

# On Neutrosophic Semi-Open sets in Neutrosophic Topological Spaces

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**Abstract** - The purpose of this paper is to define the product related neutrosophic topological space and proved some theorems based on this. We introduce the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets in neutrosophic topological spaces and derive some of their characterization. Finally, we analyze neutrosophic semi-interior and neutrosophic semi-closure operators also.

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## INTRODUCTION

Theory of Fuzzy sets [17], Theory of Intuitionistic fuzzy sets [2], Theory of Neutrosophic sets [9] and the theory of Interval Neutrosophic sets [11] can be considered as tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in [9]. In 1965, Zadeh [17] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov [2] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The neutrosophic set was introduced by Smarandache [9] and

explained, neutrosophic set is a generalization of Intuitionistic fuzzy set. In 2012, Salama, Alblowi [15], introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element.

This paper consists of six sections. The section I consists of the basic definitions and some properties which are used in the later sections. The section II, we define product related neutrosophic topological space and proved some theorem related to this definition. The section III deals with the definition of neutrosophic semi-open set in neutrosophic topological spaces and its various properties. The section IV deals with the definition of neutrosophic semi-closed set in neutrosophic topological spaces and its various properties. The section V and VI are dealt with the concepts of neutrosophic semi-interior and neutrosophic semi-closure operators.

## I. PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

**Definition 1.1** [15] Let  $X$  be a non-empty fixed set. A neutrosophic set [ NS for short ]  $A$  is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  where  $\mu_A(x)$ ,  $\sigma_A(x)$  and  $\gamma_A(x)$  which represents the degree of membership function, the degree

indeterminacy and the degree of non-membership function respectively of each element  $x \in X$  to the set  $A$ .

**Remark 1.2** [15] A neutrosophic set  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  can be identified to an ordered triple  $\langle \mu_A, \sigma_A, \gamma_A \rangle$  in  $]0, 1^+ [$  on  $X$ .

**Remark 1.3** [15] For the sake of simplicity, we shall use the symbol  $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$  for the neutrosophic set  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ .

**Example 1.4** [15] Every IFS  $A$  is a non-empty set in  $X$  is obviously on  $NS$  having the form  $A = \{ \langle x, \mu_A(x), 1 - (\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$ . Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the  $NS$   $0_N$  and  $1_N$  in  $X$  as follows:

$0_N$  may be defined as :

- (0<sub>1</sub>)  $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
- (0<sub>2</sub>)  $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$
- (0<sub>3</sub>)  $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$
- (0<sub>4</sub>)  $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

$1_N$  may be defined as :

- (1<sub>1</sub>)  $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$
- (1<sub>2</sub>)  $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$
- (1<sub>3</sub>)  $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$
- (1<sub>4</sub>)  $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

**Definition 1.5** [15] Let  $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$  be a  $NS$  on  $X$ , then the complement of the set  $A$  [  $C(A)$  for short] may be defined as three kinds of complements :

- (C<sub>1</sub>)  $C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$
- (C<sub>2</sub>)  $C(A) = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
- (C<sub>3</sub>)  $C(A) = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$

One can define several relations and operations between  $NS$ s follows :

**Definition 1.6** [15] Let  $x$  be a non-empty set, and neutrosophic sets  $A$  and  $B$  in the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$ . Then we may consider two possible definitions for subsets ( $A \subseteq B$ ).

$A \subseteq B$  may be defined as :

- (1)  $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x) \forall x \in X$
- (2)  $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x) \forall x \in X$

**Proposition 1.7** [15] For any neutrosophic set  $A$ , then the following conditions are holds :

- (1)  $0_N \subseteq A, 0_N \subseteq 0_N$
- (2)  $A \subseteq 1_N, 1_N \subseteq 1_N$

**Definition 1.8** [15] Let  $X$  be a non-empty set, and  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$  are  $NS$ s. Then

- (1)  $A \cap B$  may be defined as :
  - (I<sub>1</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$  and  $\gamma_A(x) \vee \gamma_B(x) \rangle$
  - (I<sub>2</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$  and  $\gamma_A(x) \vee \gamma_B(x) \rangle$
- (2)  $A \cup B$  may be defined as :
  - (U<sub>1</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$  and  $\gamma_A(x) \wedge \gamma_B(x) \rangle$
  - (U<sub>2</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$  and  $\gamma_A(x) \wedge \gamma_B(x) \rangle$

We can easily generalize the operations of intersection and union in Definition 1.8 to arbitrary family of  $NS$ s as follows :

**Definition 1.9** [15] Let  $\{ A_j : j \in J \}$  be a arbitrary family of  $NS$ s in  $X$ , then

- (1)  $\cap A_j$  may be defined as :
  - (i)  $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigwedge_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$
  - (ii)  $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigvee_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$

(2)  $\cup A_j$  may be defined as :

- (i)  $\cup A_j = \langle x, \bigvee, \bigvee, \bigwedge \rangle$
- (ii)  $\cup A_j = \langle x, \bigvee, \bigwedge, \bigwedge \rangle$

**Proposition 1.10** [15] For all  $A$  and  $B$  are two neutrosophic sets then the following conditions are true :

- (1)  $C(A \cap B) = C(A) \cup C(B)$
- (2)  $C(A \cup B) = C(A) \cap C(B)$ .

Here we extend the concepts of fuzzy topological space [5] and Intuitionistic fuzzy topological space [6,7] to the case of neutrosophic sets.

**Definition 1.11** [15] A neutrosophic topology [  $NT$  for short ] is a non-empty set  $X$  is a family  $\tau$  of neutrosophic subsets in  $X$  satisfying the following axioms :

- (NT<sub>1</sub>)  $0_N, 1_N \in \tau$ ,
- (NT<sub>2</sub>)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ,
- (NT<sub>3</sub>)  $\cup G_i \in \tau$  for every  $\{ G_i : i \in J \} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called a neutrosophic topological space [NTS for short]. The elements of  $\tau$  are called neutrosophic open sets [NOS for short]. A neutrosophic set  $F$  is closed if and only if  $C(F)$  is neutrosophic open.

**Example 1.12** [15] Any fuzzy topological space  $(X, \tau_0)$  in the sense of Chang is obviously a NTS in the form  $\tau = \{A : \mu_A \in \tau_0\}$  wherever we identify a fuzzy set in  $X$  whose membership function is  $\mu_A$  with its counterpart.

**Remark 1.13** [15] Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology.

**Example 1.14** [15] Let  $X = \{x\}$  and  
 $A = \{ \langle x, 0.5, 0.5, 0.4 \rangle : x \in X \}$   
 $B = \{ \langle x, 0.4, 0.6, 0.8 \rangle : x \in X \}$   
 $D = \{ \langle x, 0.5, 0.6, 0.4 \rangle : x \in X \}$   
 $C = \{ \langle x, 0.4, 0.5, 0.8 \rangle : x \in X \}$   
 Then the family  $\tau = \{0_N, A, B, C, D, 1_N\}$  of NSs in  $X$  is neutrosophic topology on  $X$ .

**Definition 1.15** [15] The complement of  $A$  [  $C(A)$  for short ] of NOS is called a neutrosophic closed set [NCS for short] in  $X$ .

Now, we define neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces :

**Definition 1.16** [15] Let  $(X, \tau)$  be NTS and  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$  be a NS in  $X$ . Then the neutrosophic closure and neutrosophic interior of  $A$  are defined by

$$NCl(A) = \cap \{K : K \text{ is a NCS in } X \text{ and } A \subseteq K\}$$

$$NInt(A) = \cup \{G : G \text{ is a NOS in } X \text{ and } G \subseteq A\}.$$

It can be also shown that  $NCl(A)$  is NCS and  $NInt(A)$  is a NOS in  $X$ .

- a)  $A$  is NOS if and only if  $A = NInt(A)$ .
- b)  $A$  is NCS if and only if  $A = NCl(A)$ .

**Proposition 1.17** [15] For any neutrosophic set  $A$  in  $(X, \tau)$  we have

- (a)  $NCl(C(A)) = C(NInt(A))$ ,
- (b)  $NInt(C(A)) = C(NCl(A))$ .

**Proposition 1.18** [15] Let  $(X, \tau)$  be a NTS and  $A, B$  be two neutrosophic sets in  $X$ . Then the following properties are holds :

- (a)  $NInt(A) \subseteq A$ ,
- (b)  $A \subseteq NCl(A)$ ,

- (c)  $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$ ,
- (d)  $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$ ,
- (e)  $NInt(NInt(A)) = NInt(A)$ ,
- (f)  $NCl(NCl(A)) = NCl(A)$ ,
- (g)  $NInt(A \cap B) = NInt(A) \cap NInt(B)$ ,
- (h)  $NCl(A \cup B) = NCl(A) \cup NCl(B)$ ,
- (i)  $NInt(0_N) = 0_N$ ,
- (j)  $NInt(1_N) = 1_N$ ,
- (k)  $NCl(0_N) = 0_N$ ,
- (l)  $NCl(1_N) = 1_N$ ,
- (m)  $A \subseteq B \Rightarrow C(B) \subseteq C(A)$ ,
- (n)  $NCl(A \cap B) \subseteq NCl(A) \cap NCl(B)$ ,
- (o)  $NInt(A \cup B) \supseteq NInt(A) \cup NInt(B)$ .

## II. PRODUCT RELATED NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we define some basic and important results which are very useful in later sections. In order topology, the product of the closure is equal to the closure of the product and product of the interior is equal to the interior of the product. But this result is not true in neutrosophic topological space. For this reason, we define the product related neutrosophic topological space. Using this definition, we prove the above mentioned result.

**Definition 2.1** A subfamily  $\beta$  of NTS  $(X, \tau)$  is called a base for  $\tau$  if each NS of  $\tau$  is a union of some members of  $\beta$ .

**Definition 2.2** Let  $X, Y$  be nonempty neutrosophic sets and  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle$  NSs of  $X$  and  $Y$  respectively. Then  $A \times B$  is a NS of  $X \times Y$  is defined by

$$(P_1) (A \times B)(x, y) = \langle (x, y), \min(\mu_A(x), \mu_B(y)), \min(\sigma_A(x), \sigma_B(y)), \max(\gamma_A(x), \gamma_B(y)) \rangle$$

$$(P_2) (A \times B)(x, y) = \langle (x, y), \min(\mu_A(x), \mu_B(y)), \max(\sigma_A(x), \sigma_B(y)), \max(\gamma_A(x), \gamma_B(y)) \rangle$$

Notice that

$$(CP_1) C((A \times B)(x, y)) = \langle (x, y), \max(\mu_A(x), \mu_B(y)), \max(\sigma_A(x), \sigma_B(y)), \min(\gamma_A(x), \gamma_B(y)) \rangle$$

$$(CP_2) C((A \times B)(x, y)) = \langle (x, y), \max(\mu_A(x), \mu_B(y)), \min(\sigma_A(x), \sigma_B(y)), \min(\gamma_A(x), \gamma_B(y)) \rangle$$

**Lemma 2.3** If  $A$  is the NS of  $X$  and  $B$  is the NS of  $Y$ , then

- (i)  $(A \times 1_N) \cap (1_N \times B) = A \times B$ ,
- (ii)  $(A \times 1_N) \cup (1_N \times B) = C(C(A) \times C(B))$ ,
- (iii)  $C(A \times B) = (C(A) \times 1_N) \cup (1_N \times C(B))$ .

**Proof :** Let  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle$ .

(i) Since  $A \times 1_N = \langle x, \min(\mu_A, 1_N), \min(\sigma_A, 1_N), \max(\gamma_A, 0_N) \rangle = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle = A$  and similarly  $1_N \times B = \langle y, \min(1_N, \mu_B), \min(1_N, \sigma_B), \max(0_N, \gamma_B) \rangle = B$ , we have  $(A \times 1_N) \cap (1_N \times B) = A(x) \cap B(y) = \langle (x, y), \mu_A(x) \wedge \mu_B(y), \sigma_A(x) \wedge \sigma_B(y), \gamma_A(x) \vee \gamma_B(y) \rangle = A \times B$ .

(ii) Similarly to (i).

(iii) Obvious by putting A, B instead of C (A), C (B) in (ii).

**Definition 2.4** Let X and Y be two nonempty neutrosophic sets and  $f : X \rightarrow Y$  be a neutrosophic function. (i) If  $B = \{ \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle : y \in Y \}$  is a NS in Y, then the pre image of B under f is denoted and defined by  $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X \}$ .

(ii) If  $A = \{ \langle x, \alpha_A(x), \delta_A(x), \lambda_A(x) \rangle : x \in X \}$  is a NS in X, then the image of A under f is denoted and defined by  $f(A) = \{ \langle y, f(\alpha_A)(y), f(\delta_A)(y), f(\lambda_A)(y) \rangle : y \in Y \}$  where  $f(\lambda_A) = C(f(C(A)))$ .

In (i), (ii), since  $\mu_B, \sigma_B, \gamma_B, \alpha_A, \delta_A, \lambda_A$  are neutrosophic sets, we explain that  $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ ,

$$\text{and } f(\alpha_A)(y) = \begin{cases} \sup \alpha_A(x) & \text{if } x \in f^{-1}(y) \\ 0 & \text{Otherwise} \end{cases}$$

**Definition 2.5** Let  $(X, \tau)$  and  $(Y, \sigma)$  be NTSs. The neutrosophic product topological space [NPTS for short] of  $(X, \tau)$  and  $(Y, \sigma)$  is the cartesian product  $X \times Y$  of NSs X and Y together with the NT  $\xi$  of  $X \times Y$  which is generated by the family  $\{P_1^{-1}(A_i), P_2^{-1}(B_j) : A_i \in \tau, B_j \in \sigma \text{ and } P_1, P_2 \text{ are projections of } X \times Y \text{ onto } X \text{ and } Y \text{ respectively}\}$  (i.e. the family  $\{P_1^{-1}(A_i), P_2^{-1}(B_j) : A_i \in \tau, B_j \in \sigma\}$  is a subbase for NT  $\xi$  of  $X \times Y$ ).

**Remark 2.6** In the above definition, since  $P_1^{-1}(A_i) = A_i \times 1_N$  and  $P_2^{-1}(B_j) = 1_N \times B_j$  and  $A_i \times 1_N \cap 1_N \times B_j = A_i \times B_j$ , the family  $\beta = \{A_i \times B_j : A_i \in \tau, B_j \in \sigma\}$  forms a base for NPTS  $\xi$  of  $X \times Y$ .

**Definition 2.7** Let  $f_1 : X_1 \rightarrow Y_1$  and  $f_2 : X_2 \rightarrow Y_2$  be the two neutrosophic functions. Then the neutrosophic product  $f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is defined by  $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$  for all  $(x_1, x_2) \in X_1 \times X_2$ .

**Definition 2.8** Let  $A, A_i (i \in J)$  be NSs in X and  $B, B_j (j \in K)$  be NSs in Y and  $f : X \rightarrow Y$  be the neutrosophic function. Then

- (i)  $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ ,
- (ii)  $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$ ,
- (iii)  $f^{-1}(1_N) = 1_N, f^{-1}(0_N) = 0_N$ ,

$$(iv) f^{-1}(C(B)) = C(f^{-1}(B)),$$

$$(v) f(\cup A_i) = \cup f(A_i).$$

**Definition 2.9** Let  $f : X \rightarrow Y$  be the neutrosophic function. Then the neutrosophic graph  $g : X \rightarrow X \times Y$  of f is defined by  $g(x) = (x, f(x))$  for all  $x \in X$ .

**Lemma 2.10** Let  $f_i : X_i \rightarrow Y_i (i = 1, 2)$  be the neutrosophic functions and A, B be NSs of  $Y_1, Y_2$  respectively. Then  $(f_1 \times f_2)^{-1} = f_1^{-1}(A) \times f_2^{-1}(B)$ .

**Proof :** Let  $A = \langle x_1, \mu_A(x_1), \sigma_A(x_1), \gamma_A(x_1) \rangle$ ,  $B = \langle x_2, \mu_B(x_2), \sigma_B(x_2), \gamma_B(x_2) \rangle$ . For each  $(x_1, x_2) \in X_1 \times X_2$ , we have  $(f_1 \times f_2)^{-1}(A, B)(x_1, x_2) = (A \times B)(f_1 \times f_2)(x_1, x_2) = (A \times B)(f_1(x_1), f_2(x_2)) = \langle (f_1(x_1), f_2(x_2)), \min(\mu_A(f_1(x_1)), \mu_B(f_2(x_2))), \min(\sigma_A(f_1(x_1)), \sigma_B(f_2(x_2))), \max(\gamma_A(f_1(x_1)), \gamma_B(f_2(x_2))) \rangle = \langle (x_1, x_2), \min(f_1^{-1}(\mu_A)(x_1), f_2^{-1}(\mu_B)(x_2)), \min(f_1^{-1}(\sigma_A)(x_1), f_2^{-1}(\sigma_B)(x_2)), \max(f_1^{-1}(\gamma_A)(x_1), f_2^{-1}(\gamma_B)(x_2)) \rangle = (f_1^{-1}(A) \times f_2^{-1}(B))(x_1, x_2)$ .

**Lemma 2.11** Let  $g : X \rightarrow X \times Y$  be the neutrosophic graph of the neutrosophic function  $f : X \rightarrow Y$ . If A is the NS of X and B is the NS of Y, then

$$g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x).$$

**Proof :** Let  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle$ . For each  $x \in X$ , we have  $g^{-1}(A \times B)(x) = (A \times B)g(x) = (A \times B)(x, f(x)) = \langle (x, f(x)), \min(\mu_A(x), \mu_B(f(x))), \min(\sigma_A(x), \sigma_B(f(x))), \max(\gamma_A(x), \gamma_B(f(x))) \rangle = \langle (x, f(x)), \min(\mu_A(x), f^{-1}(\mu_B)(x)), \min(\sigma_A(x), f^{-1}(\sigma_B)(x)), \max(\gamma_A(x), f^{-1}(\gamma_B)(x)) \rangle = (A \cap f^{-1}(B))(x)$ .

**Lemma 2.12** Let A, B, C and D be NSs in X. Then  $A \subseteq B, C \subseteq D \Rightarrow A \times C \subseteq B \times D$ .

**Proof :** Let  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ ,  $C = \langle x, \mu_C(x), \sigma_C(x), \gamma_C(x) \rangle$  and  $D = \langle x, \mu_D(x), \sigma_D(x), \gamma_D(x) \rangle$  be NSs. Since  $A \subseteq B \Rightarrow \mu_A \leq \mu_B, \sigma_A \leq \sigma_B, \gamma_A \geq \gamma_B$  and also  $C \subseteq D \Rightarrow \mu_C \leq \mu_D, \sigma_C \leq \sigma_D, \gamma_C \geq \gamma_D$ , we have  $\min(\mu_A, \mu_C) \leq \min(\mu_B, \mu_D)$ ,  $\min(\sigma_A, \sigma_C) \leq \min(\sigma_B, \sigma_D)$  and  $\max(\gamma_A, \gamma_C) \geq \max(\gamma_B, \gamma_D)$ . Hence the result.

**Lemma 2.13** Let  $(X, \tau)$  and  $(Y, \sigma)$  be any two NTSs such that X is neutrosophic product relative to Y. Let A and B be NCSs in NTSs X and Y respectively. Then  $A \times B$  is the NCS in the NPTS of  $X \times Y$ .

**Proof :** Let  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle$ . From Lemma 2.3,  $C(A \times B)(x, y) = (C(A) \times 1_N) \cup (1_N \times C(B))(x, y)$ . Since  $C(A) \times 1_N$  and  $1_N \times C(B)$  are NOSs in X and Y respectively. Hence  $C(A) \times 1_N \cup 1_N \times C(B)$  is NOS of  $X \times Y$ . Hence  $C(A \times B)$  is a NOS of  $X \times Y$  and consequently  $A \times B$  is the NCS of  $X \times Y$ .

**Theorem 2.14** If  $A$  and  $B$  are NSs of NTSs  $X$  and  $Y$  respectively, then

- (i)  $NCl(A) \times NCl(B) \supseteq NCl(A \times B)$ ,
- (ii)  $NInt(A) \times NInt(B) \subseteq NInt(A \times B)$ .

**Proof :** (i) Since  $A \subseteq NCl(A)$  and  $B \subseteq NCl(B)$ , hence  $A \times B \subseteq NCl(A) \times NCl(B)$ . This implies that  $NCl(A \times B) \subseteq NCl(NCl(A) \times NCl(B))$  and from Lemma 2.13,  $NCl(A \times B) \subseteq NCl(A) \times NCl(B)$ .  
 (ii) follows from (i) and the fact that  $NInt(C(A)) = C(NCl(A))$ .

**Definition 2.15** Let  $(X, \tau), (Y, \sigma)$  be NTSs and  $A \in \tau, B \in \sigma$ . We say that  $(X, \tau)$  is neutrosophic product related to  $(Y, \sigma)$  if for any NSs  $C$  of  $X$  and  $D$  of  $Y$ , whenever  $C(A) \not\supseteq C$  and  $C(B) \not\supseteq D \Rightarrow C(A) \times 1_N \cup 1_N \times C(B) \supseteq C \times D$ , there exist  $A_1 \in \tau, B_1 \in \sigma$  such that  $C(A_1) \supseteq C$  or  $C(B_1) \supseteq D$  and  $C(A_1) \times 1_N \cup 1_N \times C(B_1) = C(A) \times 1_N \cup 1_N \times C(B)$ .

**Lemma 2.16** For NSs  $A_i$ 's and  $B_j$ 's of NTSs  $X$  and  $Y$  respectively, we have

- (i)  $\cap \{A_i, B_j\} = \min(\cap A_i, \cap B_j)$ ;  
 $\cup \{A_i, B_j\} = \max(\cup A_i, \cup B_j)$ .
- (ii)  $\cap \{A_i, 1_N\} = (\cap A_i) \times 1_N$ ;  
 $\cup \{A_i, 1_N\} = (\cup A_i) \times 1_N$ .
- (iii)  $\cap \{1_N \times B_j\} = 1_N \times (\cap B_j)$ ;  
 $\cup \{1_N \times B_j\} = 1_N \times (\cup B_j)$ .

**Proof :** Obvious.

**Theorem 2.17** Let  $(X, \tau)$  and  $(Y, \sigma)$  be NTSs such that  $X$  is neutrosophic product related to  $Y$ . Then for NSs  $A$  of  $X$  and  $B$  of  $Y$ , we have

- (i)  $NCl(A \times B) = NCl(A) \times NCl(B)$ ,
- (ii)  $NInt(A \times B) = NInt(A) \times NInt(B)$ .

**Proof :** (i) Since  $NCl(A \times B) \subseteq NCl(A) \times NCl(B)$  (By Theorem 2.14) it is sufficient to show that  $NCl(A \times B) \supseteq NCl(A) \times NCl(B)$ . Let  $A_i \in \tau$  and  $B_j \in \sigma$ . Then  $NCl(A \times B) = \langle (x, y), \cap C(\{A_i \times B_j\}) : C(\{A_i \times B_j\}) \supseteq A \times B, \cup \{A_i \times B_j\} : \{A_i \times B_j\} \subseteq A \times B \rangle = \langle (x, y), \cap (C(A_i) \times 1_N \cup 1_N \times C(B_j)) : C(A_i) \times 1_N \cup 1_N \times C(B_j) \supseteq A \times B, \cup (A_i \times 1_N \cap 1_N \times B_j) : A_i \times 1_N \cap 1_N \times B_j \subseteq A \times B \rangle = \langle (x, y), \cap (C(A_i) \times 1_N \cup 1_N \times C(B_j)) : C(A_i) \supseteq A \text{ or } C(B_j) \supseteq B, \cup (A_i \times 1_N \cap 1_N \times B_j) : A_i \subseteq A \text{ and } B_j \subseteq B \rangle = \langle (x, y), \min(\cap \{C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(A_i) \supseteq A\}, \cap \{C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(B_j) \supseteq B\}), \max(\cup \{A_i \times 1_N \cap 1_N \times B_j : A_i \subseteq A\}, \cup \{A_i \times 1_N \cap 1_N \times B_j : B_j \subseteq B\}) \rangle$ . Since  $\langle (x, y), \cap \{C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(A_i) \supseteq A\}, \cap \{C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(B_j) \supseteq B\} \rangle \supseteq \langle (x, y), \cap \{C(A_i) \times 1_N : C(A_i) \supseteq A\}, \cap \{1_N \times C(B_j) : C(B_j) \supseteq B\} \rangle = \langle (x, y), \cap \{C(A_i) : C(A_i) \supseteq A\} \times 1_N, 1_N \times \cap \{C(B_j) : C(B_j) \supseteq B\} \rangle = \langle (x, y), NCl(A) \times$

$1_N, 1_N \times NCl(B) \rangle$  and  $\langle (x, y), \cup \{A_i \times 1_N \cap 1_N \times B_j : A_i \subseteq A, \cup \{A_i \times 1_N \cap 1_N \times B_j : B_j \subseteq B\} \rangle \subseteq \langle (x, y), \cup \{A_i \times 1_N : A_i \subseteq A\}, \cup \{1_N \times B_j : B_j \subseteq B\} \rangle = \langle (x, y), \cup \{A_i : A_i \subseteq A\} \times 1_N, 1_N \times \cup \{B_j : B_j \subseteq B\} \rangle = \langle (x, y), NInt(A) \times 1_N, 1_N \times NInt(B) \rangle$ , we have  $NCl(A \times B) \supseteq \langle (x, y), \min(NCl(A) \times 1_N, 1_N \times NCl(B)), \max(NInt(A) \times 1_N, 1_N \times NInt(B)) \rangle = \langle (x, y), \min(NCl(A), NCl(B)), \max(NInt(A), NInt(B)) \rangle = NCl(A) \times NCl(B)$ .  
 (ii) follows from (i).

### III. NEUTROSOPHIC SEMI-OPEN SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, the concepts of the neutrosophic semi-open set is introduced and also discussed their characterizations.

**Definition 3.1** Let  $A$  be NS of a NTS  $X$ . Then  $A$  is said to be neutrosophic semi-open [ written NSO ] set of  $X$  if there exists a neutrosophic open set  $NO$  such that  $NO \subseteq A \subseteq NCl(NO)$ .

The following theorem is the characterization of NSO set in NTS.

**Theorem 3.2** A subset  $A$  in a NTS  $X$  is NSO set if and only if  $A \subseteq NCl(NInt(A))$ .

**Proof :** Sufficiency: Let  $A \subseteq NCl(NInt(A))$ . Then for  $NO = NInt(A)$ , we have  $NO \subseteq A \subseteq NCl(NO)$ .  
 Necessity: Let  $A$  be NSO set in  $X$ . Then  $NO \subseteq A \subseteq NCl(NO)$  for some neutrosophic open set  $NO$ . But  $NO \subseteq NInt(A)$  and thus  $NCl(NO) \subseteq NCl(NInt(A))$ . Hence  $A \subseteq NCl(NO) \subseteq NCl(NInt(A))$ .

**Theorem 3.3** Let  $(X, \tau)$  be a NTS. Then union of two NSO sets is a NSO set in the NTS  $X$ .

**Proof :** Let  $A$  and  $B$  are NSO sets in  $X$ . Then  $A \subseteq NCl(NInt(A))$  and  $B \subseteq NCl(NInt(B))$ . Therefore  $A \cup B \subseteq NCl(NInt(A)) \cup NCl(NInt(B)) = NCl(NInt(A) \cup NInt(B)) \subseteq NCl(NInt(A \cup B))$  [ By Proposition 1.18 (o) ]. Hence  $A \cup B$  is NSO set in  $X$ .

**Theorem 3.4** Let  $(X, \tau)$  be a NTS. If  $\{A_\alpha\}_{\alpha \in \Delta}$  is a collection of NSO sets in a NTS  $X$ . Then  $\cup_{\alpha \in \Delta} A_\alpha$  is NSO set in  $X$ .

**Proof :** For each  $\alpha \in \Delta$ , we have a neutrosophic open set  $NO_\alpha$  such that  $NO_\alpha \subseteq A_\alpha \subseteq NCl(NO_\alpha)$ . Then  $\cup_{\alpha \in \Delta} NO_\alpha \subseteq \cup_{\alpha \in \Delta} A_\alpha \subseteq \cup_{\alpha \in \Delta} NCl(NO_\alpha) \subseteq NCl(\cup_{\alpha \in \Delta} NO_\alpha)$ . Hence let  $NO = \cup_{\alpha \in \Delta} NO_\alpha$ .

**Remark 3.5** The intersection of any two *NSO* sets need not be a *NSO* set in  $X$  as shown by the following example.

**Example 3.6** Let  $X = \{ a, b \}$  and

$$A = \langle (0.3, 0.5, 0.4), (0.6, 0.2, 0.5) \rangle$$

$$B = \langle (0.2, 0.6, 0.7), (0.5, 0.3, 0.1) \rangle$$

$$C = \langle (0.3, 0.6, 0.4), (0.6, 0.3, 0.1) \rangle$$

$$D = \langle (0.2, 0.5, 0.7), (0.5, 0.2, 0.5) \rangle.$$

Then  $\tau = \{ 0_N, A, B, C, D, 1_N \}$  is *NTS* on  $X$ . Now, we define the two *NSO* sets as follows:

$$A_1 = \langle (0.4, 0.6, 0.4), (0.8, 0.3, 0.4) \rangle \text{ and}$$

$$A_2 = \langle (1, 0.9, 0.2), (0.5, 0.7, 0) \rangle. \text{ Here } NInt(A_1) = A, NCl(NInt(A_1)) = 1_N \text{ and } NInt(A_2) = B,$$

$$NCl(NInt(A_2)) = 1_N. \text{ But } A_1 \cap A_2 = \langle (0.4, 0.6, 0.4), (0.5, 0.3, 0.4) \rangle \text{ is not a } NSO \text{ set in } X.$$

**Theorem 3.7** Let  $A$  be *NSO* set in the *NTS*  $X$  and suppose  $A \subseteq B \subseteq NCl(A)$ . Then  $B$  is *NSO* set in  $X$ .

**Proof :** There exists a neutrosophic open set  $NO$  such that  $NO \subseteq A \subseteq NCl(NO)$ . Then  $NO \subseteq B$ . But  $NCl(A) \subseteq NCl(NO)$  and thus  $B \subseteq NCl(NO)$ . Hence  $NO \subseteq B \subseteq NCl(NO)$  and  $B$  is *NSO* set in  $X$ .

**Theorem 3.8** Every neutrosophic open set in the *NTS*  $X$  is *NSO* set in  $X$ .

**Proof :** Let  $A$  be neutrosophic open set in *NTS*  $X$ . Then  $A = NInt(A)$ . Also  $NInt(A) \subseteq NCl(NInt(A))$ . This implies that  $A \subseteq NCl(NInt(A))$ . Hence by Theorem 3.2,  $A$  is *NSO* set in  $X$ .

**Remark 3.9** The converse of the above theorem need not be true as shown by the following example.

**Example 3.10** Let  $X = \{ a, b, c \}$  with  $\tau = \{ 0_N, A, B, 1_N \}$ . Some of the *NSO* sets are

$$A = \langle (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) \rangle$$

$$B = \langle (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) \rangle$$

$$C = \langle (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) \rangle$$

$$D = \langle (0.3, 0.5, 0.4), (0.1, 0.6, 0.2), (0.7, 0.5, 0.8) \rangle$$

$$E = \langle (0.5, 0.6, 0.1), (0.4, 0.6, 0.1), (0.9, 0.8, 0.5) \rangle$$

$$F = \langle (0.3, 0.5, 0.4), (0.1, 0.3, 0.2), (0.7, 0.5, 0.8) \rangle$$

$$G = \langle (0.4, 0.5, 0.2), (0.3, 0.6, 0.1), (0.9, 0.6, 0.8) \rangle$$

$$H = \langle (0.3, 0.5, 0.4), (0.1, 0.2, 0.2), (0.7, 0.5, 0.8) \rangle$$

$$I = \langle (0.4, 0.5, 0.2), (0.3, 0.3, 0.1), (0.9, 0.6, 0.8) \rangle$$

$$J = \langle (0.3, 0.5, 0.4), (0.1, 0.2, 0.2), (0.7, 0.5, 0.8) \rangle.$$

Here  $C, D, E, F, G, H, I$  and  $J$  are *NSO* sets but are not neutrosophic open sets.

**Proposition 3.11** If  $X$  and  $Y$  are *NTS* such that  $X$  is neutrosophic product related to  $Y$ . Then the neutrosophic product  $A \times B$  of a neutrosophic semi-open set  $A$  of  $X$  and a neutrosophic semi-open set  $B$  of  $Y$  is a neutrosophic semi-open set of the neutrosophic product topological space  $X \times Y$ .

**Proof :** Let  $O_1 \subseteq A \subseteq NCl(O_1)$  and  $O_2 \subseteq B \subseteq NCl(O_2)$  where  $O_1$  and  $O_2$  are neutrosophic open sets in  $X$  and  $Y$  respectively. Then,  $O_1 \times O_2 \subseteq A \times B \subseteq NCl(O_1) \times NCl(O_2)$ . By Theorem 2.17 (i),  $NCl(O_1) \times NCl(O_2) = NCl(O_1 \times O_2)$ . Therefore  $O_1 \times O_2 \subseteq A \times B \subseteq NCl(O_1 \times O_2)$ . Hence by Theorem 3.1,  $A \times B$  is neutrosophic semi-open set in  $X \times Y$ .

#### IV. NEUTROSOPHIC SEMI-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, the neutrosophic semi-closed set is introduced and studied their properties.

**Definition 4.1** Let  $A$  be *NS* of a *NTS*  $X$ . Then  $A$  is said to be neutrosophic semi-closed [ written *NSC* ] set of  $X$  if there exists a neutrosophic closed set  $NC$  such that  $NInt(NC) \subseteq A \subseteq NC$ .

**Theorem 4.2** A subset  $A$  in a *NTS*  $X$  is *NCS* set if and only if  $NInt(NCl(A)) \subseteq A$ .

**Proof :** Sufficiency: Let  $NInt(NCl(A)) \subseteq A$ . Then for  $NC = NCl(A)$ , we have  $NInt(NC) \subseteq A \subseteq NC$ . Necessity: Let  $A$  be *NSC* set in  $X$ . Then  $NInt(NC) \subseteq A \subseteq NC$  for some neutrosophic closed set  $NC$ . But  $NCl(A) \subseteq NC$  and thus  $NInt(NCl(A)) \subseteq NInt(NC)$ . Hence  $NInt(NCl(A)) \subseteq NInt(NC) \subseteq A$ .

**Proposition 4.3** Let  $(X, \tau)$  be a *NTS* and  $A$  be a neutrosophic subset of  $X$ . Then  $A$  is *NSC* set if and only if  $C(A)$  is *NSO* set in  $X$ .

**Proof :** Let  $A$  be a neutrosophic semi-closed subset of  $X$ . Then by Theorem 4.2,  $NInt(NCl(A)) \subseteq A$ . Taking complement on both sides,  $C(A) \subseteq C(NInt(NCl(A))) = NCl(C(NCl(A)))$ . By using Proposition 1.17 (b),  $C(A) \subseteq NCl(NInt(C(A)))$ . By Theorem 3.2,  $C(A)$  is neutrosophic semi-open. Conversely let  $C(A)$  is neutrosophic semi-open. By Theorem 3.2,

$C(A) \subseteq NCl(NInt(C(A)))$ . Taking complement on both sides,  $A \supseteq C(NCl(NInt(C(A)))) = NInt(C(NInt(C(A))))$ . By using Proposition 1.17 (b),  $A \supseteq NInt(NCl(A))$ . By Theorem 4.2,  $A$  is neutrosophic semi-closed set.

**Theorem 4.4** Let  $(X, \tau)$  be a  $NTS$ . Then intersection of two  $NSC$  sets is a  $NSC$  set in the  $NTS$   $X$ .

**Proof :** Let  $A$  and  $B$  are  $NSC$  sets in  $X$ . Then  $NInt(NCl(A)) \subseteq A$  and  $NInt(NCl(B)) \subseteq B$ . Therefore  $A \cap B \supseteq NInt(NCl(A)) \cap NInt(NCl(B)) = NInt(NCl(A) \cap NCl(B)) \supseteq NInt(NCl(A \cap B))$  [By Proposition 1.18 (n)]. Hence  $A \cap B$  is  $NSC$  set in  $X$ .

**Theorem 4.5** Let  $\{A_\alpha\}_{\alpha \in \Delta}$  be a collection of  $NSC$  sets in a  $NTS$   $X$ . Then  $\bigcap_{\alpha \in \Delta} A_\alpha$  is  $NSC$  set in  $X$ .

**Proof :** For each  $\alpha \in \Delta$ , we have a neutrosophic closed set  $NC_\alpha$  such that  $NInt(NC_\alpha) \subseteq A_\alpha \subseteq NC_\alpha$ . Then  $NInt(\bigcap_{\alpha \in \Delta} NC_\alpha) \subseteq \bigcap_{\alpha \in \Delta} NInt(NC_\alpha) \subseteq \bigcap_{\alpha \in \Delta} A_\alpha \subseteq \bigcap_{\alpha \in \Delta} NC_\alpha$ . Hence let  $NC = \bigcap_{\alpha \in \Delta} NC_\alpha$ .

**Remark 4.6** The union of any two  $NSC$  sets need not be a  $NSC$  set in  $X$  as shown by the following example.

**Example 4.7** Let  $X = \{a\}$  and

$$A = \langle (1, 0.5, 0.7) \rangle$$

$$B = \langle (0, 0.9, 0.2) \rangle$$

$$C = \langle (1, 0.9, 0.2) \rangle$$

$$D = \langle (0, 0.5, 0.7) \rangle.$$

Then  $\tau = \{0_N, A, B, C, D, 1_N\}$  is  $NTS$  on  $X$ . Now, we define the two  $NSC$  sets as follows :

$$A_1 = \langle (0.4, 0.5, 1) \rangle \text{ and}$$

$$A_2 = \langle (0.2, 0, 0.8) \rangle. \text{ Here } NCl(A_1) = \langle (0.7, 0.5, 1) \rangle, NInt(NCl(A_1)) = 0_N \text{ and } NCl(A_2) = \langle (0.2, 0.1, 0) \rangle, NInt(NCl(A_2)) = 0_N. \text{ But } A_1 \cup A_2 = \langle (0.4, 0.5, 0.8) \rangle \text{ is not a } NSC \text{ set in } X.$$

**Theorem 4.8** Let  $A$  be  $NSC$  set in the  $NTS$   $X$  and suppose  $NInt(A) \subseteq B \subseteq A$ . Then  $B$  is  $NSC$  set in  $X$ .

**Proof :** There exists a neutrosophic closed set  $NC$  such that  $NInt(NC) \subseteq A \subseteq NC$ . Then  $B \subseteq NC$ . But  $NInt(NC) \subseteq NInt(A)$  and thus  $NInt(NC) \subseteq B$ . Hence  $NInt(NC) \subseteq B \subseteq NC$  and  $B$  is  $NSC$  set in  $X$ .

**Theorem 4.9** Every neutrosophic closed set in the  $NTS$   $X$  is  $NSC$  set in  $X$ .

**Proof :** Let  $A$  be neutrosophic closed set in  $NTS$   $X$ . Then  $A = NCl(A)$ . Also  $NInt(NCl(A)) \subseteq NCl(A)$ . This implies that  $NInt(NCl(A)) \subseteq A$ . Hence by Theorem 4.2,  $A$  is  $NSC$  set in  $X$ .

**Remark 4.10** The converse of the above theorem need not be true as shown by the following example.

**Example 4.11** Let  $X = \{a, b, c\}$  with  $\tau = \{0_N, A, B, 1_N\}$  and  $C(\tau) = \{1_N, C, D, 0_N\}$  where

$$A = \langle (0.5, 0.6, 0.3), (0.1, 0.7, 0.9), (1, 0.6, 0.4) \rangle$$

$$B = \langle (0, 0.4, 0.7), (0.1, 0.6, 0.9), (0.5, 0.5, 0.8) \rangle$$

$$C = \langle (0.3, 0.4, 0.5), (0.9, 0.3, 0.1), (0.4, 0.4, 1) \rangle$$

$$D = \langle (0.7, 0.6, 0), (0.9, 0.4, 0.1), (0.8, 0.5, 0.5) \rangle.$$

$$E = \langle (0.2, 0.4, 0.9), (0, 0.2, 0.9), (0.3, 0.2, 1) \rangle.$$

Here the  $NSC$  sets are  $C, D$  and  $E$ .

Also  $E$  is  $NSC$  set but is not neutrosophic closed set.

**Proposition 4.12** If  $X$  and  $Y$  are neutrosophic spaces such that  $X$  is neutrosophic product related to  $Y$ . Then the neutrosophic product  $A \times B$  of a neutrosophic semi-closed set  $A$  of  $X$  and a neutrosophic semi-closed set  $B$  of  $Y$  is a neutrosophic semi-closed set of the neutrosophic product topological space  $X \times Y$ .

**Proof :** Let  $NInt(C_1) \subseteq A \subseteq C_1$  and  $NInt(C_2) \subseteq B \subseteq C_2$  where  $C_1$  and  $C_2$  are neutrosophic closed sets in  $X$  and  $Y$  respectively. Then  $NInt(C_1) \times NInt(C_2) \subseteq A \times B \subseteq C_1 \times C_2$ . By Theorem 2.17 (ii),  $NInt(C_1) \times NInt(C_2) = NInt(C_1 \times C_2)$ . Therefore  $NInt(C_1 \times C_2) \subseteq A \times B \subseteq C_1 \times C_2$ . Hence by Theorem 4.1,  $A \times B$  is neutrosophic semi-closed set in  $X \times Y$ .

## V. NEUTROSOPHIC SEMI-INTERIOR IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the neutrosophic semi-interior operator and their properties in neutrosophic topological space.

**Definition 5.1** Let  $(X, \tau)$  be a  $NTS$ . Then for a neutrosophic subset  $A$  of  $X$ , the neutrosophic semi-interior of  $A$  [ $NSInt(A)$  for short] is the union of all neutrosophic semi-open sets of  $X$  contained in  $A$ . That is,  $NSInt(A) = \cup \{G : G \text{ is a } NSO \text{ set in } X \text{ and } G \subseteq A\}$ .

**Proposition 5.2** Let  $(X, \tau)$  be a *NTS*. Then for any neutrosophic subsets  $A$  and  $B$  of a *NTS*  $X$  we have

- (i)  $NS\ Int(A) \subseteq A$
- (ii)  $A$  is *NSO* set in  $X \Leftrightarrow NS\ Int(A) = A$
- (iii)  $NS\ Int(NS\ Int(A)) = NS\ Int(A)$
- (iv) If  $A \subseteq B$  then  $NS\ Int(A) \subseteq NS\ Int(B)$

**Proof :** (i) follows from Definition 5.1.

Let  $A$  be *NSO* set in  $X$ . Then  $A \subseteq NS\ Int(A)$ . By using (i) we get  $A = NS\ Int(A)$ . Conversely assume that  $A = NS\ Int(A)$ . By using Definition 5.1,  $A$  is *NSO* set in  $X$ . Thus (ii) is proved.

By using (ii),  $NS\ Int(NS\ Int(A)) = NS\ Int(A)$ . This proves (iii).

Since  $A \subseteq B$ , by using (i),  $NS\ Int(A) \subseteq A \subseteq B$ . That is  $NS\ Int(A) \subseteq B$ . By (iii),  $NS\ Int(NS\ Int(A)) \subseteq NS\ Int(B)$ . Thus  $NS\ Int(A) \subseteq NS\ Int(B)$ . This proves (iv).

**Theorem 5.3** Let  $(X, \tau)$  be a *NTS*. Then for any neutrosophic subset  $A$  and  $B$  of a *NTS*, we have

- (i)  $NS\ Int(A \cap B) = NS\ Int(A) \cap NS\ Int(B)$
- (ii)  $NS\ Int(A \cup B) \supseteq NS\ Int(A) \cup NS\ Int(B)$ .

**Proof :** Since  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$ , by using Proposition 5.2 (iv),  $NS\ Int(A \cap B) \subseteq NS\ Int(A)$  and  $NS\ Int(A \cap B) \subseteq NS\ Int(B)$ . This implies that  $NS\ Int(A \cap B) \subseteq NS\ Int(A) \cap NS\ Int(B)$  -----(1). By using Proposition 5.2 (i),  $NS\ Int(A) \subseteq A$  and  $NS\ Int(B) \subseteq B$ . This implies that  $NS\ Int(A) \cap NS\ Int(B) \subseteq A \cap B$ . Now applying Proposition 5.2 (iv),  $NS\ Int(NS\ Int(A) \cap NS\ Int(B)) \subseteq NS\ Int(A \cap B)$ . By (1),  $NS\ Int(NS\ Int(A) \cap NS\ Int(B)) \subseteq NS\ Int(A \cap B)$ . By Proposition 5.2 (iii),  $NS\ Int(A) \cap NS\ Int(B) \subseteq NS\ Int(A \cap B)$  -----(2). From (1) and (2),  $NS\ Int(A \cap B) = NS\ Int(A) \cap NS\ Int(B)$ . This implies (i).

Since  $A \subseteq A \cup B$  and  $B \subseteq A \cup B$ , by using Proposition 5.2 (iv),  $NS\ Int(A) \subseteq NS\ Int(A \cup B)$  and  $NS\ Int(B) \subseteq NS\ Int(A \cup B)$ . This implies that  $NS\ Int(A) \cup NS\ Int(B) \subseteq NS\ Int(A \cup B)$ . Hence (ii).

The following example shows that the equality need not be hold in Theorem 5.3 (ii).

**Example 5.4** Let  $X = \{ a, b, c \}$  and  $\tau = \{ 0_N, A, B, C, D, 1_N \}$  where

$A = \langle (0.4, 0.7, 0.1), (0.5, 0.6, 0.2), (0.9, 0.7, 0.3) \rangle$ ,  
 $B = \langle (0.4, 0.6, 0.1), (0.7, 0.7, 0.2), (0.9, 0.5, 0.1) \rangle$ ,  
 $C = \langle (0.4, 0.7, 0.1), (0.7, 0.7, 0.2), (0.9, 0.7, 0.1) \rangle$ ,  
 $D = \langle (0.4, 0.6, 0.1), (0.5, 0.6, 0.2), (0.9, 0.5, 0.3) \rangle$ .

Then  $(X, \tau)$  is a *NTS*. Consider the *NSs* are  
 $E = \langle (0.7, 0.6, 0.1), (0.7, 0.6, 0.1), (0.9, 0.5, 0) \rangle$   
and  $F = \langle (0.4, 0.6, 0.1), (0.5, 0.7, 0.2), (1, 0.7, 0.1) \rangle$ . Then  $NS\ Int(E) = D$  and  $NS\ Int(F) = D$ . This implies that  $NS\ Int(E) \cup NS\ Int(F) = D$ . Now,

$E \cup F = \langle (0.7, 0.6, 0.1), (0.7, 0.7, 0.1), (1, 0.7, 0) \rangle$ , it follows that  $NS\ Int(E \cup F) = B$ . Then  $NS\ Int(E \cup F) \not\subseteq NS\ Int(E) \cup NS\ Int(F)$ .

## VI. NEUTROSOPHIC SEMI-CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the concept of neutrosophic semi-closure operators in a *NTS*.

**Definition 6.1** Let  $(X, \tau)$  be a *NTS*. Then for a neutrosophic subset  $A$  of  $X$ , the neutrosophic semi-closure of  $A$  [ $NS\ Cl(A)$  for short] is the intersection of all neutrosophic semi-closed sets of  $X$  contained in  $A$ . That is,  $NS\ Cl(A) = \bigcap \{ K : K \text{ is a } NSC \text{ set in } X \text{ and } K \supseteq A \}$ .

**Proposition 6.2** Let  $(X, \tau)$  be a *NTS*. Then for any neutrosophic subsets  $A$  of  $X$ ,

- (i)  $C(NS\ Int(A)) = NS\ Cl(C(A))$ ,
- (ii)  $C(NS\ Cl(A)) = NS\ Int(C(A))$ .

**Proof :** By using Definition 5.1,  $NS\ Int(A) = \bigcup \{ G : G \text{ is a } NSO \text{ set in } X \text{ and } G \subseteq A \}$ . Taking complement on both sides,  $C(NS\ Int(A)) = C(\bigcup \{ G : G \text{ is a } NSO \text{ set in } X \text{ and } G \subseteq A \}) = \bigcap \{ C(G) : C(G) \text{ is a } NSC \text{ set in } X \text{ and } C(A) \subseteq C(G) \}$ . Replacing  $C(G)$  by  $K$ , we get  $C(NS\ Int(A)) = \bigcap \{ K : K \text{ is a } NSC \text{ set in } X \text{ and } K \supseteq C(A) \}$ . By Definition 6.1,  $C(NS\ Int(A)) = NS\ Cl(C(A))$ . This proves (i).

By using (i),  $C(NS\ Int(C(A))) = NS\ Cl(C(C(A))) = NS\ Cl(A)$ . Taking complement on both sides, we get  $NS\ Int(C(A)) = C(NS\ Cl(A))$ . Hence proved (ii).

**Proposition 6.3** Let  $(X, \tau)$  be a *NTS*. Then for any neutrosophic subsets  $A$  and  $B$  of a *NTS*  $X$  we have

- (i)  $A \subseteq NS\ Cl(A)$
- (ii)  $A$  is *NSC* set in  $X \Leftrightarrow NS\ Cl(A) = A$
- (iii)  $NS\ Cl(NS\ Cl(A)) = NS\ Cl(A)$
- (iv) If  $A \subseteq B$  then  $NS\ Cl(A) \subseteq NS\ Cl(B)$

**Proof :** (i) follows from Definition 6.1.

Let  $A$  be *NSC* set in  $X$ . By using Proposition 4.3,  $C(A)$  is *NSO* set in  $X$ . By Proposition 6.2 (ii),  $NS\ Int(C(A)) = C(A) \Leftrightarrow C(NS\ Cl(A)) = C(A) \Leftrightarrow NS\ Cl(A) = A$ . Thus proved (ii).

By using (ii),  $NS\ Cl(NS\ Cl(A)) = NS\ Cl(A)$ . This proves (iii).

Since  $A \subseteq B$ ,  $C(B) \subseteq C(A)$ . By using Proposition 5.2 (iv),  $NS\ Int(C(B)) \subseteq NS\ Int(C(A))$ . Taking complement on both sides,  $C(NS\ Int(C(B))) \supseteq C(NS\ Int(C(A)))$ . By Proposition 6.2 (ii),  $NS\ Cl(A) \subseteq NS\ Cl(B)$ . This proves (iv).



**Proposition 6.4** Let  $A$  be a neutrosophic set in a  $NTS$   $X$ . Then  $NInt(A) \subseteq NSInt(A) \subseteq A \subseteq NSCl(A) \subseteq NCl(A)$ .

**Proof :** It follows from the definitions of corresponding operators.

**Proposition 6.5** Let  $(X, \tau)$  be a  $NTS$ . Then for a neutrosophic subset  $A$  and  $B$  of a  $NTS$   $X$ , we have

- (i)  $NSCl(A \cup B) = NSCl(A) \cup NSCl(B)$  and
- (ii)  $NSCl(A \cap B) \subseteq NSCl(A) \cap NSCl(B)$ .

**Proof :** Since  $NSCl(A \cup B) = NSCl(C(C(A \cup B)))$ , by using Proposition 6.2 (i),  $NSCl(A \cup B) = C(NSInt(C(A \cup B))) = C(NSInt(C(A) \cap C(B)))$ . Again using Proposition 5.3 (i),  $NSCl(A \cup B) = C(NSInt(C(A)) \cap NSInt(C(B))) = C(NSInt(C(A))) \cup C(NSInt(C(B)))$ . By using Proposition 6.2 (i),  $NSCl(A \cup B) = NSCl(C(C(A))) \cup NSCl(C(C(B))) = NSCl(A) \cup NSCl(B)$ . Thus proved (i). Since  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$ , by using Proposition 6.3 (iv),  $NSCl(A \cap B) \subseteq NSCl(A)$  and  $NSCl(A \cap B) \subseteq NSCl(B)$ . This implies that  $NSCl(A \cap B) \subseteq NSCl(A) \cap NSCl(B)$ . This proves(ii).

The following example shows that the equality need not be hold in Proposition 6.5 (ii).

**Example 6.6** Let  $X = \{ a, b, c \}$  with  $\tau = \{ 0_N, A, B, C, D, 1_N \}$  and  $C(\tau) = \{ 1_N, E, F, G, H, 0_N \}$  where  $A = \langle (0.5, 0.6, 0.1), (0.6, 0.7, 0.1), (0.9, 0.5, 0.2) \rangle$   
 $B = \langle (0.4, 0.5, 0.2), (0.8, 0.6, 0.3), (0.9, 0.7, 0.3) \rangle$   
 $C = \langle (0.4, 0.5, 0.2), (0.6, 0.6, 0.3), (0.9, 0.5, 0.3) \rangle$   
 $D = \langle (0.5, 0.6, 0.1), (0.8, 0.7, 0.1), (0.9, 0.7, 0.2) \rangle$   
 $E = \langle (0.1, 0.4, 0.5), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9) \rangle$ ,  
 $F = \langle (0.2, 0.5, 0.4), (0.3, 0.4, 0.8), (0.3, 0.3, 0.9) \rangle$ ,  
 $G = \langle (0.2, 0.5, 0.4), (0.3, 0.4, 0.6), (0.3, 0.5, 0.9) \rangle$ ,  
 $H = \langle (0.1, 0.4, 0.5), (0.1, 0.3, 0.8), (0.2, 0.3, 0.9) \rangle$ .  
 Then  $(X, \tau)$  is a  $NTS$ . Consider the  $NS$ s are  
 $I = \langle (0.1, 0.2, 0.5), (0.2, 0.3, 0.7), (0.3, 0.3, 1) \rangle$   
 and  $J = \langle (0.2, 0.4, 0.8), (0.1, 0.2, 0.8), (0.2, 0.5, 0.9) \rangle$ . Then  $NSCl(I) = G$  and  $NSCl(J) = G$ .

This implies that  $NSCl(I) \cap NSCl(J) = G$ . Now,  $I \cap J = \langle (0.1, 0.2, 0.8), (0.1, 0.2, 0.8), (0.2, 0.3, 1) \rangle$ , it follows that  $NSCl(I \cap J) = H$ . Then  $NSCl(I) \cap NSCl(J) \not\subseteq NSCl(I \cap J)$ .

**Theorem 6.7** If  $A$  and  $B$  are  $NS$ s of  $NTS$ s  $X$  and  $Y$  respectively, then

- (i)  $NSCl(A) \times NSCl(B) \supseteq NSCl(A \times B)$ ,
- (ii)  $NSInt(A) \times NSInt(B) \subseteq NSInt(A \times B)$ .

**Proof :** (i) Since  $A \subseteq NSCl(A)$  and  $B \subseteq NSCl(B)$ , hence  $A \times B \subseteq NSCl(A) \times NSCl(B)$ . This implies

that  $NSCl(A \times B) \subseteq NSCl(NSCl(A) \times NSCl(B))$  and From Proposition 4.12,  $NSCl(A \times B) \subseteq NSCl(A) \times NSCl(B)$ .

(ii) follows from (i) and the fact that  $NSInt(C(A)) = C(NSCl(A))$ .

**Lemma 6.8** For  $NS$ s  $A_i$ 's and  $B_j$ 's of  $NTS$ s  $X$  and  $Y$  respectively, we have

- (i)  $\cap \{ A_i, B_j \} = \min(\cap A_i, \cap B_j)$ ;  
 $\cup \{ A_i, B_j \} = \max(\cup A_i, \cup B_j)$ .
- (ii)  $\cap \{ A_i, 1_N \} = (\cap A_i) \times 1_N$ ;  
 $\cup \{ A_i, 1_N \} = (\cup A_i) \times 1_N$ .
- (iii)  $\cap \{ 1_N \times B_j \} = 1_N \times (\cap B_j)$ ;  
 $\cup \{ 1_N \times B_j \} = 1_N \times (\cup B_j)$ .

**Proof :** Obvious.

**Theorem 6.9** Let  $(X, \tau)$  and  $(Y, \sigma)$  be  $NTS$ s such that  $X$  is neutrosophic product related to  $Y$ . Then for  $NS$ s  $A$  of  $X$  and  $B$  of  $Y$ , we have

- (i)  $NSCl(A \times B) = NSCl(A) \times NSCl(B)$ ,
- (ii)  $NSInt(A \times B) = NSInt(A) \times NSInt(B)$ .

**Proof :** (i) Since  $NSCl(A \times B) \subseteq NSCl(A) \times NSCl(B)$  ( By Theorem 6.7 (i) ) it is sufficient to show that  $NSCl(A \times B) \supseteq NSCl(A) \times NSCl(B)$ . Let  $A_i \in \tau$  and  $B_j \in \sigma$ . Then  $NSCl(A \times B) = \langle (x, y), \cap C(\{ A_i \times B_j \}) : C(\{ A_i \times B_j \}) \supseteq A \times B, \cup \{ A_i \times B_j \} : \{ A_i \times B_j \} \subseteq A \times B \rangle = \langle (x, y), \cap (C(A_i) \times 1_N \cup 1_N \times C(B_j)) : C(A_i) \times 1_N \cup 1_N \times C(B_j) \supseteq A \times B, \cup (A_i \times 1_N \cap 1_N \times B_j) : A_i \times 1_N \cap 1_N \times B_j \subseteq A \times B \rangle = \langle (x, y), \cap (C(A_i) \times 1_N \cup 1_N \times C(B_j)) : C(A_i) \supseteq A \text{ or } C(B_j) \supseteq B, \cup (A_i \times 1_N \cap 1_N \times B_j) : A_i \subseteq A \text{ and } B_j \subseteq B \rangle = \langle (x, y), \min(\cap \{ C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(A_i) \supseteq A \}, \cap \{ C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(B_j) \supseteq B \}) , \max(\cup \{ A_i \times 1_N \cap 1_N \times B_j : A_i \subseteq A \}, \cup \{ A_i \times 1_N \cap 1_N \times B_j : B_j \subseteq B \}) \rangle$ . Since  $\langle (x, y), \cap \{ C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(A_i) \supseteq A \}, \cap \{ C(A_i) \times 1_N \cup 1_N \times C(B_j) : C(B_j) \supseteq B \} \rangle \supseteq \langle (x, y), \cap \{ C(A_i) \times 1_N : C(A_i) \supseteq A \}, \cap \{ 1_N \times C(B_j) : C(B_j) \supseteq B \} \rangle = \langle (x, y), \cap \{ C(A_i) : C(A_i) \supseteq A \} \times 1_N, 1_N \times \cap \{ C(B_j) : C(B_j) \supseteq B \} \rangle = \langle (x, y), NSCl(A) \times 1_N, 1_N \times NSCl(B) \rangle$  and  $\langle (x, y), \cup \{ A_i \times 1_N \cap 1_N \times B_j : A_i \subseteq A \}, \cup \{ A_i \times 1_N \cap 1_N \times B_j : B_j \subseteq B \} \rangle \subseteq \langle (x, y), \cup \{ A_i \times 1_N : A_i \subseteq A \}, \cup \{ 1_N \times B_j : B_j \subseteq B \} \rangle = \langle (x, y), NSInt(A) \times 1_N, 1_N \times NSInt(B) \rangle$ , we have  $NSCl(A \times B) \supseteq \langle (x, y), \min(NSCl(A) \times 1_N, 1_N \times NSCl(B)), \max(NSInt(A) \times 1_N, 1_N \times NSInt(B)) \rangle = \langle (x, y), \min(NSCl(A), NSCl(B)), \max(NSInt(A), NSInt(B)) \rangle = NSCl(A) \times NSCl(B)$ .

(ii) follows from (i).

**Theorem 6.10** Let  $(X, \tau)$  be a  $NTS$ . Then for a neutrosophic subset  $A$  and  $B$  of  $X$  we have,

- (i)  $NS Cl (A) \supseteq A \cup NS Cl (NS Int (A))$ ,
- (ii)  $NS Int (A) \subseteq A \cap NS Int (NS Cl (A))$ ,
- (iii)  $NInt (NS Cl (A)) \subseteq NInt (NCl (A))$ ,
- (iv)  $NInt (NS Cl (A)) \supseteq NInt (NS Cl (NS Int (A)))$ .

**Proof :** By Proposition 6.3 (i),  $A \subseteq NS Cl (A)$  -----

(1). Again using Proposition 5.2 (i),  $NS Int (A) \subseteq A$ . Then  $NS Cl (NS Int (A)) \subseteq NS Cl (A)$  ----- (2). By (1) & (2) we have,  $A \cup NS Cl (NS Int (A)) \subseteq NS Cl (A)$ . This proves (i).

By Proposition 5.2 (i),  $NS Int (A) \subseteq A$  ----- (1). Again using proposition 6.3 (i),  $A \subseteq NS Cl (A)$ . Then  $NS Int (A) \subseteq NS Int (NS Cl (A))$  ----- (2). From (1) & (2), we have  $NS Int (A) \subseteq A \cap NS Int (NS Cl (A))$ . This proves(ii).

By Proposition 6.4,  $NS Cl (A) \subseteq NCl (A)$ . We get  $NInt (NS Cl (A)) \subseteq NInt (NCl (A))$ . Hence (iii).

By (i),  $NS Cl (A) \supseteq A \cup NS Cl (NS Int (A))$ . We have  $NInt (NS Cl (A)) \supseteq NInt (A \cup NS Cl (NS Int (A)))$ . Since  $NInt (A \cup B) \supseteq NInt (A) \cup NInt (B)$ ,  $NInt (NS Cl (A)) \supseteq NInt (A) \cup NInt (NS Cl (NS Int (A))) \supseteq NInt (NS Cl (NS Int (A)))$ . Hence (iv).

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