Eulerian integral involving the multivariable I-function III

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ABSTRACT
In this paper, we derive two Eulerian integrals involving a product of two multivariable I-functions defined by Prasad [2], the Aleph-function of one variable, general class of polynomials of several variables and a extension of the Hurwitz-lerch Zeta-function. This general Eulerian integral formula is show to provide the key formula from which numerous others results for the multivariable I-function, H-function of several variables.

Keywords :multivariable I-function, Eulerian integral, Multivariable H-function, the Hurwitz-lerch Zeta-function, general class of polynomial,Aleph-function

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1. Introduction and preliminaries.

In this paper we establish two general Eulerian integrals concerning a product of two multivariable I-functions, the Aleph-function a general class of multivariable polynomials and a extension of the Hurwitz-lerch Zeta-function. These function generalize the multivariable H-function study by Srivastava et al [5], itself is an a generalisation of G-function of several variables. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle| \begin{array}{c} (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \\ (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (a_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (b_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) +$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \quad (1.3)$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(\,|z_1|^{\gamma'_1}, \cdots, |z_r|^{\gamma'_r}\,)\,, max(\,|z_1|, \cdots, |z_r|\,) \rightarrow 0$$

$$I(z_1, \cdots, z_r) \;= 0(\,|z_1|, \cdots, |z_r|^{\beta'_s}\,)\,, min(\,|z_1|\,, \cdots, |z_r|\,) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \quad (1.4)$$

$$W = (p', q'); \cdots; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots; (m^{(r)}, n^{(r)}) \quad (1.5)$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \cdots; (a_{(r-1)k}), \alpha'_{(r-1)k}, \alpha''_{(r-1)k}, \cdots, \alpha_{(r-1)k}^{(r-1)}) \quad (1.6)$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \cdots; (b_{(r-1)k}), \beta'_{(r-1)k}, \beta''_{(r-1)k}, \cdots, \beta_{(r-1)k}^{(r-1)}) \quad (1.7)$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{rk}, \alpha''_{rk}, \cdots, \alpha_{rk}^{r}) : \mathfrak{B} = (b_{rk}; \beta'_{rk}, \beta''_{rk}, \cdots, \beta_{rk}^{r}) \quad (1.8)$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b'_k, \beta'_k)_{1,p'}; \cdots; (b_k^{(r)}, \beta_k^{(r)})_{1,p^{(r)}} \quad (1.9)$$

The multivariable I-function write :

$$I(z_1, \cdots, z_r) = I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c|c} \mathrm{z}_1 & \text{A };\mathfrak{A};\text{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \text{B}; \mathfrak{B}; \text{B'} \end{array}\right) \quad (1.10)$$

The generalized polynomials of multivariable defined by Srivastava [3], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1, \cdots, y_s] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \quad (1.11)$$

The Aleph- function , introduced by Südland [7] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph^{M,N}_{P_i,Q_i,c_i;r}\left(z \;\middle|\; \begin{matrix} (\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r} \\ (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega^{M,N}_{P_i,Q_i,c_i;r}(s)z^{-s}\mathrm{d}s \quad (1.12)$$

for all z different to 0 and

$$\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)=\frac{\prod_{j=1}^{M}\Gamma(b_j+B_js)\prod_{j=1}^{N}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \quad (1.13)$$

With : $|argz|<\frac{1}{2}\pi\Omega$, where $\Omega=\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0 \quad ;i=1,\cdots,r$

For convergence conditions and other details of Aleph-function , see Südland et al [7]. Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph^{M,N}_{P_i,Q_i,c_i;r}(z)=\sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega^{M,N}_{P_i,Q_i,c_i,r}(s)}{B_G g!}z^{-s} \quad (1.14)$$

With $s=\eta_{G,g}=\frac{b_G+g}{B_G}, P_i<Q_i, |z|<1$ and $\Omega^{M,N}_{P_i,Q_i,c_i;r}(s)$ is given in (1.2) (1.15)

2. Required formulas

We have : $B(\alpha,\beta)=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\mathrm{d}t=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)},\ Re(\alpha)>0, Re(\beta)>0$ (2.1)

(2.1) can be rewritten in the form

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\mathrm{d}t=(b-a)^{\alpha+\beta-1}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, Re(\alpha)>0, Re(\beta)>0, b\neq a \quad (2.2)$$

The binomial expansions for $t\in[a,b]$ yields :

$$(ut+v)^{\gamma}=(au+v)^{\gamma}\sum_{m=0}^{\infty}\frac{(-\gamma)_m}{m!}\left\{\frac{-u(a-t)}{au+v}\right\}^m \quad \text{where} \quad \left|\frac{(t-a)u}{au+v}\right|<1 \quad (2.3)$$

With the help of (2.2) we obtain (see Srivastava et al [4])

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^{\gamma}\mathrm{d}t= (b-a)^{\alpha+\beta-1}B(\alpha,\beta)(at+v)^{\gamma}{}_2F_1\left(\begin{array}{c}\alpha,-\gamma\\ \alpha+\beta\end{array};-\frac{(b-a)u}{au+v}\right) \quad (2.4)$$

where $Re(\alpha)>0, Re(\beta)>0; \left|arg\left(\frac{bu+v}{au+v}\right)\right| \leqslant \pi-\epsilon(0<\epsilon<\pi), b\neq a$

3. Extension of the Hurwitz-Lerch Zeta function

The extension of the Hurwitz-Lerch Zeta function $\phi(z,S,a)$ is introduced by Srivastava et al ([6],eq.(6.2), page 503) as follows :

$$\phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{(\lambda_1,\cdots,\lambda_p,\mu_1,\cdots,\mu_q)}(z;\mathfrak{s},a)=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{(a+n)^{\mathfrak{s}}\prod_{j=1}^{q}(\mu_j)_{n\sigma_j}}\times\frac{z^n}{n!} \quad (3.1)$$

with : $p,q\in\mathbb{N}_0, \lambda_j\in\mathbb{C}(j=1,\cdots,p), a,\mu_j\in\mathbb{C}\setminus\mathbb{Z}_0^* \;\; (j=1,\cdots,q), \rho_j,\sigma_k\in\mathbb{R}^+$ $(j=1,\cdots,p;k=1,\cdots,q)$

where $\Delta>-1$ when $\mathfrak{s},z\in\mathbb{C}; \Delta=-1$ and $s\in\mathbb{C}, when|z|<\bigtriangledown^*$, $\Delta=-1$ and $Re(\chi)>\frac{1}{2}$ when $|z|=\bigtriangledown^*$

$$\bigtriangledown^*=\prod_{j=1}^{p}\rho_j^{\rho_j}\prod_{j=1}^{q}\sigma_j^{\sigma_j};\Delta=\sum_{j=1}^{q}\sigma_j-\sum_{j=1}^{p}\rho_j;\chi=\mathfrak{s}+\sum_{j=1}^{q}\mu_j-\sum_{j=1}^{p}\lambda_j+\frac{p-q}{2}$$

We denote these conditions, the conditions (f).

4. General Eulerian integral of the multivariable Aleph-function

In this section, we shall prove two main general Eulerian integrals involving the Aleph-function of one variable, general class of polynomials and several variables and multivariable Aleph-function.

We note : $a'=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]$

and $b_n=\frac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{(a+n)^{\mathfrak{s}}\prod_{j=1}^{q}(\mu_j)_{n\sigma_j}}$. We have the following result :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\,(ut+v)^{\gamma}\;(yt+z)^{\delta}\;\aleph^{M,\mathfrak{N}}_{P_i,Q_i,\mathfrak{c}_i;r'}\big(x(ut+v)^c(yt+z)^d\big)$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c}\mathrm{x}_1(ut+v)^{c_1}(yt+z)^{d_1}\\ \cdot\;\cdot\;\cdot\\ \cdot\;\cdot\;\cdot\\ \mathrm{x}_s(ut+v)^{c_s}(yt+z)^{d_s}\end{array}\right)\phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{(\lambda_1,\cdots,\lambda_p,\mu_1,\cdots,\mu_q)}\big(z'(ut+v)^{c'}(yt+z)^{d'};\mathfrak{s},a\big)$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c}\mathrm{y}_1(ut+v)^{\sigma_1}\\ \cdots\\ \cdots\\ \mathrm{y}_r(ut+v)^{\sigma_r}\end{array}\right) I_{u:P_R,Q_R;w}^{v;0,n_R;x}\left(\begin{array}{c}\mathrm{z}_1(yt+z)^{\lambda_1}\\ \cdots\\ \cdots\\ \mathrm{z}_R(yt+z)^{\lambda_R}\end{array}\right)\mathrm{d}t$$

$$=(b-a)^{\alpha+\beta-1}(au+v)^{\gamma}(by+z)^{\delta}B(\alpha,\beta)\sum_{l,m=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(\alpha)_l(\beta)_m}{(\alpha+\beta)_{l+m}l!m!}a'$$

$$\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,\mathfrak{N}}(\eta_{G,g})}{B_G g!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}\ Y^n\left\{\frac{(b-a)u}{(au+v)}\right\}^l\left\{-\frac{(b-a)y}{(by+z)}\right\}^m$$

$$I_{U:p_r+1,q_r+1;W}^{V;0,n_r+1;X}\left(\begin{array}{c}\mathrm{y}_1(au+v)^{\sigma_1}\\ \cdots\\ \cdots\\ \mathrm{y}_r(au+v)^{\sigma_r}\end{array}\left|\begin{array}{c}\mathrm{A}\ ;\ (-\gamma-c\eta_{G,g}-c'n-\sum_{i=1}^{s}c_iK_i:\sigma_1,\cdots,\sigma_r),\mathfrak{A},A'\\ \cdots\\ \mathrm{B}\ ;\ (-\gamma+l-c\eta_{G,g}-c'n-\sum_{i=1}^{s}c_iK_i:\sigma_1,\cdots,\sigma_r),B,B'\end{array}\right.\right)$$

$$I_{u:P_R+1,Q_R+1;w}^{v;0,n_R+1;x}\left(\begin{array}{c}\mathrm{z}_1(by+z)^{\lambda_1}\\ \cdots\\ \cdots\\ \mathrm{z}_R(by+z)^{\lambda_R}\end{array}\left|\begin{array}{c}\mathrm{A}_1;(-\delta-d\eta_{G,g}-d'n-\sum_{i=1}^{s}d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{A}_1;A_1'\\ \cdots\\ \mathrm{B}_1;(-\delta+m-d\eta_{G,g}-d'n-\sum_{i=1}^{s}d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{B}_1;B_1'\end{array}\right.\right)\quad(4.1)$$

where $X=x(au+v)^c\ (by+z)^d\quad X_i=x_i(au+v)^{c_i}\ (by+z)^{d_i}, i=1,\cdots,s$ and

$Y=z'(au+v)^{c'}(by+z)^{d'}$

Provided that

a) $min\{c,d,c',d',c_i,d_i,\sigma_j,\lambda_k\}>0, i=1,\cdots,s; j=1,\cdots,r; k=1,\cdots,R$

b $min\{Re(\alpha),Re(\beta)\}>0; b\neq a$, $max\left\{\left|\frac{u(b-a)}{au+v}\right|,\left|\frac{y(b-a)}{by+z}\right|\right\}<1$

c) $Re\big[\gamma+c\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+c'n+\sum_{i=1}^{r}\sigma_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}\big]>-1$

d) $Re\big[\delta+d\min_{1\leqslant j\leqslant M}\frac{b_j}{B_j}+d'n+\sum_{i=1}^{r}\lambda_i\min_{1\leqslant j\leqslant m_i}\frac{d_j'^{(i)}}{\delta_j'^{(i)}}\big]>-1$

e) $|argy_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.3) and e) $|argZ_k|<\frac{1}{2}\omega_i^{(k)}\pi$

f) The conditions (f) are satisfied

g) $|argx| < \frac{1}{2}\pi\Omega$ Where $\Omega = \sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}\alpha_j - \mathfrak{c}_i(\sum_{j=M+1}^{Q_i}\beta_{ji} + \sum_{j=\mathfrak{N}+1}^{P_i}\alpha_{ji}) > 0$

Proof

We first replace the two multivariable I-functions defined by Prasad [2] on the L.H.S of (3.1) by its Mellin-barnes contour integral respectively, the Aleph-function of one variable, the general class of polynomials of several variables and a extension of the Hurwitz-lerch Zeta-function in series using respectively (1.14), (1.11) and (2.3), Now we interchange the order of summation and integrations (which is permissible under the conditions stated). Collect the powers of $(ut+v), (yt+z)$ and apply the binomial expansion (2.3). We then use the Eulerian integral (2.2) and interpret the resulting both Mellin-Barnes contour integral as an I-function of r variables and an I-function of R variables respectively, we arrive at the desired result.

In (3.1) replace a by $-a$ and y by $-y$, we obtain the following Eulerian integral :

$$\int_{-a}^{b}(t+a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^{\gamma}\ (z-ty)^{\delta}\ \aleph_{P_i,Q_i,\mathfrak{c}_i;r'}^{M,\mathfrak{N}}\left(x(ut+v)^{c}(z-ty)^{d}\right)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix} \mathrm{x}_1(ut+v)^{c_1}(z-ty)^{d_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{x}_s(ut+v)^{c_s}(z-ty)^{d_s} \end{pmatrix}\phi_{(\lambda_1,\cdots,\lambda_p,\mu_1,\cdots,\mu_q)}^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}(z'(ut+v)^{c'}(z-ty)^{d'};\mathfrak{s},a)$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\begin{pmatrix} \mathrm{y}_1(ut+v)^{\sigma_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{y}_r(ut+v)^{\sigma_r} \end{pmatrix} I_{u:P_R,Q_R;w}^{v;0,n_R;x}\begin{pmatrix} \mathrm{z}_1(z-ty)^{\lambda_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_R(z-ty)^{\lambda_R} \end{pmatrix}\mathrm{d}t$$

$$= (b+a)^{\alpha+\beta-1}(v-au)^{\gamma}(z-by)^{\delta}B(\alpha,\beta)\sum_{l,m=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(\alpha)_l(\beta)_m}{(\alpha+\beta)_{l+m}l!m!}a'$$

$$\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,\mathfrak{N}}(\eta_{G,g})}{B_G g!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}\ Y^n\left\{\frac{(b+a)u}{(au+v)}\right\}^l\left\{\frac{(b+a)y}{(by-z)}\right\}^m$$

$$I_{U:p_r+1,q_r+1;W}^{V;0,n_r+1;X}\left(\begin{array}{c|c} \mathrm{y}_1(v-au)^{\sigma_1} & \mathrm{A}\ ;\ (\text{-}\gamma-c\eta_{G,g}-c'n-\sum_{i=1}^{s}c_iK_i:\sigma_1,\cdots,\sigma_r),\mathfrak{A},A' \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \\ \mathrm{y}_r(v-au)^{\sigma_r} & \mathrm{B}\ ;\ (\text{-}\gamma+l-c\eta_{G,g}-c'n-\sum_{i=1}^{s}c_iK_i:\sigma_1,\cdots,\sigma_r),B,B' \end{array}\right)$$

$$I_{u:P_R+1,Q_R+1;w}^{v;0,n_R+1;x}\left(\begin{array}{c} \mathrm{z}_1(z-by)^{\lambda_1} \\ \cdots \\ \cdots \\ \mathrm{z}_R(z-by)^{\lambda_R} \end{array}\middle| \begin{array}{c} \mathrm{A}_1;(-\delta-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{A}_1;A_1' \\ \cdots \\ \mathrm{B}_1;(-\delta+m-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{B}_1;B_1' \end{array}\right) \quad (4.2)$$

where $X = x(au+v)^c \ (z-by)^d \quad X_i = x_i(v-au)^{c_i} \ (z-by)^{d_i}, i = 1, \cdots, s$ and

$Y = z(v-au)^{c'}(z-by)^{d'}$

where the same notations and validity conditions that (3.1).

5. Particular case

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [5]. We have the following results.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\,(ut+v)^\gamma\ (yt+z)^\delta\ \aleph_{P_i,Q_i,\mathfrak{c}_i;r'}^{M,\mathfrak{N}}\big(x(ut+v)^c(yt+z)^d\big)$$

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} \mathrm{x}_1(ut+v)^{c_1}(yt+z)^{d_1} \\ \cdots \\ \cdots \\ \mathrm{x}_s(ut+v)^{c_s}(yt+z)^{d_s} \end{array}\right)\phi_{(\lambda_1,\cdots,\lambda_p,\mu_1,\cdots,\mu_q)}^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}(z'(ut+v)^{c'}(yt+z)^{d'};\mathfrak{s},a)$$

$$H_{p_r,q_r;W}^{0,n_r;X}\left(\begin{array}{c} \mathrm{y}_1(ut+v)^{\sigma_1} \\ \cdots \\ \cdots \\ \mathrm{y}_r(ut+v)^{\sigma_r} \end{array}\right)H_{P_R,Q_R;w}^{0,n_R;x}\left(\begin{array}{c} \mathrm{z}_1(yt+z)^{\lambda_1} \\ \cdots \\ \cdots \\ \mathrm{z}_R(yt+z)^{\lambda_R} \end{array}\right)\mathrm{dt}$$

$$= (b-a)^{\alpha+\beta-1}(au+v)^\gamma(by+z)^\delta B(\alpha,\beta)\sum_{l,m=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(\alpha)_l(\beta)_m}{(\alpha+\beta)_{l+m}l!m!}a'$$

$$\frac{(-)^g\Omega_{P_i,Q_i,\mathfrak{c}_i,r'}^{M,\mathfrak{N}}(\eta_{G,g})}{B_Gg!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}\ Y^n\left\{\frac{(b-a)u}{(au+v)}\right\}^l\left\{-\frac{(b-a)y}{(by+z)}\right\}^m$$

$$H_{p_r+1,q_r+1;W}^{0,n_r+1;X}\left(\begin{array}{c} \mathrm{y}_1(au+v)^{\sigma_1} \\ \cdots \\ \cdots \\ \mathrm{y}_r(au+v)^{\sigma_r} \end{array}\middle| \begin{array}{c} (-\gamma-c\eta_{G,g}-c'n-\sum_{i=1}^s c_iK_i:\sigma_1,\cdots,\sigma_r),\mathfrak{A},A' \\ \cdots \\ (-\gamma+l-c\eta_{G,g}-c'n-\sum_{i=1}^s c_iK_i:\sigma_1,\cdots,\sigma_r),B,B' \end{array}\right)$$

$$H^{0,n_R+1;x}_{P_R+1,Q_R+1;w}\left(\begin{array}{c} z_1(by+z)^{\lambda_1} \\ \cdots \\ \cdots \\ z_R(by+z)^{\lambda_R} \end{array}\middle| \begin{array}{c} (-\delta-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{A}_1;A'_1 \\ \cdots \\ (-\delta+m-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{B}_1;B'_1 \end{array}\right) \quad (5.1)$$

under the same notations and conditions that (4.1) with $U=V=A=B=0$

and

$$\int_{-a}^{b}(t+a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^{\gamma}\ (z-ty)^{\delta}\ \aleph^{M,\mathfrak{N}}_{P_i,Q_i,\mathfrak{c}_i;r'}(x(ut+v)^{c}(z-ty)^{d})$$

$$S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\left(\begin{array}{c} x_1(ut+v)^{c_1}(z-ty)^{d_1} \\ \cdots \\ \cdots \\ x_s(ut+v)^{c_s}(z-ty)^{d_s} \end{array}\right)\phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{(\lambda_1,\cdots,\lambda_p,\mu_1,\cdots,\mu_q)}(z'(ut+v)^{c'}(z-ty)^{d'};\mathfrak{s},a)$$

$$H^{0,n_r;X}_{p_r,q_r;W}\left(\begin{array}{c} y_1(ut+v)^{\sigma_1} \\ \cdots \\ \cdots \\ y_r(ut+v)^{\sigma_r} \end{array}\right)H^{0,n_R;x}_{P_R,Q_R;w}\left(\begin{array}{c} z_1(z-ty)^{\lambda_1} \\ \cdots \\ \cdots \\ z_R(z-ty)^{\lambda_R} \end{array}\right)\mathrm{d}t$$

$$=(b+a)^{\alpha+\beta-1}(v-au)^{\gamma}(z-by)^{\delta}B(\alpha,\beta)\sum_{l,m=0}^{\infty}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(\alpha)_l(\beta)_m}{(\alpha+\beta)_{l+m}l!m!}a'$$

$$\frac{(-)^g\Omega^{M,\mathfrak{N}}_{P_i,Q_i,\mathfrak{c}_i,r'}(\eta_{G,g})}{B_G g!}X_1^{K_1}\cdots X_s^{K_s}X^{\eta_{G,g}}\ Y^n\left\{\frac{(b+a)u}{(au+v)}\right\}^l\left\{\frac{(b+a)y}{(by-z)}\right\}^m$$

$$H^{0,n_r+1;X}_{p_r+1,q_r+1;W}\left(\begin{array}{c} y_1(v-au)^{\sigma_1} \\ \cdots \\ \cdots \\ y_r(v-au)^{\sigma_r} \end{array}\middle| \begin{array}{c} (-\gamma-c\eta_{G,g}-c'n-\sum_{i=1}^s c_iK_i:\sigma_1,\cdots,\sigma_r),\mathfrak{A},A' \\ \cdots \\ (-\gamma+l-c\eta_{G,g}-c'n-\sum_{i=1}^s c_iK_i:\sigma_1,\cdots,\sigma_r),B,B' \end{array}\right)$$

$$H^{0,n_R+1;x}_{P_R+1,Q_R+1;w}\left(\begin{array}{c} z_1(z-by)^{\lambda_1} \\ \cdots \\ \cdots \\ z_R(z-by)^{\lambda_R} \end{array}\middle| \begin{array}{c} (-\delta-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{A}_1;A'_1 \\ \cdots \\ (-\delta+m-d\eta_{G,g}-d'n-\sum_{i=1}^s d_iK_i:\lambda_1,\cdots,\lambda_R),\mathfrak{B}_1;B'_1 \end{array}\right) \quad (5.2)$$

where $X = x(au+v)^c \ (z-by)^d \quad X_i = x_i(v-au)^{c_i} \ (z-by)^{d_i}, i = 1, \cdots, s$ and

$$Y = z(v-au)^{c'}(z-by)^{d'}$$

under the same notations and conditions that (4.1) with $U = V = A = B = 0$

6. Conclusion

In this paper we have evaluated two generalized Eulerian integrals involving the Aleph-function, a class of polynomials of several variables a extension of the Hurwitz-Lerch Zeta-function and product of two multivariable I-functions defined by Prasad [2]. The integral established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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