Fractional derivatives involving multivariable I-function II

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ABSTRACT
In this document, we derive two key formulas for the fractional derivatives of the multivariable I-function defined by Prasad [2] which is defined by a multiple contour integral of Mellin-Barnes type and general a class of polynomial of several variables. Several particular cases are given

Keywords:Fractional derivative, multivariable I-function, multivariable H-function, general class of polynomials, Srivastava-Daoust function, generalized Leibnitz rule.

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1.Introduction and preliminaries.

Srivastava et al [3] have obtained a number of key formulas for the fractional derivatives of the multivariable H-function defined by Srivastava et al [6].The main of this paper is obtained two formulas for the fractional derivatives of product of the multivariable I-function defined by Prasad [2] and a class of polynomial of several variables.
The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{(1)}_j,\alpha^{(1)}_j)_{1,p^{(1)}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{(1)}_j,\beta^{(1)}_j)_{1,q^{(1)}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.3}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,),max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,),min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n_k$$

We will use these following notations in this paper :

$$U_r=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V_r=0,n_2;0,n_3;\cdots;0,n_{r-1} \tag{1.4}$$

$$W_r=(p^{(1)},q^{(1)});\cdots;(p^{(r)},q^{(r)});X_r=(m^{(1)},n^{(1)});\cdots;(m^{(r)},n^{(r)}) \tag{1.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A}=(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)}):\mathfrak{B}=(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)}) \tag{1.8}$$

$$A_1=(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};B_1=(b_k^{(1)},\beta_k^{(1)})_{1,p^{(1)}};\cdots;(b_k^{(r)},\beta_k^{(r)})_{1,p^{(r)}} \tag{1.9}$$

The multivariable I-function of r-variables write :

$$I(z_1,\cdots,z_r)=I_{U_r:p_r,q_r;W_r}^{V_r;0,n_r;X_r}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\;;\mathfrak{A};\mathrm{A}_1\\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\;\mathfrak{B};\;\mathrm{B}_1\end{array}\right) \tag{1.10}$$

$$I(z'_1,z'_2,...z'_s)=I_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}^{0,n'_2;0,n'_3;\cdots;0,n'_r:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}\left(\begin{array}{c|c} \mathrm{z'}_1 & (\mathrm{a'}_{2j};\alpha_{2j}'^{(1)},\alpha_{2j}'^{(2)})_{1,p'_2};\cdots;\\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z'}_s & (\mathrm{b'}_{2j};\beta_{2j}'^{(1)},\beta_{2j}'^{(2)})_{1,q'_2};\cdots;\end{array}\right.$$

$$\left.\begin{matrix}(\text{a'}_{sj};\alpha^{(1)\prime}{}_{sj},\cdots,\alpha'_{sj}{}^{(s)})_{1,p'_s}:(a_j'^{(1)},\alpha_j'^{(1)})_{1,p'^{(1)}};\cdots;(a'_j{}^{(s)},\alpha'_j{}^{(s)})_{1,p'^{(s)}}\\ \\ (\text{b'}_{sj};\beta'^{(1)}_{sj},\cdots,\beta'_{sj}{}^{(s)})_{1,q'_s}:(b_j'^{(1)},\beta_j'^{(1)})_{1,q'^{(1)}};\cdots;(b'_j{}^{(s)},\beta'_j{}^{(s)})_{1,q'^{(s)}}\end{matrix}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz'_i| < \frac{1}{2}\Omega'_i\pi$,

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}\alpha'_k{}^{(i)}-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'_k{}^{(i)}+\sum_{k=1}^{m'^{(i)}}\beta'_k{}^{(i)}-\sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'_k{}^{(i)}+\left(\sum_{k=1}^{n'_2}\alpha'_{2k}{}^{(i)}-\sum_{k=n_2+1}^{p'_2}\alpha'_{2k}{}^{(i)}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n'_s}\alpha'_{sk}{}^{(i)}-\sum_{k=n'_s+1}^{p'_s}\alpha'_{sk}{}^{(i)}\right)-\left(\sum_{k=1}^{q'_2}\beta'_{2k}{}^{(i)}+\sum_{k=1}^{q'_3}\beta'_{3k}{}^{(i)}+\cdots+\sum_{k=1}^{q'_s}\beta'_{sk}{}^{(i)}\right) \tag{1.13}$$

where $i=1,\cdots,s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s)=0(\,|z'_1|^{\alpha'_1},\dots,|z'_s|^{\alpha'_s}\,),max(\,|z'_1|,\cdots,|z'_s|\,)\to 0$$

$$I(z'_1,\cdots,z'_s)\;=0(\,|z'_1|^{\beta'_1}\;,\cdots,|z'_s|^{\beta'_s}\,),min(\,|z'_1|,\cdots,|z'_s|\,)\to\infty$$

where $k=1,\cdots,z:\alpha''_k=min[Re(b_j'^{(k)}/\beta_j'^{(k)})],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a_j'^{(k)}-1)/\alpha_j'^{(k)})],j=1,\cdots,n'_k$$

We will use these following notations in this paper :

$$U_s=p'_2,q'_2;p'_3,q'_3;\cdots;p'_{s-1},q'_{s-1};V_s=0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{1.14}$$

$$W_s=(p'^{(1)},q'^{(1)});\cdots;(p'^{(s)},q'^{(s)});X_s=(m'^{(1)},n'^{(1)});\cdots;(m'^{(s)},n'^{(s)}) \tag{1.15}$$

$$A'=(a'_{2k};\alpha'^{(1)}_{2k},\alpha'^{(2)}_{2k});\cdots;(a'_{(s-1)k};\alpha'^{(1)}_{(s-1)k},\alpha'^{(2)}_{(s-1)k},\cdots,\alpha'^{(s-1)}_{(s-1)k}) \tag{1.16}$$

$$B' = (b'_{2k};\beta'^{(1)}_{2k},\beta'^{(2)}_{2k});\cdots;(b'_{(s-1)k};\beta'^{(1)}_{(s-1)k},\beta'^{(2)}_{(s-1)k},\cdots,\beta'^{(s-1)}_{(s-1)k}) \tag{1.17}$$

$$\mathfrak{A}' = (a'_{sk};\alpha'^{(1)}_{sk},\alpha'^{(2)}_{sk},\cdots,\alpha'^{(s)}_{sk}) : \mathfrak{B}' = (b'_{sk};\beta'^{(1)}_{sk},\beta'^{(2)}_{k},\cdots,\beta'^{(s)}_{sk}) \tag{1.18}$$

$$A'_1 = (a'^{(1)}_k,\alpha^{(1)}_k)_{1,p^{(1)}};\cdots;(a'^{(s)}_k,\alpha'^{(s)}_k)_{1,p'^{(s)}};B'_1 = (b'^{(1)}_k,\beta'^{(1)}_k)_{1,p'^{(1)}};\cdots;(b'^{(s)}_k,\beta'^{(s)}_k)_{1,p'^{(s)}} \tag{1.19}$$

The multivariable I-function write :

$$I(z'_1,\cdots,z'_s) = I^{V_s;0,n'_s;X_s}_{U_s:p'_s,q'_s;W_s}\left(\begin{array}{c|c} z'_1 & A';\mathfrak{A}';A'_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & B';\mathfrak{B}';B'_1 \end{array}\right) \tag{1.20}$$

The Riemann-Liouville fractional derivative (or integral) of order μ is defined as follows [1 ,page 49]

$$D^{\mu}_x f(x) = \begin{cases} \frac{1}{\Gamma(-u)} \int_0^x (x-t)^{-\mu-1} f(t)\mathrm{d}t, Re(\mu) < 0 \\ \quad\cdots \\ \frac{d^m}{dx^m}\left[D^{\mu-m}_x\{f(x)\}\right], 0 \leqslant m, m \in \mathbb{N} \end{cases} \tag{1.21}$$

We have the following fractional derivative formula [1,page 67, eq. (4.4.4)] is also required :

$$D^{\mu}_x\left(x^{\lambda}\right) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)}x^{\lambda-\mu}, Re(\lambda) > -1 \tag{1.22}$$

The generalized Leibnitz formula for fractional calculus is required in the following form [1,page 76, eq.(5.5.2)]

$$D^{\mu}_x\left[f(x)g(x)\right] = \sum_{l=0}^{\infty}\binom{\mu}{l}D^{\mu-l}_x\left[f(x)\right]D^{l}_x\left[g(x)\right] \tag{1.23}$$

μ is a real or complex arbitrary number.

We have the binomial formula : $(x+a)^{\lambda} = a^{\lambda}\sum_{m=0}^{\infty}\binom{\lambda}{m}\left(\frac{x}{a}\right)^m ; \left|\frac{x}{a}\right| < 1$ (1.24)

Srivastava and Garg [4] introduced and defined a general class of multivariable polynomials as follows

$$S^{h_1,\cdots,h_u}_L[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.25}$$

2. Main results

Theorem 1

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda}S_L^{h_1,\cdots,h_u}[w_1x^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1},\cdots,w_ux^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u}]\right.$$

$$\left.I\left[z_1x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_rx^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]\right\}=\zeta^{\lambda\upsilon}x^{k-\mu}\sum_{m=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\zeta^{\upsilon\sum_{i=1}^{u}b_ih_i}x^{\sum_{i=1}^{u}\rho_ih_i}$$

$$(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}I_{U_r:p_r+2,q_r+2;W_r}^{V_r;0,n_r+2;X_r}\left(\begin{array}{c}\mathrm{z}_1\zeta^{\upsilon\sigma_1}x^{a_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\zeta^{\upsilon\sigma_r}x^{a_r}\end{array}\right|$$

$$\left.\begin{array}{l}\mathrm{A}\ ;(\text{-}\lambda-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r),\qquad(\text{-k-m}\upsilon-\sum_{i=1}^{u}R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1\\ \mathrm{B};\ (1\text{-}\lambda-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r),(\mu-m\upsilon-k-\sum_{i=1}^{u}R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1\end{array}\right) \tag{2.1}$$

Provided that :

a) $min\{\upsilon,b_i,\rho_i,\sigma_j,a_j,\}>0,i=1,\cdots,u;j=1,\cdots,r;\left|arg\left(\frac{x^{\upsilon}}{\zeta^{\upsilon}}\right)\right|<\pi$

b) $Re\left[k+\sum_{i=1}^{r}a_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>-1$

c) $|arg\,(z_i)|<\frac{1}{2}\Omega_i\pi\ \ (i=1,\cdots,r)$ where Ω_i is defined by (1.3)

Theorem 2

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda}S_L^{h_1,\cdots,h_u}[w_1x^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1},\cdots,w_ux^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u}]\right.$$

$$\left.I\left[z_1x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_rx^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]I^*\left[y_1x^{\mu_1},\cdots,y_sx^{\mu_s}\right]\right\}$$

$$=\zeta^{\lambda\upsilon}x^{k-\mu}\sum_{l,m=0}^{\infty}\binom{\mu}{l}\frac{(-x^{\upsilon}/\zeta^{\upsilon})^m}{m!}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}$$

$$\zeta^{\upsilon\sum_{i=1}^{u}b_ih_i}x^{\sum_{i=1}^{u}\rho_ih_i}I_{U_r:p_r+2,q_r+2;W_r}^{V_r;0,n_r+2;X_r}\left(\begin{array}{c|c} \mathrm{z}_1\zeta^{\upsilon\sigma_1}x^{a_1} & \mathrm{A};(\text{-}\lambda-m-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r), \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r\zeta^{\upsilon\sigma_r}x^{a_r} & \mathrm{B};(1\text{-}\lambda-m-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\left.\begin{array}{c} (\text{-m}\upsilon-\sum_{i=1}^{u}R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1 \\ \\ (\mu-m\upsilon-\sum_{i=1}^{u}R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1 \end{array}\right) I_{U_s:p_s'+1,q_s'+1;W_s}^{V_s;0,n_s'+1;X_s}\left(\begin{array}{c|c} \mathrm{y}_1x^{\mu_1} & \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{y}_sx^{\mu_s} & \end{array}\right.$$

$$\left.\begin{array}{c} \mathrm{A'};(\text{-k};\mu_1,\cdots,\mu_s),\mathfrak{A}';A_1' \\ \\ \mathrm{B'};(\mu-k-l;\mu_1,\cdots,\mu_s),\mathfrak{B}';B_1' \end{array}\right) \tag{2.2}$$

where $I^*(.,\cdots,.)$ is the s-variables I-function (occuring on the right-hand side) without the additional parameters

$(-k;\mu_1,\cdots,\mu_s)$ and $(\mu-k-l;\mu_1,\cdots,\mu_s)$

Provided that :

a) $min\{\upsilon,b_i,\rho_i,\sigma_j,a_j,\mu_k\}>0, i=1,\cdots,u; j=1,\cdots,r; k=1,\cdots,s; \left|arg\left(\frac{x^\upsilon}{\zeta^\upsilon}\right)\right|<\pi$

b) $Re\left[k+\sum_{i=1}^{s}\mu_i\min_{1\leqslant j\leqslant m_i'}\frac{b_j'^{(i)}}{\beta_j'^{(i)}}\right]>-1\ ;\sum_{i=1}^{r}a_i\min_{1\leqslant j\leqslant m_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}>-1;$

c) $|arg\,(z_i)|<\frac{1}{2}\Omega_i\pi\ \ (i=1,\cdots,r)$and $|arg\,(w_i)|<\frac{1}{2}\Omega_i'\pi\ \ (i=1,\cdots,s)$

where $\Omega_i^{(k)}$ and $\Omega_i'^{(k)}$ are defined respectively by (1.3) and (1.13)

Proof

To prove (2.1) we first replace a class of polynomial $S_L^{h_1,\cdots,h_u}[.]$ in serie with the help of (1.25) , the multivariable I-function occuring on the left-hand side by its Mellin-Barnes contour integral, collect the power of x and $(x^\upsilon+\zeta^\upsilon)$ and apply the binomial expansion with the help of (1.24). Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). We can apply the formula (1.22) and interpret the resulting Mellin-Barnes contour integral as an I-function of r-variables.

To prove (2.2), we make use of the generalized Leibniz rule for fractional derivatives with the help of (1.23).

with $f(x)=I^*\left[w_1x^{\mu_1},\cdots,w_sx^{\mu_s}\right]$ and

$$g(x) = (x^{\upsilon}+\zeta^{\upsilon})^{\lambda} S_L^{h_1,\cdots,h_u}[w_1 x^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1},\cdots,w_u x^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u}]$$

$$I\left[z_1 x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_r x^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]$$

and apply two special cases of the formula (2.1) when $\mu\rightarrow\mu-l;\lambda=0,\mu_i=a_i$ and $\sigma_i\rightarrow 0; i=1,\cdots,r$ or when $\mu=m\ (m\in\mathbb{N}_0)$, we arrive at (2.2), the s-variables I^*involved in (2.2), being identical.

3. Multivariable H-function

In this section, we use the multivariable H-function defined by Srivastava et al [6].

Let $U_r=V_r=A=B=0$, the multivariable I-function degenere in multivariable H-function

Lemme 1

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda} S_L^{h_1,\cdots,h_u}[w_1 x^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1},\cdots,w_u x^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u}]\right.$$

$$\left.H\left[z_1 x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_r x^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]\right\} = \zeta^{\lambda\upsilon}x^{k-\mu}\sum_{m=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\zeta^{\upsilon\sum_{i=1}^u b_ih_i}x^{\sum_{i=1}^u\rho_ih_i}$$

$$(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}H^{0,n_r+2;X_r}_{p_r+2,q_r+2;W_r}\left(\begin{array}{c|}z_1\zeta^{\upsilon\sigma_1}x^{a_1}\\ \cdot\\ \cdot\\ \cdot\\ z_r\zeta^{\upsilon\sigma_r}x^{a_r}\end{array}\right.$$

$$\left.\begin{array}{l}(\text{-}\lambda-\sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r),\quad (\text{-k-m}\upsilon-\sum_{i=1}^u R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1\\ (1\text{-}\lambda-\sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r),(\mu-m\upsilon-k-\sum_{i=1}^u R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1\end{array}\right) \tag{3.1}$$

under the same conditions and notations that (2.1) with $U_r=V_r=A=B=0$

Let $U_s=V_s=U_r=V_r=A=B=A'=B'=0$, the two multivariable I-functions degenere in two multivariable H-functions.

Lemme 2

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda} S_L^{h_1,\cdots,h_u}[w_1 x^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1},\cdots,w_u x^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u}]\right.$$

$$\left.H\left[z_1 x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_r x^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]H^*\left[y_1x^{\mu_1},\cdots,y_sx^{\mu_s}\right]\right\}$$

$$= \zeta^{\lambda v} x^{k-\mu} \sum_{l,m=0}^{\infty} \binom{\mu}{l} \frac{(-x^v/\zeta^v)^m}{m!} \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u \leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u} B(E;R_1,\cdots,R_u) \frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}$$

$$\zeta^{v\sum_{i=1}^u b_ih_i} x^{\sum_{i=1}^u \rho_ih_i} H^{0,n_r+2;X_r}_{p_r+2,q_r+2;W_r}\left(\begin{array}{c} \mathrm{z}_1\zeta^{v\sigma_1}x^{a_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\zeta^{v\sigma_r}x^{a_r} \end{array}\middle| \begin{array}{l} (\text{-}\lambda - m - \sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r), \\ \\ \\ (1\text{-}\lambda - m - \sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.$$

$$\left.\begin{array}{l} (\text{-m}v - \sum_{i=1}^u R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1 \\ \\ (\mu - mv - \sum_{i=1}^u R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1 \end{array}\right) H^{0,n_s'+1;X_s}_{p_s'+1,q_s'+1;W_s}\left(\begin{array}{c} \mathrm{y}_1x^{\mu_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_sx^{\mu_s} \end{array}\middle|\right.$$

$$\left.\begin{array}{l} \text{A' };(\text{-k};\mu_1,\cdots,\mu_s),\mathfrak{A}';A_1' \\ \\ \text{B'};(\mu - k - l;\mu_1,\cdots,\mu_s),\mathfrak{B}';B_1' \end{array}\right) \tag{3.2}$$

under the same conditions and notations that (2.2) with $U_s = V_s = U_r = V_r = A = B = A' = B' = 0$

4. Particular cases

If $$B(L;R_1,\cdots,R_u) = \frac{\prod_{j=1}^A (a_j)_{R_1\theta_j'+\cdots+R_u\theta_j^{(u)}} \prod_{j=1}^{B'} (b_j')_{R_1\phi_j'} \cdots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^C (c_j)_{m_1\psi_j'+\cdots+m_u\psi_j^{(u)}} \prod_{j=1}^{D'} (d_j')_{R_1\delta_j'} \cdots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u\delta_j^{(u)}}} \tag{4.1}$$

then the general class of multivariable polynomial $S_L^{R_1,\cdots,R_u}[x_1,\cdots,x_u]$ reduces to generalized Lauricella function defined by Srivastava et al [5].

$$F^{1+A:B';\cdots;B^{(u)}}_{C:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{w}_1 \\ \cdots \\ \cdots \\ \mathrm{w}_u \end{array}\middle| \begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)}),\phi^{(u)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)}),\delta^{(u)}] \end{array}\right) \tag{4.2}$$

and we have the formulas

Lemme 3

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda}F^{1+A:B';\cdots;B^{(u)}}_{C:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{w}_1 x_1^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{w}_u x_u^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u} \end{array}\right|\right.$$

$$\left.\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)}),\phi^{(u)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)}),\delta^{(u)}]\end{array}\right)$$

$$I\left[z_1x^{a_1}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_1},\cdots,z_rx^{a_r}(x^{\upsilon}+\zeta^{\upsilon})^{\sigma_r}\right]\Big\}=\zeta^{\lambda\upsilon}x^{k-\mu}\sum_{m=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\zeta^{\upsilon\sum_{i=1}^{u}b_ih_i}x^{\sum_{i=1}^{u}\rho_ih_i}$$

$$(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}I^{V_r;0,n_r+2;X_r}_{U_r:p_r+2,q_r+2;W_r}\left(\begin{array}{c}\mathrm{z}_1\zeta^{\upsilon\sigma_1}x^{a_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\zeta^{\upsilon\sigma_r}x^{a_r}\end{array}\right|$$

$$\left.\begin{array}{l}\mathrm{A}\ ;(\text{-}\lambda-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r),\qquad(\text{-k-m}\upsilon-\sum_{i=1}^{u}R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1 \\ \\ \mathrm{B};\ (1\text{-}\lambda-\sum_{i=1}^{u}R_ib_i;\sigma_1,\cdots,\sigma_r),(\mu-m\upsilon-k-\sum_{i=1}^{u}R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1\end{array}\right)\tag{4.3}$$

where $B(L;R_1,\cdots,R_u)=\dfrac{\prod_{j=1}^{A}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi'_j+\cdots+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ **(4.4)**

under the same condtions that (2.1)

Lemme 4

$$D_x^{\mu}\left\{x^k(x^{\upsilon}+\zeta^{\upsilon})^{\lambda}F^{1+A:B';\cdots;B^{(u)}}_{C:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{w}_1 x_1^{\rho_1}(x^{\upsilon}+\zeta^{\upsilon})^{b_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{w}_u x_u^{\rho_u}(x^{\upsilon}+\zeta^{\upsilon})^{b_u} \end{array}\right|\right.$$

$$\left. \begin{array}{c} [\text{(-L)};\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)}),\phi^{(u)}] \\ [\text{(c)};\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)}),\delta^{(u)}] \end{array} \right)$$

$$I\left[z_1x^{a_1}(x^\upsilon+\zeta^\upsilon)^{\sigma_1},\cdots,z_rx^{a_r}(x^\upsilon+\zeta^\upsilon)^{\sigma_r}\right]I^*\left[y_1x^{\mu_1},\cdots,y_sx^{\mu_s}\right]\Big\}$$

$$=\zeta^{\lambda\upsilon}x^{k-\mu}\sum_{l,m=0}^{\infty}\binom{\mu}{l}\frac{(-x^\upsilon/\zeta^\upsilon)^m}{m!}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{w_1^{R_1}\cdots w_u^{R_u}}{R_1!\cdots R_u!}$$

$$\zeta^{\upsilon\sum_{i=1}^u b_ih_i}x^{\sum_{i=1}^u \rho_ih_i}I_{U_r:p_r+2,q_r+2;W_r}^{V_r;0,n_r+2;X_r}\left(\begin{array}{c} \mathrm{z}_1\zeta^{\upsilon\sigma_1}x^{a_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\zeta^{\upsilon\sigma_r}x^{a_r} \end{array}\left|\begin{array}{c} \mathrm{A}\ ;(\text{-}\lambda-m-\sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r), \\ \\ \mathrm{B};\ (1\text{-}\lambda-m-\sum_{i=1}^u R_ib_i;\sigma_1,\cdots,\sigma_r), \end{array}\right.\right.$$

$$\left.\begin{array}{c} (\text{-m}\upsilon-\sum_{i=1}^u R_ib_i;a_1,\cdots,a_r),\mathfrak{A};A_1 \\ \\ (\mu-m\upsilon-\sum_{i=1}^u R_ib_i;;a_1,\cdots,a_r),\mathfrak{B};B_1 \end{array}\right)I_{U_s:p_s'+1,q_s'+1;W_s}^{V_s;0,n_s'+1;X_s}\left(\begin{array}{c} \mathrm{y}_1x^{\mu_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}_sx^{\mu_s} \end{array}\right|$$

$$\left.\begin{array}{c} \mathrm{A'}\ ;(\text{-k};\mu_1,\cdots,\mu_s),\mathfrak{A}';A_1' \\ \\ \mathrm{B'};(\mu-k-l;\mu_1,\cdots,\mu_s),\mathfrak{B}';B_1' \end{array}\right) \tag{4.2}$$

where $$B(L;R_1,\cdots,R_u)=\frac{\prod_{j=1}^{A}(a_j)_{R_1\theta_j'+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{C}(c_j)_{m_1\psi_j'+\cdots+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}} \quad \textbf{(4.6)}$$

Under the same conditions that (2.2)

3. Conclusion

In this paper we have evaluated two formulas concerning the fractional derivatives of product of the multivariable I-function defined by Prasad [2] and a class of polynomial of several variables. The two formulaes established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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