Some generalized expectations associated with probability density functions of various multivariable distributions II

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ABSTRACT
In this paper, we employ the multivariable I-function defined by Prasad [3] to derive various generalized expectations associated with probability density functions, of various multivariate distributions (beta, gamma, Dirichlet distibutions).

KEYWORDS : I-function of several variables, generalized expectations, probability density functions, multivariate distributions.

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1. Introduction

Recently Chandel and Gupta [2] are employed the multivariable H-function defined by Srivastava and Panda [4] inderiving expectations associated with probability density functions of various multivariate distributions. Here in the present document, we extend the work with the multivariable I-function defined by Prasad [3]. The I-function of several variables generalize the multivariable H-function defined by Srivastava et al [4] , itself is a generalisation of G-function of several variables. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, ...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left. \begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r} : (a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r} : (b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\alpha'_1}, \cdots, |z_r|^{\alpha'_r}) , max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\beta'_1}, \cdots, |z_r|^{\beta'_r}) , min(|z_1|, \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots ; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots ; 0, n_{s-1} \tag{1.4}$$

$$W = (p', q'); \cdots ; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots ; (m^{(r)}, n^{(r)}) \tag{1.5}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \cdots ; (a_{(r-1)k}), \alpha'{}_{(r-1)k}, \alpha''_{(r-1)k}, \cdots, \alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \cdots ; (b_{(r-1)k}), \beta'{}_{(r-1)k}, \beta''_{(r-1)k}, \cdots, \beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A} = (a_{sk}; \alpha'{}_{rk}, \alpha''_{rk}, \cdots, \alpha_{rk}^{r}) : \mathfrak{B} = (b_{rk}; \beta'{}_{rk}, \beta''_{rk}, \cdots, \beta_{rk}^{r}) \tag{1.8}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots ; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b'_k, \beta'_k)_{1,p'}; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,p^{(r)}} \tag{1.9}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_r) = I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c|c} z_1 & \text{A} ;\mathfrak{A};\text{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & \text{B}; \mathfrak{B}; \text{B'} \end{array} \right) \tag{1.10}$$

2. Required results

In this section, we write following results a) Liouville's theorem (see Chandel [1], page 83,eq .3.1)

$$\int_0^\infty \cdots \int_0^\infty f(x_1+\cdots+x_r)x_1^{\mu_1-1}\cdots x_r^{\mu_r-1}\mathrm{d}x_1\cdots \mathrm{d}x_r$$

$$=\frac{\Gamma(\mu_1)\cdots\Gamma(\mu_r)}{\Gamma(\mu_1+\cdots+\mu_r)}\int_0^\infty f(t)t^{\mu_1+\cdots+\mu_r-1}\mathrm{d}t\,,Re(\mu_i)>0,i=1,\cdots,r \tag{2.1}$$

b)Gamma function

$$\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}\mathrm{d}t \qquad \mathrm{Re}(z)>0 \tag{2.2}$$

c) Beta function

$$B(\alpha,\beta)=\int_0^\infty \frac{t^{\alpha-1}}{(1+t)^{\alpha+\beta}}\mathrm{d}t\,,Re(\alpha)>0,Re(\beta)>0 \tag{2.3}$$

3. Multivariate Gamma distribution

$$f(x_1,\cdots,x_r)=\frac{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\lambda^{\mu+\mu_1+\cdots+\mu_r}}{\Gamma(\mu+\mu_1+\cdots+\mu_r)}e^{-(x_1+\cdots+x_r)}(x_1+\cdots+x_r)^{\mu}x_1^{\mu_1-1}\cdots x_r^{\mu_r-1} \tag{3.1}$$

where $Re(\lambda)>0,Re(\mu)>0,x_i\geqslant 0,Re(\mu_i)>0,i=1,\cdots,r$ and $f(x_1,\cdots,x_r)=0$ elsewhere.

With the help of the equations (2.1) and (2.2) , we obtain the following result.

$$\int_0^\infty \cdots \int_0^\infty f(x_1+\cdots+x_r)\,\mathrm{d}x_1\cdots \mathrm{d}x_r=1 \tag{3.2}$$

Therefore $f(x_1,\cdots,x_r)$ is a probability density function for multivariate gamma distribution.

4. Expectations associated with multivariate gamma distribution

Corresponding to density function $f(x_1,\cdots,x_r)$ defined in (3.1), the expectation value of the function $g(x_1,\cdots,x_r)$ is defined as

$$\langle g(x_1,\cdots,x_r)\rangle=\int_0^\infty \cdots \int_0^\infty f(x_1,\cdots,x_r)g(x_1,\cdots,x_r)\mathrm{d}x_1\cdots \mathrm{d}x_r \tag{4.1}$$

Consider $$g_1(x_1,\cdots,x_r)=I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c|c} \mathrm{z}_1x_1^{\alpha_1^1}\cdots x_r^{\alpha_r^1}(x_1+\cdots+x_r)^{\upsilon_1} & \mathrm{A}\ ;\ \mathfrak{A}\ ;\ \mathrm{A}' \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \mathrm{z}_rx_1^{\alpha_1^r}\cdots x_r^{\alpha_r^r}(x_1+\cdots+x_r)^{\upsilon_r} & \mathrm{B}\ ;\ \mathfrak{B};\ \mathrm{B}' \end{array}\right) \tag{4.2}$$

where $I_{U:p_r,q_r;W}^{V;0,n_r;X}$ is the I-function of r variables.

With the help of (4.1), (2.1) and (2.2), the expectation of $g_1(x_1,\cdots,x_r)$ corresponding to density function $f(x_1,\cdots,x_r)$ defined by (3.1), is given by

$$\langle g_1(x_1,\cdots,x_r)\rangle = \frac{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\lambda^{\mu+\mu_1+\cdots+\mu_r}}{\Gamma(\mu+\mu_1+\cdots+\mu_r)}\int_0^\infty\cdots\int_0^\infty e^{-(x_1+\cdots+x_r)}(x_1+\cdots+x_r)^\mu$$

$$x_1^{\mu_1-1}\cdots x_r^{\mu_r-1} I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{z}_1 x_1^{\alpha_1^1}\cdots x_r^{\alpha_r^1}(x_1+\cdots+x_r)^{\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x_1^{\alpha_1^r}\cdots x_r^{\alpha_r^r}(x_1+\cdots+x_r)^{\upsilon_r} \end{array}\left|\begin{array}{c} \mathrm{A}\ ;\ \mathfrak{A}\ ;\ \mathrm{A'} \\ \cdots \\ \cdots \\ \mathrm{B}\ ;\ \mathfrak{B};\ \mathrm{B'} \end{array}\right.\right)\mathrm{d}x_1\cdots\mathrm{d}x_r$$

$$= \frac{\Gamma(\mu_1)\cdots\Gamma(\mu_r)}{\Gamma(\mu+\mu_1+\cdots+\mu_r)} I_{U:p_r+r+1,q_r+1;W}^{V;0,n_r+r+1;X}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\left|\begin{array}{c} [1\text{-}\mu_j : \alpha_j^1,\cdots,\alpha_j^r]_{1,r}, \\ \cdots \\ \cdots \\ (1\text{-}\mu_1-\cdots-\mu_r;\alpha_1^1+\cdots+\alpha_r^1,\cdots,\alpha_1^r+\cdots+\alpha_r^r), \end{array}\right.\right.$$

$$\left.\begin{array}{c} \left(1-\mu-\mu_1-\cdots-\mu_r;\upsilon_1+\alpha_1^1+\cdots+\alpha_r^1,\cdots,\upsilon_r+\alpha_1^r+\cdots+\alpha_r^r\right),\mathfrak{A};A' \\ \cdots \\ \cdots \\ \mathfrak{B}\ ;\ \mathrm{B'} \end{array}\right) \tag{4.3}$$

Provided that

a) $Re(\lambda)>0, Re(\mu_i)>0, Re(\upsilon_i)>0, Re(\alpha_j^i)>0, x_i\geqslant 0, i,j=1,\cdots,r$

b) $\left|\dfrac{arg z_k}{\lambda^{\upsilon_i+\alpha_1^i+\cdots+\alpha_r^i}}\right| < \dfrac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.3)

c) $f(x_1,\cdots,x_r)=0$ elsewhere.

5. Multivariate Beta distribution

Consider the function

$$F(x_1,\cdots,x_r) = \frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)(x_1+\cdots+x_r)^\alpha}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\beta)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}$$

$$\frac{1}{(1+x_1+\cdots+x_r)^{\alpha+\beta+\mu_1+\cdots+\mu_r}} \tag{5.1}$$

where $Re(\alpha)>0, Re(\beta)>0, Re(\mu_i)>0, i=1,\cdots,r$ and $F(x_1,\cdots,x_r)=0$ elsewhere. Now use to (2.1) and (2.3) , we have

$$\int_0^\infty \cdots \int_0^\infty F(x_1,\cdots,x_r)\mathrm{d}x_1\cdots\mathrm{d}x_r = 1 \tag{5.2}$$

6. Expectation associated with multivariate Beta distribution

Corresponding to probability density function $F(x_1,\cdots,x_r)$ defined by (5.1) consider the function

$$G(x_1,\cdots,x_r)=G_1(x_1,\cdots,x_r)=I^{V;0,n_r;X}_{U:p_r,q_r;W}\left(\begin{array}{c|c} \frac{z_1 x_1^{\alpha_1^1}\cdots x_r^{\alpha_r^1}(x_1+\cdots+x_r)^{\eta_1}}{(1+x_1+\cdots+x_r)^{\alpha_1^1+\cdots+\alpha_r^1+\eta_1}} & \mathrm{A}\ ;\ \mathfrak{A}\ ;\ \mathrm{A'} \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \frac{z_r x_1^{\alpha_1^r}\cdots x_r^{\alpha_r^r}(x_1+\cdots+x_r)^{\eta_r}}{(1+x_1+\cdots+x_r)^{\alpha_1^r+\cdots+\alpha_r^r+\eta_r}} & \mathrm{B}\ ;\ \mathfrak{B};\ \mathrm{B'} \end{array}\right) \tag{6.1}$$

Therefore expectation of $G_1(x_1,\cdots,x_r)$ is given by

$$\langle G_1(x_1,\cdots,x_r)\rangle=\frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)(x_1+\cdots+x_r)^{\alpha}}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\beta)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}$$

$$\int_0^\infty \cdots \int_0^\infty \frac{(x_1+\cdots+x_r)^{\alpha}x_1^{\mu_1-1}\cdots x_r^{\mu_r-1}}{(1+x_1+\cdots+x_r)^{\alpha+\beta+\mu_1+\cdots+\mu_r}} I^{V;0,n_r;X}_{U:p_r,q_r;W}\left(\begin{array}{c|c} \frac{z_1 x_1^{\alpha_1^1}\cdots x_r^{\alpha_r^1}(x_1+\cdots+x_r)^{\eta_1}}{(1+x_1+\cdots+x_r)^{\alpha_1^1+\cdots+\alpha_r^1+\eta_1}} & \mathrm{A}\ ;\ \mathfrak{A}\ ;\ \mathrm{A'} \\ \cdot & \cdots \\ \cdot & \cdots \\ \cdot & \\ \frac{z_r x_1^{\alpha_1^r}\cdots x_r^{\alpha_r^r}(x_1+\cdots+x_r)^{\eta_r}}{(1+x_1+\cdots+x_r)^{\alpha_1^r+\cdots+\alpha_r^r+\eta_r}} & \mathrm{B}\ ;\ \mathfrak{B};\ \mathrm{B'} \end{array}\right)$$

$$\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)(x_1+\cdots+x_r)^{\alpha}}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}$$

$$I^{V;0,n_r+r+1;X}_{U:p_r+r+1,q_r+2;W}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\ \left[1\text{-}\mu_j:\alpha_j^1,\cdots,\alpha_j^r\right]_{1,r} \\ \cdot & \cdots \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B}\ ;\ (1\text{-}\mu_1-\cdots-\mu_r;\alpha_1^1+\cdots+\alpha_r^1,\cdots,\alpha_1^r+\cdots+\alpha_r^r) \end{array}\right)$$

$$\left.\begin{matrix}(1-\alpha-\mu_1-\cdots-\mu_r;\eta_1+\alpha_1^1+\cdots+\alpha_r^1,\cdots,\eta_r+\alpha_1^r+\cdots+\alpha_r^r),\mathfrak{A};A' \\ \cdots \\ (1-\beta-\alpha-\mu_1-\cdots-\mu_r;\eta_1+\alpha_1^1+\cdots+\alpha_r^1,\cdots,\eta_r+\alpha_1^r+\cdots+\alpha_r^r),\mathrm{B};B'\end{matrix}\right) \tag{6.2}$$

Provided that

a) $Re(\alpha)>0, Re(\beta)>0, Re(\mu_i)>0, Re(v_i)>0, Re(\alpha_i)>0, x_i\geqslant 0, i=1,\cdots,r$

b) $|arg z_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.3)

c) $F(x_1,\cdots,x_r)=0$ elsewhere, $\alpha_i^j,\eta_i(i,j=1,\cdots,r)$ are non-negative real numbers

8. Multivariate Dirichlet distribution

Consider the function defined by

$$H(x_1,\cdots,x_r)=\frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)(x_1+\cdots+x_r)^{\alpha}}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\beta)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}x_1^{\mu_1-1}\cdots x_r^{\mu_r-1}$$

$$(x_1+\cdots+x_r)^{\alpha}\left[1-(x_1+\cdots+x_r)\right]^{\beta-1} \tag{8.1}$$

at ani point in the domain $x_i\geqslant 0, x_1+\cdots+x_r\leqslant 1, Re(\mu_i)>0, Re(\alpha)>0, Re(\beta)>0, i=1,\cdots,r$

and $H(x_1,\cdots,x_r)=0$ elsewhere.

Therefore $\int_0^{\infty}\cdots\int_0^{\infty}H(x_1,\cdots,x_r)\mathrm{d}x_1\cdots\mathrm{d}x_r=\frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\beta)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}$

$$\int_0^{\infty}\cdots\int_0^{\infty}x_1^{\mu_1-1}\cdots x_r^{\mu_r-1}(x_1+\cdots+x_r)^{\alpha}\left[1-(x_1+\cdots+x_r)\right]^{\beta-1}\mathrm{d}x_1\cdots\mathrm{d}x_r$$

$$=\frac{\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)}{\Gamma(\alpha+\mu_1+\cdots+\mu_r)\Gamma(\beta)}\int_0^{\infty}t^{\alpha+\mu_1+\cdots+\mu_r}(1-t)^{\beta-1}\mathrm{d}t=1 \tag{8.2}$$

Hence $H(x_1,\cdots,x_r)$ is probability density function for multivariate Dirichlet distribution.

9. Expectation associated with multivariate Dirichlet distribution

Corresponding to density function $H(x_1,\cdots,x_r)$ defined in (8.1), the expectation value of the function is defined as $h(x_1,\cdots,x_r)$

$$\langle h(x_1,\cdots,x_r)\rangle=\int_0^{\infty}\cdots\int_0^{\infty}h(x_1,\cdots,x_r)H(x_1,\cdots,x_r)\mathrm{d}x_1\cdots\mathrm{d}x_r \tag{9.1}$$

Further consider $h_1(x_1,\cdots,x_r)$ defined by

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} z_1 x_1^{\alpha_1^1}\cdots x_r^{\alpha_r^1}(x_1+\cdots+x_r)^{\beta_1}(1-x_1-\cdots-x_r)^{\gamma_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x_1^{\alpha_1^r}\cdots x_r^{\alpha_r^r}(x_1+\cdots+x_r)^{\beta_r}(1-x_1-\cdots-x_r)^{\gamma_r} \end{array}\middle| \begin{array}{c} \mathrm{A}\ ;\ \mathfrak{A}\ ;\ \mathrm{A'} \\ \cdots \\ \cdots \\ \mathrm{B}\ ;\ \mathfrak{B};\ \mathrm{B'} \end{array}\right) \tag{9.2}$$

Thus corresponding to probability density function $H(x_1,\cdots,x_r)$ defined by (8.1) ,the espectation value of $h_1(x_1,\cdots,x_r)$ is given by

$$\langle h_1(x_1,\cdots,x_r)\rangle=\frac{\Gamma(\mu_1+\cdots+\mu_r)\Gamma(\alpha+\beta+\mu_1+\cdots+\mu_r)}{\Gamma(\mu_1)\cdots\Gamma(\mu_r)\Gamma(\beta)\Gamma(\alpha+\mu_1+\cdots+\mu_r)}\, I_{U:p_r+r+2,q_r+2;W}^{V;0,n_r+r+2;X}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle|\right.$$

$$\begin{array}{c}[1\text{-}\mu_j:\alpha_j^1,\cdots,\alpha_j^r]_{1,r},(1-\beta;\gamma_1,\cdots,\gamma_r) \\ \cdots \\ (1\text{-}\mu_1-\cdots-\mu_r;\alpha_1^1+\cdots+\alpha_r^1,\cdots,\alpha_1^r+\cdots+\alpha_r^r)\end{array}$$

$$\left.\begin{array}{c}(1-\alpha-\mu_1-\cdots-\mu_r;\beta_1+\alpha_1^1+\cdots+\alpha_r^1,\cdots,\beta_r+\alpha_1^r+\cdots+\alpha_r^r)\ ,\mathfrak{A};A' \\ \cdots \\ (1-\beta-\alpha-\mu_1-\cdots-\mu_r;\beta_1+\alpha_1^1+\cdots+\alpha_r^1,\cdots,\beta_r+\alpha_1^r+\cdots+\alpha_r^r)\ ,\mathrm{B}\ ;\mathrm{B'}\end{array}\right) \tag{9.3}$$

Provided that

a) $Re(\alpha)>0, Re(\beta)>0, Re(\mu_i)>0, x_i\geqslant 0, i=1,\cdots,r$

b) $|arg z_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.3)

c) $F(x_1,\cdots,x_r)=0$ elsewhere, $\alpha_i^j,\gamma_i(i,j=1,\cdots,r)$ are non-negative real numbers

Remark

If $U=V=A=B=0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [4], for more details, see Chandel and Gupta [2].

10. Conclusion

In this paper we have evaluated the expectation concerning three various multivariate distributions involving the multivariable I-function defined by Prasad [3]. The formulas established in this paper is of very general nature as it contains multivariable H-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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