On multiple eulerian integral involving the multivariable I-function

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ABSTRACT
Recently, Raina and Srivastava [2] and Srivastava and Hussain [4] have provided closed-form expressions for a number of a general eulerian integrals involving multivariable H-functions. Motivated by these recent works, we aim at evaluating a general class of multiple eulerian integrals involving a multivariable I-function defined by Prasad [3] with general arguments. These integrals will serve as a key formula from which one can deduce numerous useful integrals.

Keywords :Multiple eulerian integral , Multivariable I-function .

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1.Introduction and preliminaries.

The object of this document is to evaluate a multiple Eulerian integrals involving the I-function of several variablesdefined by Prasad [3]. These function generalize the multivariable H-function study by Srivastava et al [5], itself is an a generalisation of G of multiple variables. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,...z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(t_1,\cdots,t_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}}\alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}}\beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)} + \left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right) +$$

$$+\left(\sum_{k=1}^{n_r}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}\alpha_{rk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_r}\beta_{rk}^{(i)}\right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(\,|z_1|^{\alpha'_1} \ldots |z_r|^{\alpha'_r}\,), max(\,|z_1| \ldots |z_r|\,) \rightarrow 0$

$\aleph(z_1, \cdots, z_s)\ = 0(\,|z_1|^{\beta'_1} \ldots |z_r|^{\beta'_r}\,), min(\,|z_1| \ldots |z_r|\,) \rightarrow \infty$

where $k = 1, \cdots, z : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{r-1}, q_{r-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.4}$$

$$W = (p', q'); \cdots; (p^{(r)}, q^{(r)}); X = (m', n'); \cdots; (m^{(r)}, n^{(r)}) \tag{1.5}$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \cdots; (a_{(r-1)k}), \alpha'{}_{(r-1)k}, \alpha''_{(r-1)k}, \cdots, \alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \cdots; (b_{(r-1)k}), \beta'{}_{(r-1)k}, \beta''_{(r-1)k}, \cdots, \beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A} = (a_{sk}; \alpha'{}_{rk}, \alpha''_{rk}, \cdots, \alpha_{rk}^{r}) : \mathfrak{B} = (b_{rk}; \beta'{}_{rk}, \beta''_{rk}, \cdots, \beta_{rk}^{r}) \tag{1.8}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}}; B' = (b'_k, \beta'_k)_{1,p'}; \cdots; (b_k^{(r)}, \beta_k^{(r)})_{1,p^{(r)}} \tag{1.9}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_r) = I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}\ ;\mathfrak{A};\mathrm{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & \mathrm{B};\ \mathfrak{B};\ \mathrm{B'} \end{array}\right) \tag{1.10}$$

2. Main integral

In this document, we shall establish the following Eulerian multiple integral of multivariable Aleph-function and we shall use the following notations (2.1) and (2.2).

Let $f(t_j) = (b_j - a_j) + \rho_j(t_j - a_j) + \sigma_j(b_j - t_j)$ (2.1)

$$g^{(i)}(t_j)=\frac{(t_j-a_j)^{\gamma_j^{(i)}}(b_j-t_j)^{\delta_j^{(i)}}\left\{f(t_j)\right\}^{1-\gamma_j^{(i)}-\delta_j^{(i)}}}{\beta_j(b_j-a_j)+(\beta_j\rho_j+\alpha_j-\beta_j)(t_j-a_j)+\beta_j\sigma_j(b_j-t_j)} \tag{2.2}$$

$j=1,\cdots,n;i=1,\cdots,r$

Formula 1 ([1] p.287)

$$\int_a^b\frac{(t-a)^{\alpha-1}(b-t)^{\beta-1}}{\{b-a+\lambda(t-a)+\mu(b-t)\}^{\alpha+\beta}}\mathrm{d}t=\frac{(1+\lambda)^{-\alpha}(1+\mu)^{-\beta}\Gamma(\alpha)\Gamma(\beta)}{(b-a)\Gamma(\alpha+\beta)} \tag{2.3}$$

with $t\in[a;b]\quad a\neq b, Re(\alpha)>0, Re(\beta)>0,\ \ \eta+\lambda(t-a)+\mu(b-t)\neq 0$

Formula 2

$$\int_{a_1}^{b_1}\cdots\int_{a_n}^{b_n}\prod_{i=1}^{n}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j+\mu_j+2}}I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c}\mathrm{z}_1\prod_{j=1}^n\left[g'(t_j)\right]^{v_j'}\\ \cdots\\ \mathrm{z}_r\prod_{j=1}^n\left[g^{(r)}(t_j)\right]^{v_j^{(r)}}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_n$$

$$=\prod_{j=1}^{n}\left\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\sum_{r_j=0}^{\infty}\frac{\left\{(\beta_j-\alpha_j)/\beta_j\right\}^{r_j}(1+\rho_j)^{-r_j}\right\}}{r_j!}$$

$$I_{U:p_r+3n,q_r+2n;W}^{V;0,n_r+3n;X}\left(\begin{array}{c}\mathrm{z}_1\prod_{j=1}^n\left\{\beta_j(1+\rho_j)^{\gamma_j'}(1+\sigma_j)^{\delta_j'}\right\}^{-v_j'}\\ .\\ .\\ .\\ \mathrm{z}_r\prod_{j=1}^n\left\{\beta_j(1+\rho_j)^{\gamma_j^{(n)}}(1+\sigma_j)^{\delta_j^{(n)}}\right\}^{-v_j^{(r)}}\end{array}\left|\begin{array}{l}\mathrm{A}:[1\text{-}\mathrm{r}_j;v_j',\cdots,v_j^{(r)}]_{1,n}\\ \cdots\\ \cdots\\ \mathrm{B}:[1\ ;\ \mathrm{v'}_j,\cdots,v_j^{(r)})]_{1,n}\end{array}\right.\right.$$

$$\left.\begin{array}{c}[\text{-}\lambda_j-r_j;\gamma_j'v_j',\cdots,\gamma_j^{(r)}v_j^{(r)})]_{1,n},(-\mu_j;\delta_j'v_j',\cdots,\delta_j^{(r)}v_j^{(r)})]_{1,n},\mathfrak{A},A'\\ \cdots\\ \cdots\\ [-\lambda_j-\mu_j-r_j-j;(\gamma_j'+\delta_j')v_j',\cdots,(\gamma_j^{(r)}+\delta_j^{(r)})v_j^{(r)}]_{1,n},\mathfrak{B};B'\end{array}\right) \tag{2.4}$$

Provided that

a) $v_j^{(i)}>0,\gamma_j^{(i)}>0,\delta_j^{(i)}>0,\beta_j\neq 0,b_j-a_j\neq 0,\rho_j\neq -1,\sigma_j\neq -1,j=1,\cdots,n,i=1,\cdots,r$

b) $(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j)\neq 0,t_j\in[a_j;b_j]$

c) $|argz_k|<\frac{1}{2}\Omega_i^{(k)}\pi$, where $\Omega_i^{(k)}$ is given in (1.5)

d) $|(\beta_j-\alpha_j)(t_j-a_j)|<|\beta_j(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j)|$

e)$Re[\lambda_j + \sum_{i=1}^{r} \gamma_j^{(i)} v_j^{(i)} \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] + 1 > 0\,;\, Re[\mu_j + \sum_{i=1}^{r} \delta_j^{(i)} v_j^{(i)} \min_{1 \leqslant j \leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}] + 1 > 0$

with $j = 1, \cdots, n, i = 1, \cdots, r$

f) the multiple serie on the R.H.S of (2.4) converges absolutly

Proof

Let $M = \frac{1}{(2\pi\omega)^n} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_n) \prod_{k=1}^{r} \zeta_k(s_k)$, we have

$$\int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} \prod_{i=1}^{n} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + 2}} \aleph \left(\begin{array}{c} z_1 \prod_{j=1}^{n} \left[g'(t_j)\right]^{v_j'} \\ \cdots \\ z_r \prod_{j=1}^{n} \left[g^{(r)}(t_j)\right]^{v_j^{(r)}} \end{array} \right) \mathrm{d}t_1 \cdots \mathrm{d}t_n$$

$$= \int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} \prod_{i=1}^{n} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + 2}} M \{ \prod_{i=1}^{r} [z_i^{s_i} \prod_{j=1}^{n} [g^{(i)}(t_j)]^{v_j^{(i)} s_i}] \mathrm{d}s_1 \cdots \mathrm{d}s_r \} \mathrm{d}t_1 \cdots \mathrm{d}t_n$$

Now, changing the order of multiple integral (wich is justified under the conditions of (2.4)), we find that

$$M \{ \prod_{i=1}^{r} [z_i^{s_i}] \int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} \prod_{i=1}^{n} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + 2}} \prod_{j=1}^{n} [g^{(i)}(t_j)]^{v_j^{(i)} s_i}] \mathrm{d}t_1 \cdots \mathrm{d}t_n \} \mathrm{d}s_1 \cdots \mathrm{d}s_r$$

$$= M \{ \prod_{i=1}^{r} z_i^{s_i} \prod_{j=1}^{n} [\int_{a_j}^{b_j} (t_j - a_j)^{\lambda_j + \sum_{i=1}^{r} \gamma_j^{(i)} v_j^{(i)} s_i} \frac{(b_j - t_j)^{\mu_j + \sum_{i=1}^{r} \delta_j^{(i)} v_j^{(i)} s_i}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + \sum_{i=1}^{r} (\gamma_j^{(i)} + \delta_j^{(i)}) v_j^{(i)} s_i + 2}}$$

$$\{1 - \frac{(\beta_j - \alpha_j)(t_j - a_j)}{\beta_j f(t_j)}\}^{-\sum_{i=1}^{r} v_j^{(i)} s_i} dt_j]\} \mathrm{d}s_1 \cdots \mathrm{d}s_r \qquad (2.5)$$

If $|(\beta_j - \alpha_j)(t_j - a_j)| < |\beta_j f(t_j)|$, then we can use binomial expansion and we thus find from (2.5)

$$\int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} \prod_{i=1}^{n} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + 2}} I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c} z_1 \prod_{j=1}^{n} \left[g'(t_j)\right]^{v_j'} \\ \cdots \\ z_r \prod_{j=1}^{n} \left[g^{(r)}(t_j)\right]^{v_j^{(r)}} \end{array} \right) \mathrm{d}t_1 \cdots \mathrm{d}t_n$$

$$= \prod_{j=1}^{n} \sum_{r_j=0}^{\infty} \frac{\{(\beta_j - \alpha_j)/\beta_j\}^{r_j}\}}{r_j!} M\{ \prod_{i=1}^{r} [z_i^{s_i} \beta_j^{-\sum_{i=1}^{r} v_j^{(i)}} \frac{\Gamma(r_j + \sum_{i=1}^{n} v_j^{(i)} s_i)}{\Gamma(\sum_{i=1}^{n} v_j^{(i)} s_i)}$$

$$\int_{a_j}^{b_j} \frac{(t_j - a_j)^{\lambda_j + r_j + \sum_{i=1}^{r} \gamma_j^{(i)} v_j^{(i)} s_i}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + \sum_{i=1}^{r} (\gamma_j^{(i)} + \delta_j^{(i)}) v_j^{(i)} s_i + 2}} (b_j - t_j)^{\mu_j + \sum_{i=1}^{r} \delta_j^{(i)} v_j^{(i)} s_i} \mathrm{d}t_j] \} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{2.6}$$

provided that the order of summation and integration can be inversed. Now evaluating the inner-integral in (2.6) with the help of equation (2.1). We finally obtain the formula (2.4)

3. Particular cases

a) For $n = 1$, the equation (2.4) reduces in the following formula after making slight ajustement in parameters.

$$\int_a^b \frac{(t-a)^{\lambda}(b-t)^{\mu}}{\left[f(t)\right]^{\lambda+\mu+2}} I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c} z_1 \left[g'(t)\right]^{v'} \\ \cdots \\ z_r \left[g^{(r)}(t)\right]^{v^{(r)}} \end{array} \right) \mathrm{d}t_1 \cdots \mathrm{d}t_n$$

$$= \{ (b-a)^{-1} (1+\rho)^{-\lambda-1} (1+\sigma)^{-\mu-1} \sum_{r'=0}^{\infty} \frac{\{(\beta-\alpha)/\beta\}^{r'} (1+\rho)^{-r'}\}}{r'!}$$

$$I_{U:p_r+3,q_r+2;W}^{V;0,n_r+3;X} \left(\begin{array}{c} z_1 \{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\}^{-v'} \\ . \\ . \\ . \\ z_r \{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\}^{-v^{(r)}} \end{array} \left| \begin{array}{c} \mathrm{A} ; (1\text{-}r' ; v'_1, \cdots, v_1^{(r)}), \\ \cdots \\ \cdots \\ \mathrm{B} ; (1 ; v'_1, \cdots, v_1^{(r)}), \end{array} \right. \right.$$

$$\left. \begin{array}{c} (-\lambda - r'; \gamma' v', \cdots, \gamma^{(r)} v^{(r)}), (-\mu; \delta' v', \cdots, \delta^{(r)} v^{(r)}), \mathfrak{A}; A' \\ \cdots \\ \cdots \\ (-\lambda - \mu - r' - 1; (\gamma' + \delta') v', \cdots, (\gamma^{(r)} + \delta^{(r)}) v^{(r)}), \mathfrak{B}; B' \end{array} \right) \tag{3.1}$$

which holds true under the same conditions from (2.4) with $n = 1$

b)Taking $\beta_j = \alpha_j, j = 1, \cdots, n$ in the formula (2.4), we get

$$\int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} \prod_{i=1}^{n} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j + \mu_j + 2}} I_{U:p_r,q_r;W}^{V;0,n_r;X} \left(\begin{array}{c} z_1 \prod_{j=1}^{n} \left[g'(t_j)\right]^{v'_j} \\ \cdots \\ z_r \prod_{j=1}^{n} \left[g^{(r)}(t_j)\right]^{v_j^{(r)}} \end{array} \right) \mathrm{d}t_1 \cdots \mathrm{d}t_n$$

$$=\prod_{j=1}^{n}\left\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}$$

$$I_{U:p_r+2n,q_r+n;W}^{V;0,n_r+2n;X}\left(\begin{array}{c|c} z_1\prod_{j=1}^{n}\left\{\beta_j(1+\rho_j)^{\gamma'_j}(1+\sigma_j)^{\delta'_j}\right\}^{-v'_j} & A\ ; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r\prod_{j=1}^{n}\left\{\beta_j(1+\rho_j)^{\gamma_j^{(n)}}(1+\sigma_j)^{\delta_j^{(n)}}\right\}^{-v_j^{(r)}} & B; \end{array}\right.$$

$$\left.\begin{array}{c} [-\lambda_j;\gamma'_jv'_j,\cdots,\gamma_j^{(r)}v_j^{(r)}]_{1,n},[-\mu_j;\delta'_jv'_j,\cdots,\delta_j^{(r)}v_j^{(r)}],\mathfrak{A};A' \\ \cdots \\ \cdots \\ [-\lambda_j-\mu_j-1;(\gamma'_j+\delta'_j)v'_j,\cdots,(\gamma_j^{(r)}+\delta_j^{(r)})v^{(r)}]_{1,n},\mathfrak{B};B' \end{array}\right) \tag{3.2}$$

which holds true under the same conditions from (2.4)

c) For $\sigma=\rho=0$ and $z_i=(b-t)^{\gamma+\delta-1)v^{(i)}}$, (3.1) becomes

$$\int_a^b\frac{(t-a)^{\lambda}(b-t)^{\mu}}{\left[(b-a)\right]^{\lambda+\mu+2}}I_{U:p_r,q_r;W}^{V;0,n_r;X}\left(\begin{array}{c} z_1\left\{(b-a)/\beta\right\}^{v'} \\ \cdots \\ z_r\left\{(b-a)/\beta\right\}^{v^{(r)}} \end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_n$$

$$=\left\{(b-a)^{-1}\sum_{r'=0}^{\infty}\frac{\left\{(\beta-\alpha)/\beta\right\}^{r'}\}}{r'!}I_{U:p_r+3,q_r+2;W}^{V;0,n_r+3;X}\left(\begin{array}{c|c} \left\{(\text{b-a})/\beta\right\}^{(\gamma+\delta-1)v'} & A\ ; \\ \cdot & \\ \cdot & \\ \cdot & \\ \left\{(\text{b-a})/\beta\right\}^{(\gamma+\delta-1)v^{(r)}} & B; \end{array}\right.$$

$$\left.\begin{array}{cc} (1\text{-r}';\ \text{v}'_1,\cdots,v_1^{(r)}),(-\lambda-r';\gamma'v',\cdots,\gamma^{(r)}v^{(r)}), & (-\mu;\delta'v',\cdots,\delta^{(r)}v^{(r)}),\mathfrak{A};A' \\ \cdots & \cdots \\ \cdots & \cdots \\ (1\ ;\ \text{v}'_1,\cdots,v_1^{(r)}),\ (-\lambda-\mu-r'-1;(\gamma'+\delta')v',\cdots, & (\gamma^{(r)}+\delta^{(r)})v^{(r)}),\mathfrak{B};B' \end{array}\right) \tag{3.3}$$

which holds true under the same conditions from (2.4) with $n=1$

4. Particular case

If $U=V=A=B=0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [5]. We have the following result.

$$\int_{a_1}^{b_1}\cdots\int_{a_n}^{b_n}\prod_{i=1}^{n}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{\left[f(t_j)\right]^{\lambda_j+\mu_j+2}}H_{p_r,q_r;W}^{0,n_r;X}\left(\begin{array}{c}\mathrm{z}_1\prod_{j=1}^{n}\left[g'(t_j)\right]^{v'_j}\\ \cdots\\ \mathrm{z}_r\prod_{j=1}^{n}\left[g^{(r)}(t_j)\right]^{v_j^{(r)}}\end{array}\right)\mathrm{d}t_1\cdots\mathrm{d}t_n$$

$$=\prod_{j=1}^{n}\left\{(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\sum_{r_j=0}^{\infty}\frac{\left\{(\beta_j-\alpha_j)/\beta_j\right\}^{r_j}(1+\rho_j)^{-r_j}\right\}}{r_j!}$$

$$H_{p_r+3n,q_r+2n;W}^{0,n_r+3n;X}\left(\begin{array}{c}\mathrm{z}_1\prod_{j=1}^{n}\left\{\beta_j(1+\rho_j)^{\gamma'_j}(1+\sigma_j)^{\delta'_j}\right\}^{-v'_j}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\prod_{j=1}^{n}\left\{\beta_j(1+\rho_j)^{\gamma_j^{(n)}}(1+\sigma_j)^{\delta_j^{(n)}}\right\}^{-v_j^{(r)}}\end{array}\left|\begin{array}{c}[1\text{-r}_j;v'_j,\cdots,v_j^{(r)}]_{1,n}\\ \cdots\\ \cdots\\ [1;v'_j,\cdots,v_j^{(r)})]_{1,n}\end{array}\right.\right.$$

$$\left.\begin{array}{c}[\text{-}\lambda_j-r_j;\gamma'_jv'_j,\cdots,\gamma_j^{(r)}v_j^{(r)})]_{1,n},(-\mu_j;\delta'_jv'_j,\cdots,\delta_j^{(r)}v_j^{(r)})]_{1,n},\mathfrak{A},A'\\ \cdots\\ \cdots\\ [-\lambda_j-\mu_j-r_j-j;(\gamma'_j+\delta'_j)v'_j,\cdots,(\gamma_j^{(r)}+\delta_j^{(r)})v_j^{(r)}]_{1,n},\mathfrak{B};B'\end{array}\right) \quad (4.1)$$

under the same notations and conditions that (2.4) with $U=V=A=B=0$

5. Conclusion

The I-function of several variables defined by Prasad [3] presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions of several variables such as , multivariable H-function , definedby Srivastava et al [5].

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