On some generalized results of fractional derivatives II

$F.Y. \ AYANT^{1}$

1 Teacher in High School , France

ABSTRACT
The purpose of the present document is to derive a number of key formulas for fractional derivatives of multivariables I-function defined by Prasad [5] and generalized multivariable polynomials. Some of the applications of the key formulas provide potentially useful generalizations of know results in the theory of fractional calculus.

KEYWORDS : I-function of several variables,general fractional derivative formulae, special function, general class of polynomials.

1. Introduction and preliminaries.

The object of this document is to study the fractional derivative formula from the multivariables I-function defined by Prasad [5]. These function generalize the multivariable H-function recently study by C.K. Srivastava et al [6] , itself is an a generalisation of G-function of several variables. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [5]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)}=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.3}$$

where $i = 1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r) = 0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,, max(\,|z_1|,\cdots,|z_r|\,) \rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,, min(\,|z_1|,\cdots,|z_r|\,) \rightarrow \infty$$

where $k = 1,\cdots,r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1,\cdots,m_k$ and

$$\beta'_k = max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})], j = 1,\cdots,n_k$$

We will use these following notations in this paper :

$$U = p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V = 0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{1.4}$$

$$W = (p',q');\cdots;(p^{(r)},q^{(r)});X = (m',n');\cdots;(m^{(r)},n^{(r)}) \tag{1.5}$$

$$A = (a_{2k},\alpha'_{2k},\alpha''_{2k});\cdots;(a_{(r-1)k}),\alpha'{}_{(r-1)k},\alpha''_{(r-1)k},\cdots,\alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B = (b_{2k},\beta'_{2k},\beta''_{2k});\cdots;(b_{(r-1)k}),\beta'{}_{(r-1)k},\beta''_{(r-1)k},\cdots,\beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A} = (a_{sk};\alpha'{}_{rk},\alpha''_{rk},\cdots,\alpha_{rk}^{r}) : \mathfrak{B} = (b_{rk};\beta'{}_{rk},\beta''_{rk},\cdots,\beta_{rk}^{r}) \tag{1.8}$$

$$A' = (a'_k,\alpha'_k)_{1,p'};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};B' = (b'_k,\beta'_k)_{1,p'};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,p^{(r)}} \tag{1.9}$$

The multivariable I-function write :

$$I(z_1,\cdots,z_r) = I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\;;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\;\mathfrak{B};\;\mathrm{B'} \end{array}\right) \tag{1.10}$$

Srivastava and Garg introduced and defined a general class of multivariable polynomials [9] as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]= \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \tag{1.13}$$

The fractional derivative of a function f (x) of a complex order μ is defined by Oldham et al[4], (1974 , page 49) in the followin manner :

$$ {}_aD_x^{\mu} f(x) = \frac{1}{\Gamma(-\mu)} \int_a^x (x-y)^{-\mu-1} f(y)\,\mathrm{dy} \text{ if } Re(\mu) < 0\,; \ \frac{d^m}{dx^m}\ {}_aD_x^{\mu-m} f(x) \text{ if } 0 \leqslant Re(\mu) < m $$

where m is a positive integer.

For simplicity , the special ense of the fractional derivative operator ${}_aD_x^{\mu}$ when $a=0$, will be written as D_x^{μ}

Also we have :

$$ D_x^{\mu}\left(x^{\lambda}\right) = \frac{d^{\mu}}{dx^{\mu}}(x^{\lambda}) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)} x^{\lambda-\mu} \quad , Re(\lambda) > -1 \tag{1.14} $$

and the binomial expansion

$$ (x+\mu)^{\lambda} = \mu^{\lambda} \sum_{m=0}^{\infty} \binom{\lambda}{m} \left(\frac{x}{\mu}\right)^m \ , \quad \left|\frac{x}{\mu}\right| < 1 \tag{1.15} $$

For $0 \leqslant \alpha < 1, \beta, \eta, x \in \mathbb{R}; m \in \mathbb{N}$, the generalized modified fractional derivative operator due to Saigo is defined in Samko, Kilbas and Marichev [6] as

$$ D_{0,x,m}^{\alpha,\beta,\eta} f(x) = \frac{d}{dz}\left(\frac{z^{-m(\beta-\eta)}}{\Gamma(1-\alpha)} \int_a^x (x^m - t^m)^{-\alpha} F(\beta-\alpha, 1-\eta; 1-\alpha; 1-t^m/x^m) \right) f(t) \mathrm{d}t^m \tag{1.16} $$

the multiplicity of $t^m - x^m$ is above equation is removed by requiring $log(t^m - x^m)$ as real for $t^m - x^m > 0$ and is assumed to be well defined in the unit disk.

We have . $D_{0,x,1}^{\alpha,\alpha,\eta} f(x) = D_x^{\alpha} f(x)$ (1.17)

Where D_x^{α} is the familiar Riemann-Liouville fractional derivative operator.

For $0 \leqslant \alpha < 1, \beta, \eta, x \in \mathbb{R}; m \in \mathbb{N}, \mu > max(0, \beta-\eta)$, the refined form due to Bhatt and Raina [1] is given by.

$$ D_{0,x,m}^{\alpha,\beta,\eta}\{x^{(\mu-1)m}\} = \frac{\Gamma(\mu)\Gamma(\mu+\eta-\beta)}{\Gamma(\mu-\beta)\Gamma(\mu+\eta-\alpha)} x^{(\mu-\beta-1)m} \tag{1.18} $$

2.Formulas

In these section, we give three formulas fractional derivatives of multivariable Aleph-function.

Formula 1

$$ D_{x_1}^{\mu_1} \cdots D_{x_r}^{\mu_r} \Big[x_1^{m_1}(x_1^{v_1}+\zeta_1)^{\lambda_1} \cdots x_r^{m_r}(x_r^{v_r}+\zeta_r)^{\lambda_r} $$

$$ I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c} \mathrm{z}_1 x_1^{\rho_1'}(x_1^{v_1}+\zeta_1)^{-\sigma_1'} \cdots x_r^{\rho_r'}(x_r^{v_r}+\zeta_r)^{-\sigma_r'} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_n x_1^{\rho_1^n}(x_1^{v_1}+\zeta_1)^{-\sigma_1^n} \cdots x_r^{\rho_r^n}(x_r^{v_r}+\zeta_r)^{-\sigma_r^n} \end{array} \right) \Bigg] = \zeta_1^{\lambda_1} \cdots \zeta_r^{\lambda_r} x_1^{m_1-\mu_1} \cdots x_r^{m_r-\mu_r} $$

$$\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-x_1^{v_1}/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-x_r^{v_r}/\zeta_r)^{N_r}}{N_r!}I_{U:p_r+2r,q_r+2r;W}^{V;0,n_r+2r;X}\left(\begin{array}{c|}\mathrm{z}_1A_1\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_nA_n\end{array}\right.$$

$$\left.\begin{array}{ll}\mathrm{A}\ ;\ [1+\lambda_j-N_j:\sigma'_j,\cdots,\sigma_j^n]_{1,r}, & [1\text{-}\mu_j-v_1N_j-k_1K:\rho'_j,\cdots,\rho_j^n]_{1,r},\mathfrak{A};A'\\ \quad\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \quad\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ \mathrm{B}\ ;\ [1+\lambda_j:\sigma'_j,\cdots,\sigma_j^n]_{1,r}, & [1-\mu_j+\beta_j-v_jN_j-k_1K:\rho'_j,\cdots,\rho_j^n]_{1,r},\mathfrak{B};B'\end{array}\right) \tag{2.1}$$

Where $A_i=\dfrac{x_1^{\rho_1^i}\cdots x_r^{\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}},i=1,\cdots,n$

Provided

a) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i)>0,i=1,\cdots,n$

b) $max[|arg(x_1^{v_1}/\zeta_1)|,\cdots,|arg(x_r^{v_r}/\zeta_r)|]<\pi$

c) $Re[m_1+\sum_{i=1}^{n}\rho_1^i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1,\cdots,Re[m_r+\sum_{i=1}^{n}\rho_r^i\min\limits_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>-1$

Proof of (2.1)

Let $M=\;=\dfrac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)$

Where $\psi(s_1,\cdots,s_r),\theta_k(s_k)$ are defined respectively by (1.2) and (1.3), thefore

$$D_{x_1}^{\mu_1}\cdots D_{x_r}^{\mu_r}\{M[x_1^{m_1}(x_1^{v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r}(x_r^{v_r}+\zeta_r)^{\lambda_r}\cdots[z_1x_1^{\rho'_1}(x_1^{v_1}+\zeta_1)^{-\sigma'_1}\cdots x_r^{\rho'_r}(x_r^{v_r}+\zeta_r)^{-\sigma'_r}]^{s_1}$$

$$[z_nx_1^{\rho_1^n}(x_1^{v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{\rho_r^n}(x_r^{v_r}+\zeta_r)^{-\sigma_r^n}]^{s_n}\,\mathrm{d}s_1\cdots\mathrm{d}s_n\}$$

Using the formulas (1.14) and (1.15), we obtain.

$$\Bigg[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma'_1s_1+\cdots+\sigma_1^ns_n}\zeta_1^{\sigma'_rs_1+\cdots+\sigma_r^ns_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}\frac{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^is_i+N_1)}{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^is_i)}$$

$$\cdots\frac{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^is_i+N_r)}{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^is_i)}\frac{\Gamma(h_1+\sum_{i=1}^n\sigma_1^is_i+v_1N_1+1)}{\Gamma(h_1+\sum_{i=1}^n\sigma_1^is_i+v_1N_1-\mu_1)}\cdots$$

$$\frac{\Gamma(h_r+\sum_{i=1}^n\sigma_r^is_i+v_rN_r+1)}{\Gamma(h_r+\sum_{i=1}^n\sigma_r^is_i+v_rN_r-\mu_r)}\;x_1^{h_1+\sum_{i=1}^n\rho_1^is_i+v_1N_1-\mu_1}\cdots x_1^{h_1+\sum_{i=1}^n\rho_r^is_i+v_rN_r-\mu_r}\mathrm{d}s_1\cdots\mathrm{d}s_n]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

Formula 2

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\,[x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n}\end{array}\right)\Bigg]$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-x_1^{m_1v_1}/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-x_r^{m_rv_r}\zeta_r)^{N_r}}{N_r!}$$

$$I_{U:p_r+3r,q_r+3r;W}^{V;0,n_r+3r;X}\left(\begin{array}{c|c} \mathrm{z}_1B_1 & \mathrm{A}\ ;\ [1\text{-}\mu_j-v_1N_j:\rho_j',\cdots,\rho_j^n)]_{1,r}, \\ \cdot\ \cdot & \cdots\cdots \\ \cdot\ \cdot\ \cdot & \cdots \\ \mathrm{z}_nB_n & \mathrm{B}\ ;[1+\beta_j-\mu_j-v_1N_j:\rho_j',\cdots,\rho_j^n]_{1,r}, \end{array}\right.$$

$$\left.\begin{array}{cc} [1\text{-}\mu_j-\eta_j+\beta_j-v_jN_j:\rho_j',\cdots,\rho_j^n]_{1,r}, & [1\text{+}\lambda_j-N_1:\sigma_j',\cdots,\sigma_j^n]_{1,r},\mathfrak{A};A' \\ \cdots\cdots & \cdots\cdots \\ [1-\mu_j-\eta_j+\alpha_j-v_jN_j:\rho_j',\cdots,\rho_j^n]_{1,r}, & [1+\lambda_j:\sigma_j',\cdots,\sigma_j^n]_{1,r},\mathfrak{B};B' \end{array}\right) \tag{2.2}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\quad i=1,\cdots,n$

Provided that

a) For $0\leqslant\alpha_i<1,\beta_i,\eta_i,x_i\in\mathbb{R};m_i\in\mathbb{N},\mu_i>max(0,\beta_i-\eta_i)$,

b) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i)>0,i=1,\cdots,n$

c) $max[|arg(x_1^{m_1v_1}/\zeta_1)|,\cdots,|arg(x_r^{m_rv_r}/\zeta_r)|]<\pi$

d) $Re[\mu_1-1+\sum_{i=1}^{n}\rho_1^i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0,\cdots,Re[\mu_r-1+\sum_{i=1}^{n}\rho_r^i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

Proof of (2.2)

Let $M==\dfrac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)$

Where $\psi(s_1,\cdots,s_r),\theta_k(s_k)$ are defined respectively by (1.2) and (1.3)

Therefore

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}[Mx_1^{(mu_1+\sum_{i=1}^{n}\rho_1^in_i)m_1}(x^{m_1v_1}+\zeta_1)^{\lambda_1-(\sum_{i=1}^{n}\rho_1^in_i)}$$

$$[x_1^{(mu_r+\sum_{i=1}^{n}\rho_r^i n_i)m_r}(x^{m_r v_r}+\zeta_r)^{\lambda_r-(\sum_{i=1}^{n}\rho_r^i n_i)}]\mathrm{d}s_1\cdots\mathrm{d}s_n]$$

Using the formulas (1.15) and (1.18), we obtain.

$$\Big[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1' s_1+\cdots+\sigma_1^n s_n}\zeta_1^{\sigma_r' s_1+\cdots+\sigma_r^n s_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}\frac{\Gamma(-\lambda_1+\sum_{i=1}^{n}\sigma_1^i s_i+N_1)}{\Gamma(-\lambda_1+\sum_{i=1}^{n}\sigma_1^i s_i)}$$

$$\cdots\frac{\Gamma(-\lambda_r+\sum_{i=1}^{n}\sigma_r^i s_i+N_r)}{\Gamma(-\lambda_r+\sum_{i=1}^{n}\sigma_r^i s_i)}\frac{\Gamma(\mu_1+\sum_{i=1}^{n}\rho_1^i s_i+v_1 N_1)}{\Gamma(\mu_1+\sum_{i=1}^{n}\sigma_1^i s_i+v_1 N_1-\beta_1)}\cdots$$

$$\cdots\frac{\Gamma(\mu_r+\sum_{i=1}^{n}\rho_r^i s_i+v_r N_r)}{\Gamma(\mu_r+\sum_{i=1}^{n}\sigma_r^i s_i+v_1 N_r-\beta_r)}\frac{\Gamma(\mu_1+\sum_{i=1}^{n}\rho_1^i s_i+v_1 N_1+\eta_1-\beta_1)}{\Gamma(\mu_1+\sum_{i=1}^{n}\sigma_1^i s_i+v_1 N_1+\eta_1-\alpha_1)}$$

$$\frac{\Gamma(\mu_r+\sum_{i=1}^{n}\rho_r^i s_i+v_r N_r+\eta_r-\beta_r)}{\Gamma(\mu_r+\sum_{i=1}^{n}\sigma_r^i s_i+v_r N_r+\eta_r-\alpha_r)}\quad x_1^{m_1(\mu_1-1+\sum_{i=1}^{n}\rho_1^i s_i+v_1 N_1-\beta_1+\sum_{i=1}^{s}k_1^i R_i)}\cdots$$

$$x_r^{m_r(\mu_r-1+\sum_{i=1}^{n}\rho_1^i s_i+v_r N_r-\beta_r+\sum_{i=1}^{s}k_r^i R_i)}\mathrm{d}s_1\cdots\mathrm{d}s_n]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

Formula 3

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1)m_1}(x_1^{m_1 v_1}+\zeta_1)^{\lambda_1}\cdots x^{m_r(\mu_r-1)}(x_r^{m_r v_r}+\zeta_r)^{\lambda_r}$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1 v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_r v_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1 v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_r v_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right)$$

$$S_L^{F_1,\cdots,F_s}[w_1 x_1^{k_1' m_1}\cdots x_r^{k_r' m_r},\cdots,w_s x_1^{k_1^s m_1}\cdots x_r^{k_r^s m_r}]\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1/\zeta_r)^{N_r}}{N_r!}\sum_{R_1,\cdots,R_s=0}^{h_1 R_1+\cdots h_s R_s\leqslant L}$$

$$(-L)_{h_1 R_1+\cdots+h_s R_s}B(L;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}x_1^{m_1(v_1 N_r+\sum_{i=1}^{s}k_1^i R_i)}\cdots x_r^{m_r(v_r N_r+\sum_{i=1}^{s}k_r^i R_i)}$$

$$I_{U:p_r+3r,q_r+3r;W}^{V;0,n_r+3r;X}\left(\begin{array}{c|c} z_1B_1 & \mathrm{A}\ ;\ [1+\lambda_j-N_1:\sigma_j',\cdots,\sigma_j^n]_{1,r}, \\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \\ z_nB_n & \mathrm{B}\ ;\ [1+\lambda_1:\sigma_1',\cdots,\sigma_1^n]_{1,r}, \end{array}\right.$$

$$\begin{array}{c} [1\text{-}\mu_j-\eta_j+\beta_j-v_jN_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r}, \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ [1-\mu_j-\eta_j+\alpha_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r}, \end{array}$$

$$\left.\begin{array}{c} [1\text{-}\mu_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{A};A' \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ [1-\mu_j+\beta_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{B};B' \end{array}\right) \tag{2.3}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\ i=1,\cdots,n$

Provided that

a) For $0\leqslant\alpha_i<1,\beta_i,\eta_i,x_i\in\mathbb{R};m_i\in\mathbb{N},\mu_i>max(0,\beta_i-\eta_i)$,

b) $min(v_1,\cdots,v_r;\rho_1^i,\cdots,\rho_r^i;\sigma_1^i,\cdots,\sigma_r^i)>0,i=1,\cdots,n$

c) $max[|arg(x_1^{m_1v_1}/\zeta_1)|,\cdots,|arg(x_r^{m_rv_r}/\zeta_r)|]<\pi$

d) $Re[\mu_1-1+\sum_{i=1}^n\rho_1^i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0,\cdots,Re[\mu_r-1+\sum_{i=1}^n\rho_r^i\min_{1\leqslant j\leqslant m_i}\dfrac{d_j^{(i)}}{\delta_j^{(i)}}]>0$

Proof of (2.3)

Let $M=\ =\dfrac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)$

Where $\psi(s_1,\cdots,s_r),\theta_k(s_k)$ are defined respectively by (1.2) and (1.3)

Use the formula (1.15), the left hand side of (2.3) is given by

$$\left[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1's_1+\cdots+\sigma_1^ns_n}\zeta_1^{\sigma_r's_1+\cdots+\sigma_r^ns_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}(-\lambda_1+\sum_{i=0}^n\sigma_1^is_i,N_1)\cdots\right.$$

$$(-\lambda_r+\sum_{i=0}^n\sigma_r^is_i,N_r)\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}$$

$$\left.D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1+\sum_{i=1}^n\rho_1^is_i+\sum_{i=1}^s k_1^iR_i)m_1}\cdots x_r^{(\mu_r-1+\sum_{i=1}^n\rho_r^is_i+\sum_{i=1}^s k_r^iR_i)m_r}\}\mathrm{d}s_1\cdots\mathrm{d}s_n\right]$$

Use the formula (1.18), we get

$$\Big[M\frac{z_1^{s_1}\cdots z_n^{s_n}\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}}{\zeta_r^{\sigma_1' s_1+\cdots+\sigma_1^n s_n}\zeta_1^{\sigma_r' s_1+\cdots+\sigma_r^n s_n}}\sum_{N_1,\cdots,N_r=0}^{\infty}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\frac{(-)^{N_1+\cdots+N_n}}{N_1!\cdots N_r!\zeta_1^{N_1}\cdots\zeta_r^{N_r}}$$

$$(-\lambda_1+\sum_{i=0}^{n}\sigma_1^i s_i,N_1)\cdots(-\lambda_r+\sum_{i=0}^{n}\sigma_r^i s_i,N_r)(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}$$

$$\frac{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^i s_i+N_1)}{\Gamma(-\lambda_1+\sum_{i=1}^n\sigma_1^i s_i)}\cdots\frac{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^i s_i+N_r)}{\Gamma(-\lambda_r+\sum_{i=1}^n\sigma_r^i s_i)}$$

$$\frac{\Gamma(\mu_1+\sum_{i=1}^n\rho_1^i s_i+v_1N_1+\sum_{i=1}^s k_i'R_i)}{\Gamma(\mu_1+\sum_{i=1}^n\sigma_1^i s_i+v_1N_1-\beta_1+\sum_{i=1}^s k_i'R_i)}\cdots\frac{\Gamma(\mu_r+\sum_{i=1}^n\rho_r^i s_i+v_1N_r+\sum_{i=1}^s k_i'R_i)}{\Gamma(h_r+\sum_{i=1}^n\sigma_1^i s_i+v_rN_r-\beta_r+\sum_{i=1}^s k_i'R_i)}$$

$$\cdots\frac{\Gamma(\mu_1+\sum_{i=1}^n\rho_1^i s_i+v_1N_1+\sum_{i=1}^s k_1^sR_i+\eta_1-\beta_1)}{\Gamma(\mu_1+\sum_{i=1}^n\sigma_1^i s_i+v_1N_1+\eta_1-\alpha_1+\sum_{i=1}^s k_1^sR_i)}\cdots$$

$$\frac{\Gamma(\mu_r+\sum_{i=1}^n\rho_r^i s_i+v_rN_r+\sum_{i=1}^s k_i'R_i)}{\Gamma(\mu_r+\sum_{i=1}^n\sigma_r^i s_i+v_1N_1-\beta_r+\sum_{i=1}^s k_i'R_i)}x_1^{m_1(\mu_1-1+\sum_{i=1}^n\rho_1^i s_i+v_1N_1-\beta_1+\sum_{i=1}^s k_1^iR_i)}\cdots$$

$$x_r^{m_r(\mu_r-1+\sum_{i=1}^n\rho_1^i s_i+v_rN_r-\beta_r+\sum_{i=1}^s k_r^iR_i)}\mathrm{d}s_1\cdots\mathrm{d}s_n\Big]$$

Finally, interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

3. Particular case

a) If $B(L;R_1,\cdots,R_s)=\dfrac{\prod_{j=1}^{A}(a_j)_{R_1\theta_j'+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(v)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j'+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(v)})_{R_s\delta_j^{(s)}}}$ (3.1)

then the general class of multivariable polynomial $S_L^{R_1,\cdots,R_s}[x_1,\cdots,x_s]$ reduces to generalized Lauricella function defined by Srivastava et al [8].

$$F_{C:D';\cdots;D^{(s)}}^{1+A:B';\cdots;B^{(s)}}\left(\begin{array}{c|c}\mathrm{w}_1x_1^{k_1'm_1}\cdots x_r^{k_r'm_r} & [\text{(-L)}: \mathrm{R}_1,\cdots,R_s],[(a):\theta',\cdots,\theta^{(s)}];[(b'):\phi'];\cdots;[(b^{(s)}):\phi^{(s)}]\\ \cdots & \\ \mathrm{w}_sx_s^{k_1^sm_1}\cdots x_r^{k_r^sm_r} & [\text{(c)}:\ \psi',\cdots,\psi^{(s)}];[(d'):\delta'];\cdots;[(b)^{(s)}:\delta^{(s)}]\end{array}\right)$$

The formula (2.3) write

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right)$$

$$F_{C:D';\cdots;D^{(s)}}^{1+A:B';\cdots;B^{(s)}}\left(\begin{array}{c} \mathrm{w}_1 x_1^{k_1'm_1}\cdots x_r^{k_r'm_r} \\ \cdots \\ \mathrm{w}_s x_s^{k_1^sm_1}\cdots x_r^{k_r^sm_r} \end{array}\left|\begin{array}{c} [\text{(-L)}: \mathrm{R}_1,\cdots,R_s],[(a):\theta',\cdots,\theta^{(s)}];[(b'):\phi'];\cdots;[(b^{(s)}):\phi^{(s)}] \\ [\text{(c)}: \psi',\cdots,\psi^{(s)}];[(d'):\delta'];\cdots;[(b)^{(s)}:\delta^{(s)}] \end{array}\right.\right)\Bigg\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1\zeta_r)^{N_r}}{N_r!}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$(-L)_{h_1R_1+\cdots+h_sR_s}B(L;R_1,\cdots,R_s)\frac{w_1^{R_1}\cdots w_s^{R_s}}{R_1!\cdots R_s!}x_1^{m_1(v_1N_r+\sum_{i=1}^s k_1^iR_i)}\cdots x_r^{m_r(v_rN_r+\sum_{i=1}^s k_r^iR_i)}$$

$$I_{U:p_r+3r,q_r+3r;W}^{V;0,n_r+3r;X}\left(\begin{array}{c} \mathrm{z}_1B_1 \\ \cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \mathrm{z}_nB_n \end{array}\left|\begin{array}{c} \mathrm{A}\ ;\ [1+\lambda_j-N_1:\sigma_j',\cdots,\sigma_j^n]_{1,r}, \\ \cdots\cdots \\ \cdots \\ \mathrm{B}\ ;\ [1+\lambda_1:\sigma_1',\cdots,\sigma_1^n]_{1,r}, \end{array}\right.\right.$$

$$\begin{array}{c}[1\text{-}\mu_j-\eta_j+\beta_j-v_jN_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r}, \\ \cdots\cdots \\ [1-\mu_j-\eta_j+\alpha_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\end{array}$$

$$\left.\begin{array}{c}[1\text{-}\mu_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{A};A' \\ \cdots\cdots \\ [1-\mu_j+\beta_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{B};B'\end{array}\right) \quad (3.2)$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}$ $i=1,\cdots,n$ and $B(E;R_1,\cdots,R_s)$ is defined by (3.1)

which holds true under the same conditions as needed in (2.3)

b) If $x_2=\cdots,x_s=0$, then $S_L^{R_1,\cdots,R_s}[x_1,\cdots,x_s]$ degenere to $S_N^M(x)$ defined by Srivastava [7] and we have

$$D_{0,x_1,m_1}^{\alpha_1,\beta_1,\eta_1}\cdots D_{0,x_r,m_r}^{\alpha_r,\beta_r,\eta_r}\{x_1^{(\mu_1-1)m_1}(x_1^{m_1v_1}+\zeta_1)^{\lambda_1}\cdots x_r^{m_r(\mu_r-1)}(x_r^{m_rv_r}+\zeta_r)^{\lambda_r}$$

$$I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} \mathrm{z}_1 x_1^{m_1\rho_1'}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1'}\cdots x_r^{m_r\rho_r'}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r'} \\ \cdots \\ \cdots \\ \mathrm{z}_n x_1^{m_1\rho_1^n}(x_1^{m_1v_1}+\zeta_1)^{-\sigma_1^n}\cdots x_r^{m_r\rho_r^n}(x_r^{m_rv_r}+\zeta_r)^{-\sigma_r^n} \end{array}\right) S_N^M[wx_1^{k_1'm_1}\cdots x_r^{k_r'm_r}]\Bigg\}$$

$$=\zeta_1^{\lambda_1}\cdots\zeta_r^{\lambda_r}x_1^{(\mu_1-\beta_1-1)m_1}\cdots x_r^{(\mu_r-\beta_r-1)m_r}\sum_{N_1,\cdots,N_r=0}^{\infty}\sum_{K=0}^{[N/M]}\frac{(-1/\zeta_1)^{N_1}}{N_1!}\cdots\frac{(-1\zeta_r)^{N_r}}{N_r!}$$

$$\frac{(-N)_{MK}}{K!}A_{N,K}\frac{w^K}{K!}\;x_1^{m_1(v_1N_r+k_1'K)}\cdots x_r^{m_r(v_rN_r+k_r'K)}$$

$$I_{U:p_r+3r,q_r+3r;W}^{V;0,n_r+3r;X}\left(\begin{array}{c|c} \mathrm{z}_1B_1 & \mathrm{A}\;;\;[1+\lambda_j-N_1:\sigma_j',\cdots,\sigma_j^n]_{1,r}, \\ \cdot\;\cdot & \cdots\cdots \\ \cdot\;\cdot\;\cdot & \cdots \\ \mathrm{z}_nB_n & \mathrm{B}\;;\;[1+\lambda_1:\sigma_1',\cdots,\sigma_1^n]_{1,r}, \end{array}\right.$$

$$\begin{array}{c} [1\text{-}\mu_j-\eta_j+\beta_j-v_jN_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r}, \\ \cdots\cdots \\ [1-\mu_j-\eta_j+\alpha_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r}, \end{array}$$

$$\left.\begin{array}{c} [1\text{-}\mu_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{A};A' \\ \cdots\cdots \\ [1-\mu_j+\beta_j-v_1N_j-\sum_{i=1}^s k_j^iR_i:\rho_j',\cdots,\rho_j^n]_{1,r},\mathfrak{B};B' \end{array}\right) \tag{3.3}$$

Where $B_i=\dfrac{x_1^{m_1\rho_1^i}\cdots x_r^{m_r\rho_r^i}}{\zeta_1^{\sigma_1^i}\cdots\zeta_r^{\sigma_r^i}}\quad i=1,\cdots,n$

which holds true under the same conditions as needed in (2.3)

Remark : If $U=V=A=B=0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [3], for more details, see Chandel et al [2].

4.Conclusion

The I-function of several variables defined by Prasad presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions such as, multivariable H-function defined by Srivastava et al [10].

References

[1] Bhatt S. and Raina R.K. A new class of analytic function involving certain fractional derivatives operator. Acta.

Math. Univ. Comenianee, vol 68,1, 1999, p.179-193.

[2] Chandel R.C.S and Kumar . On some generalized results of fractional derivatives. Jnanabha vol36, 2006, p.105-112

[3] Chandel R.C.S and Gupta V. On some generalized fractional derivatives formulas. Jnanabha Vol41, 2011, p.109-130

[4] Oldham K.B.and Spanier J. The fractional calculus. Academic Press , New York 1974.

[5] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[6] Samko S.G. Kilbas A.A. And Marichev O.I. Fractional integrals and derivatives, Theory and Applications, Gordon and Beach, New-York, 1993

[7] Srivastava H.M., A contour integral involving Fox's H-function. Indian J.Math. 14(1972), page1-6.

[8] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser. A72 = Indag. Math, 31, (1969), p 449-457.

[9] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[10] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE