Some Applications of Labelled Graphs

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Abstract : *This paper presents some applications of labelled graphs of different kinds of labellings alongwith various kinds of graphs serving as models for the applications.*

Keywords : *graceful, felicitous, harmonious, magic, Antimagic, inner magic, inner antimagic, sequential.*

AMS Classification: 05C78

1. Introduction

In this paper we look at some practical applications of labelled graphs in particular animagic labelling which was given by [5], *inner magic* and *inner antimagic* labellings for planar graphs introduced by [8], and other labelling schemes like harmonious, graceful etc. An *antimagic* labelling of a graph G(V,E) is a bijection $f : E \rightarrow \{1,2,\ldots, |E|\}$ such that all vertex weights are distinct where the vertex weight is the sum of all edge labels incident on a vertex. A graph is said to be antimagic if it has antimagic labelling. An inner magic labelling is concerned with labelling the p vertices, q edges and the f internal faces of a planar graph such that the weights of the faces form an arithmetic progression with common difference d. If $d = 0$ then the graph is said to have an inner magic labelling and if $d \neq 0$ then it is inner antimagic labelling. The unbound outer regions is not considered for labelling. The graph which exhibits inner magic labelling is called inner magic graph and exhibiting inner antimagic labelling is called inner antimagic graph. A *graceful* labeling for a graph with p vertices and q edges is an injection $g : V(G) \rightarrow \{0,1,2,\ldots q\}$ such that the induced function given by $g^*(x,y) = |g(x) - g(y)|$ for all edges xy is injective. A *harmonious* labelling is a function $f : V(G) \rightarrow Zq = \{0,1,2,\ldots,(q-1)\}$ so that the induced edge label is given by $(f(x) + f(y)) \bmod q$, repetition of one vertexlabel is allowed in a tree. Whereas a *felicitous* labelling is given by $f : V(G) \rightarrow Zq = \{0,1,2,\ldots,q\}$ so that the induced edge label is given by (f(x) +f(y) mod q}. A *sequential* labelling is given by $f: V(G) \rightarrow Zq = \{0,1,2,\ldots,q\}$ and the induced edge label is given by $(f(x) + f(y))$.

2. Main Results

The antimagic graphs namely double wheel, centreless wheels $CW2_n$ and $CW3_n$ shown in [11] and antimagic helm, regular actinia in [13] could serve as models for applications in some civil engineering and urban planning applications. The graph double wheel is formed by adding another cycle to the wheel. Centreless wheel $CW2_n$ is obtained from double wheel by deleting its central vertex. This graph could also be viewed as the cartesian product of cycle C_n and path P_2. The centreless wheel $CW3_n$ is obtained by adding another cycle to CW2n. This graph may also be viewed as cartesian product of C_n and P_3.

[11] gives the alogorithms, proofs and formulas for the antimagic labelling of double wheel, centreless wheels 2 and 3 ($CW2_n$ and $CW3_n$). [13] gives proofs, algorithms and formulas for helm, regular actinia and web graphs. The following figures are taken from these works.

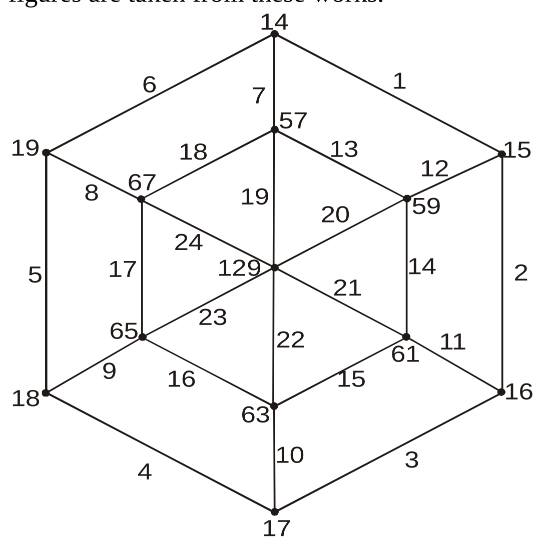


Figure 1 : Antimagic double wheel D_6

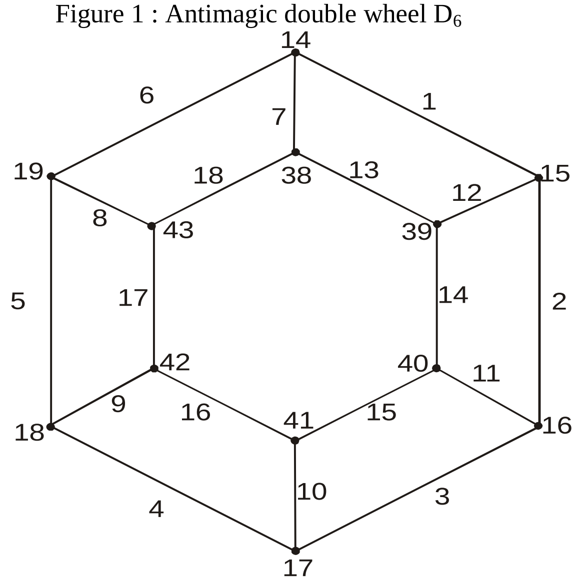


Figure 2 : Antimagic centreless wheel-2 $CW2_6$

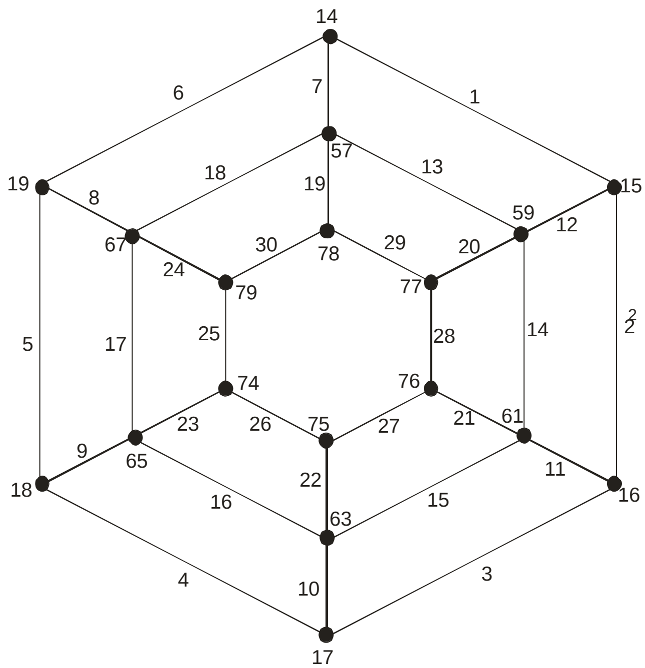


Figure 3 : Antimagic centreless wheel-3 $CW3_6$

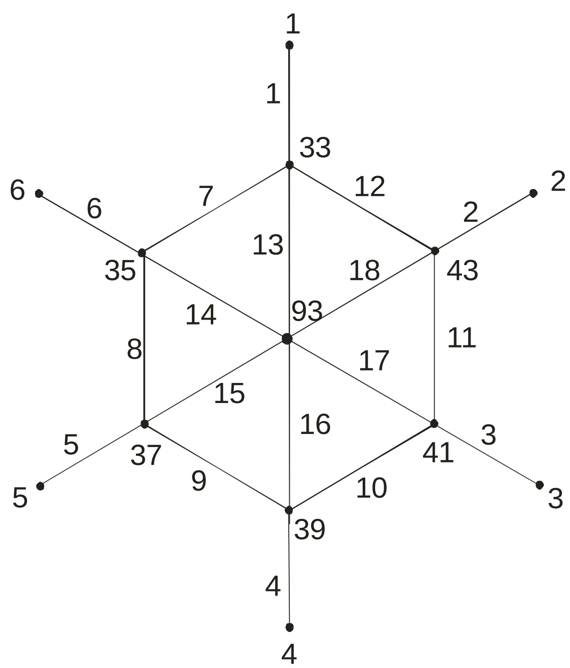


Figure 4: Antimagic helm

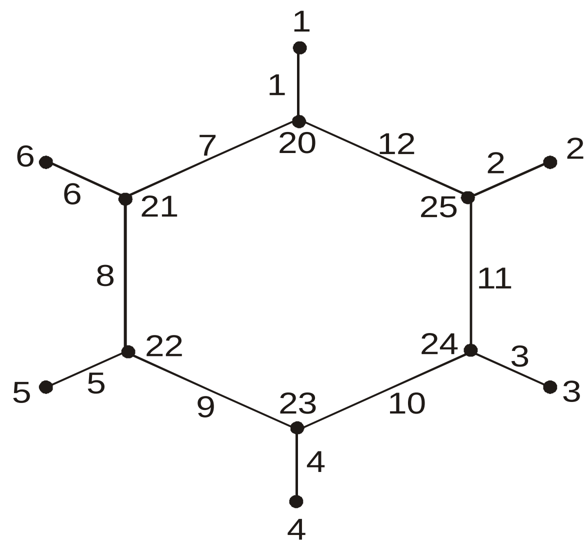


Figure 5 : Antimagic regular actinia

Application :

In the antimagic graphs shown in figures 1,2,3,4,5 the vertices could represent rooms in a building and the antimagic weighted edges could represent routes or ways to reach those rooms in a legitimate or allowed ways resulting in the allowed access as the antimagic label of the particular room or vertex. A removal of a single route would disrupt the whole antimagic graph. This antimagic graph could serve as a model for some sensitive area of the building or on a larger scale the vertices could represent cities and the antimagic weighted edges could represent various kind of legitimate routes to reach them.
Thus, these antimatgic graphs could serve as a surveillance or security model also for various kinds of buildings.

In double wheel and helm, the central antimagic vertex could serve as the central supervision room or the centre supervising all the rooms /cities or the vertices and their routes or edges being the antimagic weighted edges. The antimagic centreless wheels $CW2_n$, $CW3_n$ and regular actinia could serve as model for a surveillance or security system without centralized control. The concepts of vertex connectivity (minimum number of vertices deleted to disconnect the graph) and edge connectivity (minimum number of edges removed to disconnect the graph) could also be combined to check for vulnerability.

The graphs studied in [8] are wheels, flower-1 and flower-2. The planar graph flower-1 has one central vertex and rest all being outer vertices and all the inernal faces are bound by four edges. Flower-2 is a planar graph with one central vertex and rest all being outer vertices and all internal vertices are bound by five edges. Wheels are found to have inner magic as well as inner antimagic labellings and flower-1 and flower-2 have inner antimagic labellings. These graphs could also serve as models for surveillance or security systems, electrical switchboards, circuit design and communication networks.

Inner Magic wheel could be used in a locking security system where matching of all the weights to the inner magic weight number could lead to unlock the security system.

In case of some bigger graphs in flower-1 and flower-2 it may need to be verified whether inner antimagic labellings exist or not. Other graphs could also be examined for these labellings and applications.

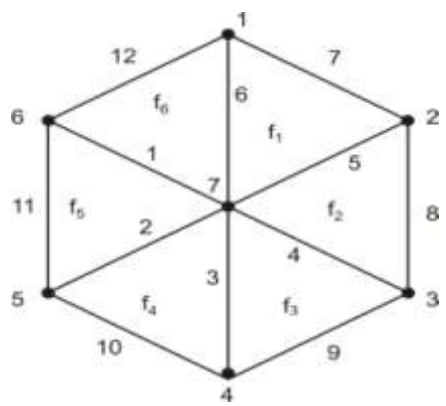


Figure 6 : Inner Magic and Inner Antimagic Wheel,Inner Magic internal face labels :

f_1 =6, f_2 =5, f_3 =4, f_4 =3, f_5 =2, f_6 =1

Inner Magic weight number = 34

Inner Antimagic internal face labels :

f_1 =1, f_2 =2, f_3 =3, f_4 =4, f_5 =5, f_6 =6

Inner Antimagic internal face weights :

29, 31, 33, 35, 37, 39

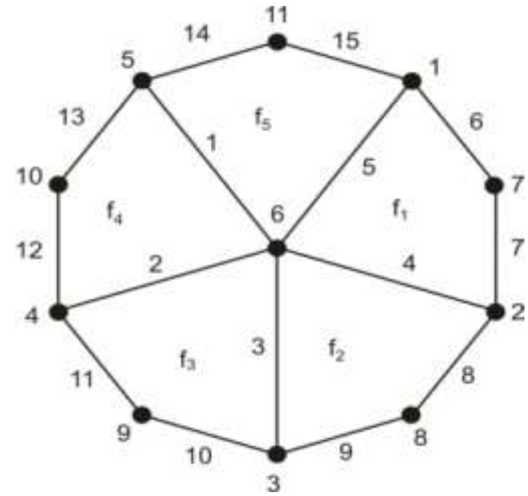


Figure 7 : Inner Antimagic flower-1

Inner Antimagic internal face weight and internal face labels are as follows :

(1) 39, 45, 51, 57, 63

f_1 =1, f_2 =2, f_3 =3, f_4 =4, f_5 =5

(2) 43, 47, 51, 55, 59

f_1 =5, f_2 =4, f_3 =3, f_4 =2, f_5 =1

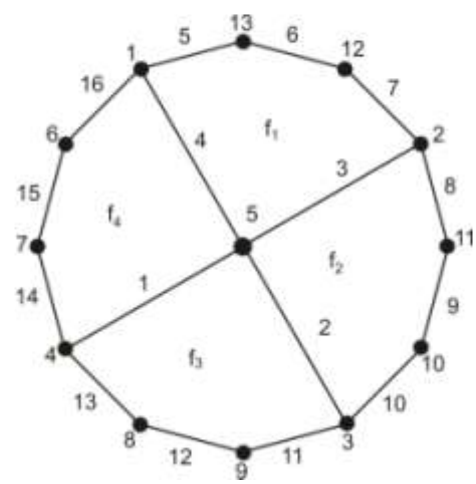
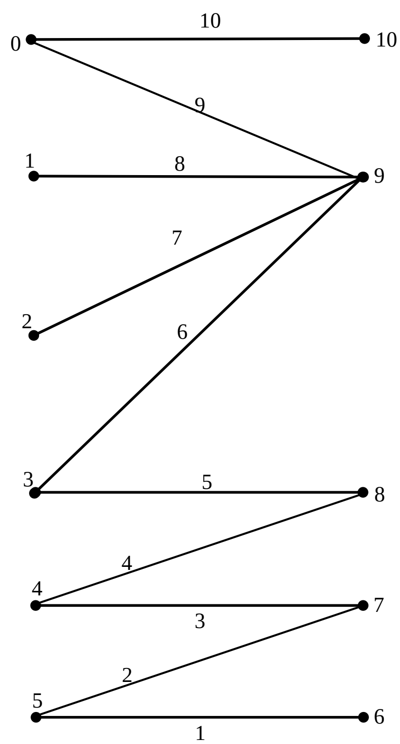


Figure 8 : Inner Antimagic flower -2

Inner Antimagic internal face weights and infernal face labels are as follows :
(1) 59, 65, 71, 77; f_1 =1, f_2 =2, f_3 =3, f_4 =4
(2) 62, 66, 70, 74; f_1 =4, f_2 =3, f_3 =2, f_4 =1

[7] gives graph labelling algorithms namely haramonious, sequential, felicitous, graceful, antimagic for bipartite trees. The algorithm G for graceful labellings in this work was an attempt to

directly solve the graceful tree conjecture for trees which was finally answered in [12].



T(11,10)

Figure 9 : Graceful bipartite tree

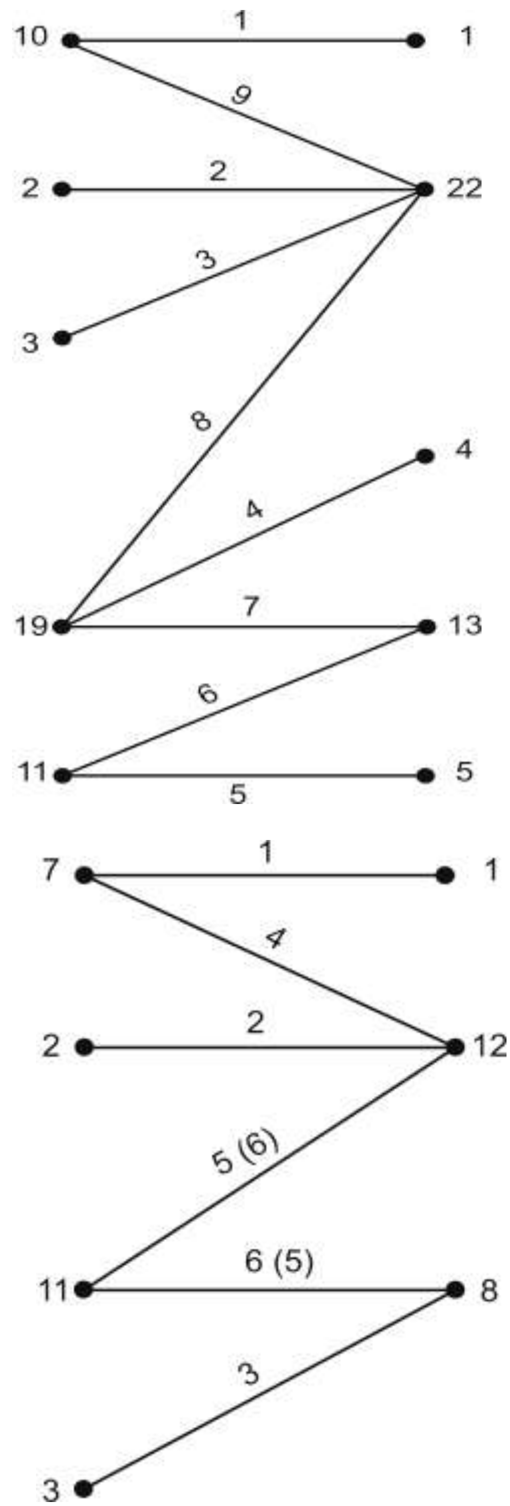


Figure 10 : Antimagic bipartite trees : the edge labels in parentheses for T (7,6) show the interchanged edge labels as in step 4 of the algorithm AM.

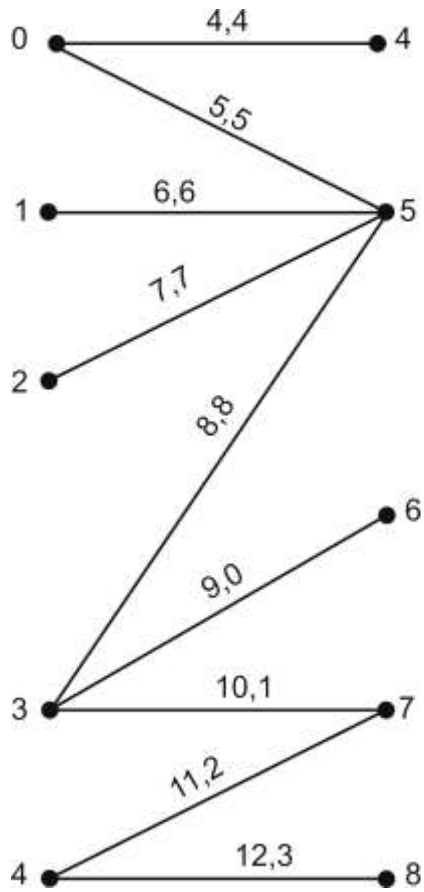


Figure 11 : Harmonious and Sequential bipartite tree

Application :

The labelled bipartite trees could be used for matching or assignment of resources and persons which could follow certain regulations depending on addition (harmonious, sequential, felicitous labelling) and difference (graceful labelling).

As mentioned in [1], antimagic properties are relevant to applications that require unbalanced loading. In a scenario of urban planning, the bipartite antimagic trees as shown in figure 11 could serve in such an application of relationship of demand and supply of distinct quantities to build a structure between vendors and buyers.

3. Conclusion

Some applications of antimagic, inner magic, inner antimagic, graceful , harmonious graphs have been discussed which could serve as model of surveillance or security system in civil engineering and urban planning, circuit design, communication networks, electrical switchboards. Applications of antimagic bipartite trees have been discussed which could be used in demand and supply scenario. Graceful, harmonious, sequential, felicitous bipartite tree could be used for matching or assignment of resources and persons. Depending upon the viewing angle and the subject background of the person looking at these graphs, further applications could emerge as well.

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