Eulerian integral involving the multivariable I-function II

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ABSTRACT
In this paper, we derive a key Eulerian integral involving the multivariable I-function defined by Prasad [4], the Aleph-function of one variable,a general class of polynomials of several variables and a generalized multiple-index Mittag-Leffler function. This general Eulerian integral formula is show to provide the key formula from which numerous others results for the multivariable I-function and multivariable H-function

Keywords :multivariable : Eulerian integral, Multivariable I-function, a generalized multiple-index Mittag-Leffler function, general class of polynomial,Aleph-function, multivariable H-function

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1. Introduction and preliminaries.

In this paper we establish a general Eulerian integral concerning the multivariable I—function defined by Prasad [4], the Aleph-function, a general class of multivariable polynomials and a generalized multiple-index Mittag-Leffler function. The I-function of several variables generalize the multivariable H-function defined by Srivastava et al [6] , itself is an a generalisation of G-function of several variables. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)}=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.3}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\gamma_1'},\cdots,|z_r|^{\gamma_r'}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|,\cdots,|z_r|^{\beta_s'}\,),min(\,|z_1|\,,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,z:\alpha_k'=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta_k'=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper :

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V=0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{1.4}$$

$$W=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}) \tag{1.5}$$

$$A=(a_{2k},\alpha_{2k}',\alpha_{2k}'');\cdots;(a_{(r-1)k}),\alpha'_{(r-1)k},\alpha''_{(r-1)k},\cdots,\alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B=(b_{2k},\beta_{2k}',\beta_{2k}'');\cdots;(b_{(r-1)k}),\beta'_{(r-1)k},\beta''_{(r-1)k},\cdots,\beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A}=(a_{sk};\alpha'_{rk},\alpha''_{rk},\cdots,\alpha_{rk}^{r}):\mathfrak{B}=(b_{rk};\beta'_{rk},\beta''_{rk},\cdots,\beta_{rk}^{r}) \tag{1.8}$$

$$A'=(a_k',\alpha_k')_{1,p'};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};B'=(b_k',\beta_k')_{1,p'};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,p^{(r)}} \tag{1.9}$$

The multivariable I-function write :

$$I(z_1,\cdots,z_r)=I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\;;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\;\mathfrak{B};\;\mathrm{B'} \end{array}\right) \tag{1.10}$$

The generalized polynomials of multivariable defined by Srivastava [5], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.11}$$

The Aleph- function , introduced by Südland [8] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z) = \aleph^{M,N}_{P_i,Q_i,c_i;r}\left(z \;\middle|\; \begin{matrix} (\mathrm{a}_j, A_j)_{1,\mathfrak{n}}, [c_i(a_{ji}, A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j, B_j)_{1,m}, [c_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix} \right) = \frac{1}{2\pi\omega}\int_L \Omega^{M,N}_{P_i,Q_i,c_i;r}(s) z^{-s} \mathrm{d}s \quad (1.12)$$

for all z different to 0 and

$$\Omega^{M,N}_{P_i,Q_i,c_i;r}(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.13)$$

With : $|argz| < \frac{1}{2}\pi\Omega$, where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - c_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=N+1}^{P_i}\alpha_{ji}) > 0 \quad ; i = 1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [8]. Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph^{M,N}_{P_i,Q_i,c_i;r}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_G g!}z^{-s} \quad (1.14)$$

With $s = \eta_{G,g} = \frac{b_G+g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.15)

2. Required formulas

We have : $B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\mathrm{d}t = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \quad Re(\alpha) > 0, Re(\beta) > 0$ (2.1)

(2.1) can be rewritten in the form

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t = (b-a)^{\alpha+\beta-1}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, Re(\alpha) > 0, Re(\beta) > 0, b \neq a \quad (2.2)$$

The binomial expansions for $t \in [a,b]$ yields :

$$(ut+v)^\gamma = (au+v)^\gamma\sum_{m=0}^{\infty}\frac{(-\gamma)_m}{m!}\left\{\frac{-u(a-t)}{au+v}\right\}^m \quad \text{where} \quad \left|\frac{(t-a)u}{au+v}\right| < 1 \quad (2.3)$$

With the help of (2.2) we obtain (see Srivastava et al [6])

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^\gamma\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(at+v)^\gamma{}_2F_1\left(\begin{matrix}\alpha,-\gamma\\ \alpha+\beta\end{matrix}; -\frac{(b-a)u}{au+v}\right) \quad (2.4)$$

where $Re(\alpha) > 0, Re(\beta) > 0; \left|arg\left(\frac{bu+v}{au+v}\right)\right| \leqslant \pi - \epsilon(0 < \epsilon < \pi), b \neq a$

3. Generalized multiple-index Mittag-Leffler function

A further generalization of the Mittag-Leffler functions is proposed recently in Paneva-Konovska [2]. These are 3m-parametric Mittag-Leffler type functions generalizing the Prabhakar [3] 3-parametric function , defined as:

$$E^{(\gamma_i),m}_{(\alpha_i),(\beta_i)}(z) = \sum_{k=0}^{\infty} \frac{(\gamma_1)_k \cdots (\gamma_m)_k}{\Gamma(\alpha_1 k + \beta_1) \cdots \Gamma(\alpha_m k + \beta_m)} \frac{z^k}{k!} \tag{3.1}$$

where $\alpha_i, \beta_i, \gamma_i \in \mathbb{C}, i = 1, \cdots, m, Re(\alpha_i) > 0$

4. General Eulerian integral of the multivariable Aleph-function

In this section, we shall prove one main general Eulerian integral involving the Aleph-function of one variable, general class of polynomials of several variables and multivariable Aleph-function.We note :

$$a' = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} A[N_1, K_1; \cdots ; N_s, K_s] \text{ and } \; b_k = \frac{(\gamma_1)_k \cdots (\gamma_m)_k}{\Gamma(\alpha_1 k + \beta_1) \cdots \Gamma(\alpha_m k + \beta_m)}$$

We have the following result :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \, (u_1t+v_1)^{r_1} \, (u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}$$

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(x(u_1t+v_1)^{c}(u_2t+v_2)^{d}(y_1t+z_1)^{e}(y_2t+z_2)^{f}\right)$$

$$E^{(\gamma_i),m}_{(\alpha_i),(\beta_i)}\left(z(u_1t+v_1)^{c'}(u_2t+v_2)^{d'}(y_1t+z_1)^{e'}(y_2t+z_2)^{f'}\right)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix} \mathrm{x}_1(u_1t+v_1)^{c_1}(u_2t+v_2)^{d_1}(y_1t+z_1)^{e_1}(y_2t+z_2)^{f_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{x}_s(u_1t+v_1)^{c_s}(u_2t+v_2)^{d_s}(y_1t+z_1)^{e_s}(y_2t+z_2)^{f_s} \end{pmatrix}$$

$$I^{V;0,n_r;X}_{U:p_r,q_r;W}\begin{pmatrix} \mathrm{Z}_1(u_1t+v_1)^{\rho_1}(u_2t+v_2)^{\rho'_1}(y_1t+z_1)^{\sigma_1}(y_2t+z_2)^{\sigma'_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_r(u_1t+v_1)^{\rho_r}(u_2t+v_2)^{\rho'_r}(y_1t+z_1)^{\sigma_r}(y_2t+z_2)^{\sigma'_r} \end{pmatrix}\mathrm{d}t$$

$$= (b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2} \sum_{l_1,l_2,l_3,l_4=0}^{\infty} \sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty} \frac{b_k z^k}{k!}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{B(\alpha+l_1+l_3,\beta+l_2+l_4)}{l_1!l_2!l_3!l_4!}a'\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(\eta_{G,g})}{B_G g!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}Y^k$$

$$\left\{\frac{(b-a)u_1}{(au_1+v_1)}\right\}^{l_1}\left\{-\frac{(b-a)y_1}{(by_1+z_1)}\right\}^{l_2}\left\{-\frac{(b-a)u_2}{(au_2+v_2)}\right\}^{l_3}\left\{\frac{(b-a)y_2}{(by_2+z_2)}\right\}^{l_4}I_{U:p_r+4,q_r+4;W}^{V;0,n_r+4;X}\left(\begin{array}{c|c}Z'_1 & A\ ;\\ \cdots & \cdots\\ \cdots & \cdots\\ Z'_r & B;\end{array}\right.$$

$$(-\delta_1-e\eta_{G,g}-e'k-\textstyle\sum_{i=1}^s e_iK_i:\sigma_1,\cdots,\sigma_r),\qquad(-r_1-c\eta_{G,g}-c'k-\sum_{i=1}^s c_iK_i:\rho_1,\cdots,\rho_r),$$

$$\cdots\qquad\cdots$$

$$(-\delta_1+l_2-e\eta_{G,g}-e'k-\textstyle\sum_{i=1}^s e_iK_i:\sigma_1,\cdots,\sigma_r),(-r_1+l_1-c\eta_{G,g}-c'k-\sum_{i=1}^s c_iK_i:\rho_1,\cdots,\rho_r),$$

$$(r_2-d\eta_{G,g}-d'k-\textstyle\sum_{i=1}^s d_iK_i:\rho'_1,\cdots,\rho'_r),\qquad(\delta_2-f\eta_{G,g}-f'k-\sum_{i=1}^s f_iK_i:\sigma'_1,\cdots,\sigma'_r),$$

$$\cdots\qquad\cdots$$

$$(r_2+l_3-d\eta_{G,g}-d'k-\textstyle\sum_{i=1}^s d_iK_i:\rho'_1,\cdots,\rho'_r),(\delta_2+l_4-f\eta_{G,g}-f'k-\sum_{i=1}^s f_iK_i:\sigma'_1,\cdots,\sigma'_r),$$

$$\left.\begin{array}{c}\mathfrak{A};A'\\ \cdots\\ \mathfrak{B}\ ;\ B'\end{array}\right)\tag{4.1}$$

where $X=x(au_1+v_1)^c\ (au_2+v_2)^d(by_1+z_1)^e(by_2+z_2)^f$

$Y=z(au_1+v_1)^{c'}\ (au_2+v_2)^{d'}(by_1+z_1)^{e'}(by_2+z_2)^{f'}$

$X_i=\ x_i(au_1+v_1)^{c_i}\ (au_2+v_2)^{d_i}(by_1+z_1)^{e_i}(by_2+z_2)^{f_i},i=1,\cdots,s$ and

$Z'_i=Z_i(au_1+v_1)^{\rho_i}(au_2+v_2)^{-\rho'_i}(by_1+z_1)^{\sigma_i}(by_2+z_2)^{-\sigma'_i},i=1,\cdots,r$

Provided

a) $min\{c,d,e,f,c',d',e',f',c_i,d_i,e_i,f_i,\rho_j,\rho'_j,\sigma_j,\sigma'_j\}>0,i=1,\cdots,s;j=1,\cdots,r$

b $min\{Re(\alpha),Re(\beta)\}>0,b\neq a$

c) $max\left\{\left|\frac{u_1(b-a)}{au_1+v_1}\right|,\left|\frac{y_1(b-a)}{by_1+z_1}\right|,\left|\frac{(b-a)u_2}{au_2+v_2}\right|,\left|\frac{(b-a)y_2}{by_2+z_2}\right|\right\}<1$

d) $Re\big[r_1+c\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+c'k+\sum\limits_{i=1}^r\rho_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

e) $Re\big[r_2+d\min\limits_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+d'k+\sum\limits_{i=1}^r\rho'_i\min\limits_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

f) $Re[\delta_1 + e \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + e'k + \sum_{i=1}^{r} \sigma_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

g) $Re[\delta_2 + f \min_{1\leqslant j\leqslant M} \frac{b_j}{B_j} + f'k + \sum_{i=1}^{r} \sigma'_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] > -1$

h) $|argZ_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.3)

i) $|argx| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} \alpha_j - c_i(\sum_{j=M+1}^{Q_i} \beta_{ji} + \sum_{j=N+1}^{P_i} \alpha_{ji}) > 0$

j) $\alpha_i, \beta_i, \gamma_i \in \mathbb{C}, i = 1, \cdots, m, Re(\alpha_i) > 0$

Proof

We first replace the multivariable I-function defined by Prasad [4] on the L.H.S of (3.1) by its Mellin-barnes contour integral (1.1), the Aleph-function , a general class of polynomials of several variables and the generalized multiple-index Mittag-Leffler function.in series using respectively (1.13), (1.11) and (3.1). Now we interchange the order of summation and integrations (which is permissible under the conditions stated). Collect the powers of $(u_1t + v_1), (u_2t + v_2), (y_1t + z_1), (y_2t + z_2)$, and apply the binomial expansion (2.3). We then use the Eulerian integral (2.2) and interpret the resulting Mellin-Barnes contour integral as a I-function of r variables, we arrive at the desired result.

5. Particular case

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [7]. We have the following result.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} (u_1t+v_1)^{r_1} \ (u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}$$

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}\big(x(u_1t+v_1)^c(u_2t+v_2)^d(y_1t+z_1)^e(y_2t+z_2)^f\big)$$

$$E_{(\alpha_i),(\beta_i)}^{(\gamma_i),m} \big(z(u_1t+v_1)^{c'}(u_2t+v_2)^{d'}(y_1t+z_1)^{e'}(y_2t+z_2)^{f'}\big)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{x}_1(u_1t+v_1)^{c_1}(u_2t+v_2)^{d_1}(y_1t+z_1)^{e_1}(y_2t+z_2)^{f_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{x}_s(u_1t+v_1)^{c_s}(u_2t+v_2)^{d_s}(y_1t+z_1)^{e_s}(y_2t+z_2)^{f_s} \end{pmatrix}$$

$$H_{p_r,q_r;W}^{0,n_r;X}\begin{pmatrix} \mathrm{Z}_1(u_1t+v_1)^{\rho_1}(u_2t+v_2)^{\rho'_1}(y_1t+z_1)^{\sigma_1}(y_2t+z_2)^{\sigma'_1} \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ \mathrm{Z}_r(u_1t+v_1)^{\rho_r}(u_2t+v_2)^{\rho'_r}(y_1t+z_1)^{\sigma_r}(y_2t+z_2)^{\sigma'_r} \end{pmatrix}\mathrm{d}t$$

$$= (b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}\sum_{l_1,l_2,l_3,l_4=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{k=0}^{\infty}\frac{b_k z^k}{k!}$$

$$\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{B(\alpha+l_1+l_3,\beta+l_2+l_4)}{l_1!l_2!l_3!l_4!}a'\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_G g!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}Y^k$$

$$\left\{\frac{(b-a)u_1}{(au_1+v_1)}\right\}^{l_1}\left\{-\frac{(b-a)y_1}{(by_1+z_1)}\right\}^{l_2}\left\{-\frac{(b-a)u_2}{(au_2+v_2)}\right\}^{l_3}\left\{\frac{(b-a)y_2}{(by_2+z_2)}\right\}^{l_4}H^{0,n_r+4;X}_{p_r+4,q_r+4;W}\left(\begin{array}{c}\text{Z'}_1\\ \cdots\\ \cdots\\ \text{Z'}_r\end{array}\right|$$

$$(\text{-}\delta_1-e\eta_{G,g}-e'k-\textstyle\sum_{i=1}^{s}e_iK_i:\sigma_1,\cdots,\sigma_r),\qquad(\text{-r}_1-c\eta_{G,g}-c'k-\sum_{i=1}^{s}c_iK_i:\rho_1,\cdots,\rho_r),$$
$$\cdots\qquad\qquad\cdots$$
$$(\text{-}\delta_1+l_2-e\eta_{G,g}-e'k-\textstyle\sum_{i=1}^{s}e_iK_i:\sigma_1,\cdots,\sigma_r),(\text{-r}_1+l_1-c\eta_{G,g}-c'k-\sum_{i=1}^{s}c_iK_i:\rho_1,\cdots,\rho_r),$$

$$(\text{r}_2-d\eta_{G,g}-d'k-\textstyle\sum_{i=1}^{s}d_iK_i:\rho'_1,\cdots,\rho'_r),\qquad(\delta_2-f\eta_{G,g}-f'k-\sum_{i=1}^{s}f_iK_i:\sigma'_1,\cdots,\sigma'_r),$$
$$\cdots\qquad\qquad\cdots$$
$$(\text{r}_2+l_3-d\eta_{G,g}-d'k-\textstyle\sum_{i=1}^{s}d_iK_i:\rho'_1,\cdots,\rho'_r),(\delta_2+l_4-f\eta_{G,g}-f'k-\sum_{i=1}^{s}f_iK_i:\sigma'_1,\cdots,\sigma'_r),$$

$$\left.\begin{array}{c}\mathfrak{A};\text{A'}\\ \cdots\\ \mathfrak{B};\text{B'}\end{array}\right)\tag{5.1}$$

6. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the Aleph-function, a class of polynomials of several variables, the generalized multiple-index Mittag-Leffler function and the multivariable I-function defined by Prasad [4]. The integral established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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