A New Approach to (1, 2)-j-open sets in Bitopological Spaces

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Abstract

The aim of this present paper is to introduce a new class of set namely (1,2)-j-open, (1,2)-j-closed in bitopological spaces. We investigate the several properties and study their relationship with other existing sets.

Keywords*: (1,2)-j-open sets , (1,2)-j-closed sets*

1 Introduction

Kelly[6] initiated the study of bitopological spaces in 1963. A nonempty set X equipped with two topological spaces τ_1,τ_2 is called a bitopological space and is denoted by (X,τ_1,τ_2). Using the notation of pre open set in 1990 D.Andrijievic and M. Ganster[1] defined the concept of γ-open set in topological spaces. S.N. Maheshwariand R. Prasad[10] extened the notion of semi-open sets and semi-continuity to the bitopological setting in 1977. B.P. Dvalishvli[4] introduced concepts of (1,2)-open domain and (1,2)-boundaries in bitopological space. B. Bhattacharya and A. Paul [2] introduced γ-open set in bitopological spaces and studied their properties. The concept of pre open sets in topological space was initiated by Mashhouret. Al[11]. S. Raychaudhui and M.N. Mukherjee[12] have introduced the notion of δ-preopen sets and δ-almost continuity in topological spaces.The class of δ-preopen sets is larger than that of preopen setsThe purpose of this paper is to define some properties by using (1,2)-j-open, (1,2)-j-closed in bitopological space and analyse the relationships between them.

2 Preliminaries

2.1 Definition

Let X be a non empty set and τ_1,τ_2be the topologies on X. A triple(X,τ_1,τ_2) is said to be a bitopological space**.**

2.2 Definition

A subset A of a bitopological space (X,τ_1,τ_2) is called a (1,2)-semi open if $A \subset cl_2(int_1(A))$and it is (1,2)-semi closed if $cl_2(int_1(A)) \subset A$

2.3 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X$, A is said to be (1,2)-p-open set if $A \subset int_1(cl_2(A))$ and A is (1,2)-p-closed if $X\backslash A$ is (1,2)-p-open

2.4 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be (2,1)-p-open set if $A \subset int_2(cl_1(A))$

2.5 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 1-p-open set if $A \subset int_1(cl_1(A))$

2.6 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 2-p-open set if $A \subset int_2(cl_2(A))$

2.7 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2) then, the union all (1,2)-p-open sets contained in A is called (1,2)-p-int(A)

2.8 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2) then, the intersection all (1,2)-p-closed sets containing in A is called (1,2)-p-cl(A)

2.9 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2) then, A subset N of a bitopological space (X,τ_1,τ_2) is called (1,2)-p- neighbourhood of a subset A of

X if there exists on (1,2)-p-open set U such that $A \subseteq U \subseteq N$

2.10 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2) then, (1,2)-γ-open set if for any non empty (1,2)-p-open set B such that $A \cap B \subseteq int_1(cl_2(A \cap B))$

3 (1,2)-j-open sets

3.1 Definition

Let A be a subset of a bitopological space(X,τ_1,τ_2) then, A is said to be (1,2)-j-open set if A $\subset int_1(Pcl_2(A))$.

3.2 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be (2,1)-j-open set if A $\subset int_2(Pcl_1(A))$.

3.3 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 1-j-open set if A $\subset int_1(Pcl_1(A))$.

3.4 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 2-j-open set if A $\subset int_2(Pcl_2(A))$.

3.5 Definition

Let $X = \{a,b,c\}$,$\tau_1 = \{\phi, X, \{b\}, \{b,c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{a,c\}, \{c\}\}$ then (1,2)-jO(X)=$\{\phi, X, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$ and(2,1)jO(X)={ ϕ,X,{a},{b},{c},{a,b}, {b,c},{a,c}}It is clear that $\{a\}\{a\,,b\}$ are (2,1)-jO(X) but not (1,2)-jO(X)

3.6 Example

Let $X = \{a,b,c\}, \tau_1 = \{\phi, X, \{b\}, \{b,c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{a,c\}, \{c\}\}$then (1,2)-jO(X)=$\{\phi, X, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$ and 1-jO(X) =$\{\phi, X, \{b\}, \{a,b\}, \{b,c\}\}$ It is clear that $\{c\}\{a\,,c\}$ are (1,2)-jO(X) but not 1-jO(X) and $\{a,b\}$ is 1-jO(X) but not (1,2)-jO(X)

3.7 Example

Let $X = \{a,b,c\}$,$\tau_1 = \{\phi, X, \{b\}, \{b,c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{a,c\}, \{c\}\}$ then (1,2)-jO(X)=$\{\phi, X, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$ and 2-jO(X)=$\{\phi, X, \{a\}, \{a,c\}, \{c\}\}$ It is clear that $\{b\}\{b\,,c\}$ are (1,2)-jO(X) but not 2-jO(X) and $\{a\}$ is 2-jO(X) but not (1,2)-jO(X)

3.8 Theorem

Every 1-open sets is (1,2)-j-open

Every 2-open sets is (2,1)-j-open

Proof

Let A be any 1-open set in bitopological space (X,τ_1,τ_2) then $A = int_1(A)$ and also $A \subseteq Pcl_2(A)$.

Therefore,A $\subset int_1(Pcl_2(A))$.

Let A be any 2-open set in bitopological space (X,τ_1,τ_2) then $A = int_2(A)$ and also $A \subseteq Pcl_1(A)$.

Therefore,A $\subset int_2(Pcl_1(A))$.

4 Interior, Closure and neighbourhood in bitopological spaces

4.1 Definition

LetA be a subset of a bitopological space (X,τ_1,τ_2) then, the union all (1,2)-j-open sets contained in A is called (1,2)-j-int(A).

4.2 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2) then, the intersection all (1,2)-j-closed sets containing in A is called (1,2)-j-cl(A).

4.3 Definition

Let A be a subset of a bitopological space (X,τ_1,τ_2). A subset N of a bitopological space (X,τ_1,τ_2) is called (1,2)-j- neighbourhood of a subset A of X if there exists on (1,2)-j-open set U such that $A \subseteq U \subseteq N$

4.4 Theorem

For any subset A & B of a bitopological space X, the following statements are true:

(i) The (1,2)-j-int(A) is the largest (1,2)-j-open set contained in A.

(ii) The (1,2)-j-int(A) is an (1,2)-j-open set in X contained in A.

(iii) A is an (1,2)-j-open set iff A=(1,2)-j-int(A)

(iv) (1,2)-j-int (ϕ)= ϕ

(v) (1,2)-j-int (X)= X

(vi) IfA ⊆ B, the (1,2)-j-int (A) ⊆(1,2)-j-int(B)

(vii) (1,2)-j-int($A \cap B$) ⊆(1,2)-j-int (A)∩(1,2)-j-int(B)

(viii) (1,2)-j-int (A)∪(1,2)-j-int(B) ⊆ (1,2)-j-int($A \cup B$)

Proof

(i) Since (1,2)-j-int(A) = ∪{G:G is (1,2)-j-open, G⊂A}contains every j-open subset G of A. It is therefore largest open subset of A.

(ii) Since (1,2)-j-int(A)is the largest (1,2)-j-open set of A and (1,2)-j-int(A) = ∪{G:G is (1,2)-j-open set of A}so (1,2)-j-int(A) is an (1,2)-j-open set of A.

(iii) Let A=(1,2)-j-int(A) and since(1,2)-j-int(A) is an (1,2)-j-open set of A. So, A is also (1,2)-j-open. LetA is (1,2)-j-open then A is largest (1,2)-j-open set of A.Hence A=(1,2)-j-int(A).

(vi) Let $A \subset B$, Let x∈(1,2)-j-int (A),∃ a (1,2)-j-open set G Such that x∈ $G \subset A$. Since $A \subseteq$ B, B is also have x∈ $G \subset B$which impliesx∈(1,2)-j-int (A). Thus x∈(1,2)-j-int (A) ⇒(1,2)-j-int(B). Therefore, (1,2)-j-int (A) ⊆(1,2)-j-int(B).

(vii) Let x∈(1,2)-j-int($A \cap B$), ∃ a (1,2)-j-open set G in X such that x∈($A \cap B$), x∈ A and x∈ B,x∈ $G \subset A$ and x∈ $G \subset B$.So,x ∈(1,2)-j-int (A) and x ∈(1,2)-j-int (B). Therefore, (1,2)-j-int (A)∩(1,2)-j-int(B).

(viii) Let xϵ(1,2)-j-int (A)∪(1,2)-j-int(B), xϵ(1,2)-j-int (A) or $x\epsilon$(1,2)-j-int(B). If xϵ(1,2)-j-int (A), ∃ (1,2)-j-open set G in X such that x∈ $G \subset A$ ⇒x∈ $G \subset A \cup B$ ⇒x∈ (1,2)-j-int($A \cup B$) Therefore**,** (1,2)-j-int (A)∪(1,2)-j-int(B) ⊆ (1,2)-j-int($A \cup B$).

4.5 Example

(1,2)-j-int(A)∩(1,2)-j-int(B)≠(1,2)-j-int($A \cap B$)

Let X=$\{a,b,c\}$,τ_1=$\{\phi,X,\{b,c\}\}$ $\tau_2 = \{\phi,X,\{a,b\}\}$ then (1,2)-jO(X)=$\{\phi,X,\{a,b\},\{b,c\}\}$. If we take A=$\{a,b\}$ and B=$\{b,c\}$ then (1,2)-j-int(A)=$\{a,b\}$,(1,2)-j-int(B)=$\{b,c\}$ So (1,2)-j-int (A)∩(1,2)-j-int(B)=$\{b\}$,(1,2)-j-int($A \cap B$)=(1,2)-j-int($\{b\}$)=$\{\phi\}$. Hence (1,2)-j-int (A)∩(1,2)-j-int(B) ≠(1,2)-j-int($A \cap B$).

4.6 Example

(1,2)-j-int (A) ∪(1,2)-j-int(B) ≠(1,2)-j-int ($A \cup B$).

LetX={a,b,c}, τ_1={ $\phi,X,\{a\}$} τ_2={ $\phi,X,\{c\},\{a,b\}$} then (1,2)-jO(X)=$\{\phi,X,\{a\}\}$**.** If we take A=$\{a,b\}$ and B=$\{b\}$ then (1,2)-j-int(A)=$\{a\}$,(1,2)-j-int(B)=$\{\phi\}$So (1,2)-j-int (A)∪(1,2)-j-int(B)=$\{a\}$,(1,2)-j-int($A \cup B$)=(1,2)-j-int($\{X\}$)= X.Hence (1,2)-j-int (A)∪(1,2)-j-int(B) ≠(1,2)-j-int($A \cup B$).

4.7 Lemma

For any subset A we have

(i) 1-int (A) ⊆ (1,2)-j-int(A), but (1,2)-j-int(A) ≠1-int (A).

(ii) 2-int (A) ⊆ (2,1)-j-int(A), but (2,1)-j-int(A) ≠2-int (A).

Proof

Follows from the fact that every 1-open set is (1,2)-j-open.

The converse of the above lemma is not true which is shown in the following example:

4.8 Example

Let X={a,b,c}, τ_1={ $\phi,X,\{a\}$} τ_2={ $\phi,X,\{c\},\{a,b\}$} then (1,2)-jO(X) =$\{\phi,X,\{a\}\}$**.** 1-jO(X)={ $\phi,X,\{a\},\{a,b\},\{a,c\}$}. If we take A=$\{a,c\}$ and B = $\{b\}$ then (1,2)-j-int(A)=$\{a\}$,1-j-int(A)=$\{a,c\}$ ⇒(1,2)-j-int(A) ≠1-j-int(A)

5 (1,2)-j-closed sets

5.1 Definition

Let A is said to be (1,2)-j-closed if $int_1(Pcl_2(A)) \subset A$.

5.2 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be (2,1)-j-closed set if $int_2(Pcl_1(A)) \subset A$.

5.3 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 1-j-closed set if $int_1(Pcl_1(A)) \subset A$.

5.4 Definition

Let (X,τ_1,τ_2) be a bitopological space, $A \subset X, A$ is said to be 2-j-closed set if A ⊂ $int_2(Pcl_2(A)) \subset A$

5.5Theorem

For any subset A & B of a bitopological space X, the following statements are true:

(i) The (1,2)-j-cl(A) is the smallest (1,2)-j-closed set containing A.
(ii) A is an (1,2)-j-closed set iff A= (1,2)-j-cl(A)
(iii) (1,2)-j-cl (ϕ)= ϕ
(iv) (1,2)-j-cl(X)= X
(v) A ⊆ (1,2)-j-cl (A)
(vi) IfA ⊆ B, then (1,2)-j-cl (A) ⊆(1,2)-j-cl (B)
(vii) (1,2)-j-cl($A \cap B$) ⊆(1,2)-j-cl (A)∩(1,2)-j-cl (B)
(viii) (1,2)-j-cl (A)∪(1,2)-j-cl (B) ⊆ (1,2)-j-cl ($A \cup B$)

Proof

(i) (1,2)-j-cl (A)is intersection of all(1,2)-j-closed set containing A so (1,2)-j-cl (A) is the smallest (1,2)-j-closed set containing A.

(ii) If A is (1,2)-j-closed set, then A itself is the smallest (1,2)-j-closed set containing A. Hence (1,2)-j-cl(A)=A

Conversely,

If (1,2)-j-cl (A)=A,Since (1,2)-j-cl(A) is (1,2)-j-closed and so A is also (1,2)-j-closed.

(v) Since (1,2)-j-cl (A) is the smallest (1,2)-j-closed set containing A, and A ⊆ (1,2)-j-cl (A)

(vi) Let $A \subset B$, and by the pervious theorem we have B ⊂ (1,2)-j-cl (B).Since $A \subset B$, we have A ⊂ (1,2)-j-cl (B) but (1,2)-j-cl (B) is (1,2)-j-closed set.Thus (1,2)-j-cl (B) is (1,2)-j-closed set containing A.Since (1,2)-j-cl (A) is the smallest (1,2)-j-closed set containing A.We have (1,2)-j-cl (A) ⊆(1,2)-j-cl(B) so, A ⊆ B ⇒ (1,2)-j-cl (A) ⊆(1,2)-j-cl (B)

(vii)A ∩ B⊂A⇒ (1,2)-j-cl ($A \cap B$) ⊂(1,2)-j-cl (A) and A ∩ B⊂ B ⇒ (1,2)-j-cl ($A \cap B$) ⊂(1,2)-j-cl (B). Hence (1,2)-j-cl ($A \cap B$) ⊆(1,2)-j-cl (A)∩(1,2)-j-cl (B)

5.2 Example

(1,2)-j-cl (A) ∩(1,2)-j-cl (B) ≠(1,2)-j-cl ($A \cap B$)

Let X=$\{a,b,c\}$,τ_1=$\{\phi,X,\{b,c\}\}$,$\tau_2 = \{\phi,X,\{a,b\}\}$ then (1,2)-jO(X)=$\{\phi,X,\{a,b\},\{b,c\}\}$, (1,2)-jO(X)=$\{\phi,X,\{a\},\{b\},\{c\},\{a,c\}\}$. If we take A=$\{a,b\}$ and B=$\{b,c\}$ then (1,2)-j-cl (A)= X and (1,2)-j-cl (B)= X. So (1,2)-j-cl (A)∩(1,2)-j-cl (B)=$\{b\}$,(1,2)-j-cl ($A \cap B$)=(1,2)-j-cl ($\{b\}$)=$\{b\}$. Hence (1,2)-j-cl (A)∩(1,2)-j-cl (B) ≠(1,2)-j-cl ($A \cap B$)

5.3 Example

(1,2)-j-cl (A)∪(1,2)-j-cl (B) ≠(1,2)-j-cl ($A \cup B$)

LetX=$\{a,b,c\}$,τ_1=$\{\phi,X,\{b,c\}\}$, $\tau_2 = \{\phi,X,\{a,b\}\}$ then (1,2)-jO(X)=$\{\phi,X,\{a,b\},\{b,c\}\}$, (1,2)-jC(X)=$\{\phi,X,\{a\},\{b\},\{c\},\{a,c\}\}$. If we take A=$\{a\}$ and B=$\{b\}$ then (1,2)-j-cl (A)=$\{a\}$and (1,2)-j-cl (B)=$\{b\}$ so, (1,2)-j-cl (A)∪(1,2)-j-cl (B)=$\{a,b\}$,(1,2)-j-cl ($A \cup B$)=(1,2)-j-cl ($\{a,b\}$)= X. Hence (1,2)-j-cl (A)∪(1,2)-j-cl (B) ≠(1,2)-j-cl ($A \cup B$)

5.4 Proposition

A subset A is (1,2)-j-closed iff $Pcl_1 int_2(A) \subset A$

Proof

Suppose that A is(1,2)-j-closed set in a bitopological space X, the $X \setminus A$ is (1,2)-j-open. Hence $X \setminus A \subset int_1 Pcl_2(X \setminus A)$. But $Pcl_2(X \setminus A) = X \setminus Pint_2(A)$, so $X \setminus A \subset Pint_1(X \setminus A) \subset int_2(A))$. Again $Pint_1(X \setminus int_2(A)) = X \setminus Pcl_1 int_2(A)$. Therefore, we get $X \setminus A \subset X \setminus Pcl_1 int_2(A)$. Taking complement of both sides, we obtain $Pcl_1 int_2(A) \subset A$.

Conversely,

Suppose the $Pcl_1 int_2(A) \subset A$, then by taking complement of both sides we obtain $X \setminus A \subset Pint_1 Pcl_2(X \setminus A)$ which implies that $X \setminus A$ is (1,2)-j-open. Hence A is (1,2)-j-closed.

5.5 Proposition

Every 1-closed subset of a bitopological space (X,τ_1,τ_2) is (1,2)-j-closed

Proof

Let A be 1-closed subset of bitopological space (X,τ_1,τ_2). Then A=$Pcl_1(A)$.

Hence $Pcl_1 int_2(A) \subset A$, so A is (1,2)-j-closed.

5.6 Proposition

Every 2-closed subset of a bitopological space (X,τ_1,τ_2) is (2,1)-j-closed

Proof

Let A be 2-closed subset of bitopological space (X,τ_1,τ_2). Then A=$Pcl_2(A)$.

Hence$Pcl_1 int_2(A) \subset A$, so A is (2,1)-j-closed.

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