}$, $v_g(a) = v_g(b) = v_g(c) = \frac{(p+1)p}{4}$.

i.e.$v_g(0) = v_g(a) = v_g(b) = v_g(c) = \frac{p}{4}$.

Moreover $e_g(0) = v_f(0) + \frac{pq}{4} + \frac{pp}{4} = \frac{1}{4}[pq + q +$ *p2=Q4=ega=egb=egc.*

Therefore,H is a balanced V_4-cordial graph.

i.e. $\bar{G}^*$is balanced V_4-cordial.

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