Banach fixed point theorem for Contraction Mapping Principle in a Cone b-pentagonal metric spaces

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Abstract

In this paper, we introduce new space cone b-pentagonal metric spaces and use this space to prove Banach fixed point theorem for Contraction Mapping Principle in cone b-pentagonal metric spaces without assuming the normality condition.Our results improve and extend recent known results.

Keywords:Cone b-pentagonal metric space,b-metric space,fixed points, contraction mapping principle,ordered Banach space.

1 Introduction

In many branches of science,economics,computer science,engineering and the development of nonlinear analysis,the fixed point theory is one of the most important tool.

in 1989, Backhtin[9] introduced the concept of b-metric space.In 1993,Czerwik[10] extended the results of b-metric spaces.Using this idea many researcher presented generalization of the renowned banach fixed point theorem in the b-metric spaces.

In 2007,Long-Guang and Xian[6]introduced the concept of a cone metric space,they replaced set of real numbers by an ordered Banach space and proved some fixed point theorems for contractive type conditions in cone metric spaces.Later on many authors have(for e.g;[1,5,8])proved some fixed point theorems for different contractive type conditions in cone metric spaces.

Recently,Azam et al.[3] introduced the notion of cone rectangular metric space and proved Banach contraction mapping principle in a cone rectangular metric space setting.In 2012,Rashwan and Saleh[7] improve and extended the result of Azam et al.[3]by removing the normality condition.

Very recently ,Garg and Agarwal[4]introduced the notionof cone pentagonal metric space and proved Banach contraction mapping principle in a cone pentagonal metric space setting using the normality condition.

Motivated by these results of [4,7],it is our purpose in this paper to continue the study of fixed point theorem in cone pentagonal metric space setting.Our results improve and extend the results 0f [4,7].

We want to extend some well known Banach fixed point theorems which are also valid in cone b-pentagonal metric space.

2 Preliminaries

We present some definitions introduced in [2,3,4,6,7,9,10] which will be needed in the sequel.

Definition 2.1. *Let E be a real Banach space and P subset of E.P is called a cone if and only if:*
(1) P is closed ,nonempty and $P \neq \{0\}$.
(2) $a,b \in R$, $a,b \geq 0$ *and* $x,y \in P \implies ax+by \in P$.
(3) $x \in P$ *and* $-x \in P \implies x=0$.

Definition 2.2. *Given a cone* $P \subseteq E$*,we defined a partial ordering* $\leq$ *with respect to P by* $x \leq y$ *if and only if* $y-x \in P$*.We shall write* $x < y$ *to indicate that* $x \leq y$ *but* $x \neq y$*,while* $x \ll y$ *will stand for* $y-x \in int(P)$*, where int(P) denotes the interior of P.*

Definition 2.3. *A cone P is called normal if there is a number* $k \geq 1$ *such that for all* $x,y \in E$*, the inequality*

$$0 \leq x \leq y \implies \| x \| \leq k \| y \|$$

The least positive number k satisfying (1) is the normal constant of P.

In this paper,we always suppose that E is a real Banach space and P is a cone in E with $int(P) \neq \phi$ and $\leq$ is a partial ordering with respect to P.

Definition 2.4. *Let X be a nonvoid set.Suppose that the mapping* $d : X \times X \to E$ *satisfies:*
(1) $0 < d(x,y)$ *for every* $x,y \in X$ *and* $d(x,y) = 0$ *if and only if* $x = y$.
(2) $d(x,y) = d(y,x)$ *for all* $x,y \in X$.
(3) $d(x,y) \leq d(x,z) + d(z,y)$
Then d is a called a cone metric on X and (X,d) is called a cone metric space.

Remark The concept of a cone metric space is more general than that of a metric space,because each metric space is a cone metric space where E=R and P=$[0,\infty)$ (e.g;see[6]).

Definition 2.5. *Let X be a nonvoid set.Suppose that the mapping $d : X \times X \to E$ satisfies:*
(1) $0 < d(x,y)$ for every x,y $\in X$ and $d(x,y) = 0$ if and only if $x = y$.
(2) $d(x,y) = d(y,x)$ for all x,y $\in X$.
(3) $d(x,y) \leq d(x,w)+d(w,z)+d(z,y)$ for all x,y,z$\in$ X and for all distinct points w,z$\in$ X-$\{x,y\}$[Rectangular property].
Then d is called a cone rectangular metric on X and (X,d) is called a cone rectangular metric space.

Definition 2.6. *Let X be a nonvoid set.Suppose that the mapping $d : X \times X \to E$ satisfies:*
(1) $0 < d(x,y)$ for every x,y $\in X$ and $d(x,y) = 0$ if and only if $x = y$.
(2) $d(x,y) = d(y,x)$ for all x,y $\in X$.
(3) $d(x,y) \leq d(x,z)+d(z,w)+d(w,u)+d(u,y)$ for all x,y,z,w,u$\in$ X and for all distinct points u,w,z$\in$ X-$\{x,y\}$[Penatagonal property].
Then d is called a cone Pentagonal metric on X and (X,d) is called a cone Pentagonal metric space.

Definition 2.7. *Let X be a nonvoid set and s≥ 1 be a given real number.Suppose that the mapping $d : X \times X \to E$ is said to be cone b-metric space then the following conditions are hold:*
(1) $0 < d(x,y)$ for every x,y $\in X$ and $d(x,y) = 0$ if and only if $x = y$.
(2) $d(x,y) = d(y,x)$ for all x,y $\in X$.
(3) $d(x,y) \leq s[d(x,z) + d(z,y)]$
Then d is a called a cone b-metric on X and (X,d) is called a cone b-metric space.

Now we define new space cone b-pentagonal metric space as follows:

Definition 2.8. *Let X be a nonvoid set and s≥ 1 be a given real number.Suppose that the mapping $d : X \times X \to E$ is said to be cone b-pentagonal metric space then the following conditions hold:*
(1) $0 < d(x,y)$ for every x,y $\in X$ and $d(x,y) = 0$ if and only if $x = y$.
(2) $d(x,y) = d(y,x)$ for all x,y $\in X$.
(3) $d(x,y) \leq s[d(x,z) + d(z,w) + d(w,u) + d(u,y)]$ for all x,y,z,w,u$\in$ X and for all distinct points u,w,z$\in$ X-$\{x,y\}$.
Then d is called a cone b-Pentagonal metric on X and (X,d) is called a cone b-Pentagonal metric space.

Definition 2.9. *Let (X,d) be a cone b-pentagonal metric space.Let $\{x_n\}$ be a sequence in (X,d) and x$\in$X.If for every c$\in$E with $0 \ll c$ there exist $n_0 \in N$ and that for every $n > n_0$, $d(x_n,x) \ll c$,then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x and x is the limit of $\{x_n\}$.We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.*

Definition 2.10. *If for every c$\in$ E with $0 \ll c$ there exist $n_0 \in N$ such that for all n>n_0 ,d(x_n,x)$\ll$ c, then $\{x_n\}$ is called Cauchy sequence in(X,d).*

Definition 2.11. *If every Cauchy sequence is convergent in(X,d),then (X,d) is a called a complete cone b-pentagonal metric space.*

Definition 2.12. *Let P be a cone defined as above and let ϕ be the set of non decreasing continuous functions $\varphi : P \rightarrow P$ satisfying:*

(1)0<$\varphi(t)$ <t for every t$\in$P $\backslash\{0\}$.
(2)the series $\Sigma_{n\geq 0}$ $\varphi^n(t)$ converge for all t$\in$P $\backslash\{0\}$.
From (1),we have $\varphi(0) = 0$ and from (2),we have $lim_{n\rightarrow 0}\varphi^n(t)$=0 for all t$\in$P $\backslash\{0\}$.

Lemma 2.1. *Let (X,d) be a cone metric space with cone P not necessary to be normal.Then for a,c,u,v,w $\in$ E, we have:*
(1)If a$\leq$ha and h$\in$[0,1) then a=0.
(2)If 0$\leq$u$\ll$c for each 0$\ll$c,then u=0.
(3)If u$\leq$v and v$\ll$w,then u$\ll$w.

Now,we give the main result of our work which is based on new space cone b-pentagonal metric space.

3 Main Results

Example 1 Let E=R^2, P = $\{(x, y) \in E\ x, y \geq 0\}$, X=[0,4] and $d : X \times X \rightarrow E$ such that d(x,y)=($| x - y |^3, | x - y |^3$).Then,d is a cone b-pentagonal metric space on X.Indeed,d(x,y)$\leq$2[d(x,z)+dd(z,w)+d(w,u)+d(u,y)].
However d(0,4)=(64,64)>(56,56)=2[d(0,1)+d(1,4)]and so d is not a cone b-metric on X.

Theorem 3.1. *Let (X,d) be a complete cone b-Pentagonal metric space.Suppose the mapping T:X$\rightarrow$X satisfy the following:*

$$d(Tx, Ty) \leq \varphi d(x, y), \tag{1}$$

for all x,y$\in$X,where $\varphi \in \Phi$.Then T has a unqiue fixed point in X.

Proof. Let x_0 be an arbitrary point in X.Define a sequence x_n in X such that

x_{n+1}=Tx_n for every n=0,1,2,.....

we assume that $x_n \neq x_{n+1}$ for every n$\in$ N.Then,from(1),it follows that

$$\begin{aligned}
d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) \\
&\leq \varphi d(x_{n-1}, x_n) \\
&= d(Tx_{n-2}, Tx_{n-1}) \\
&\leq \varphi^2 d(x_{n-2}, x_{n-1}) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\leq \varphi^n d(x_0, x_1). \qquad (2)
\end{aligned}$$

It again follows that

$$\begin{aligned}
d(x_n, x_{n+2}) &= d(Tx_{n-1}, Tx_{n+1}) \\
&\leq \varphi d(x_{n-1}, x_{n+1}) \\
&= d(Tx_{n-2}, Tx_n) \\
&\leq \varphi^2 d(x_{n-2}, x_n) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\leq \varphi^n d(x_0, x_2). \qquad (3)
\end{aligned}$$

It further follows that

$$\begin{aligned}
d(x_n, x_{n+3}) &= d(Tx_{n-1}, Tx_{n+2}) \\
&\leq \varphi d(x_{n-1}, x_{n+2}) \\
&= d(Tx_{n-2}, Tx_{n+1}) \\
&\leq \varphi^2 d(x_{n-2}, x_{n+1}) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\leq \varphi^n d(x_0, x_3). \qquad (4)
\end{aligned}$$

Similarly,for k=1,2,3....,we get

$$d(x_n, x_{n+3k+1}) \leq \varphi^n d(x_0, x_{3k+1}) \qquad (5)$$
$$d(x_n, x_{n+3k+2}) \leq \varphi^n d(x_0, x_{3k+2}) \qquad (6)$$
$$d(x_n, x_{n+3k+3}) \leq \varphi^n d(x_0, x_{3k+3}) \qquad (7)$$

By using(2)and cone b-pentagonal property,we have

$$\begin{aligned} d(x_0,x_4) &\leq s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_4)] \\ &\leq s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1))+\varphi^3(d(x_0,x_1))] \\ &\leq s[\Sigma_{i=0}^{3}\varphi^i(d(x_0,x_1))] \end{aligned}$$

Similarly,

$$\begin{aligned} d(x_0,x_7) &\leq s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_4)+d(x_4,x_5)+d(x_5,x_6)+ \\ &\quad d(x_6,x_7)] \\ &\leq s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1))+\varphi^3(d(x_0,x_1))+\varphi^4(d(x_0,x_1))+ \\ &\quad \varphi^5(d(x_0,x_1))+\varphi^6(d(x_0,x_1))] \\ &\leq s[\Sigma_{i=0}^{6}\varphi^i(d(x_0,x_1))] \end{aligned}$$

Now,by induction,we obtain for each k=1,2,3,....

$$d(x_0,x_{3k+1}) \leq s[\Sigma_{i=0}^{3k}\varphi^i(d(x_0,x_1))] \tag{8}$$

Also,by using (2),(3)and b-pentagonal property,we have

$$\begin{aligned} d(x_0,x_5) &\leq s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_5)] \\ &\leq s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1))+\varphi^3(d(x_0,x_2))] \\ &\leq s[\Sigma_{i=0}^{2}\varphi^i(d(x_0,x_1))+\varphi^3(d(x_0,x_2))] \end{aligned}$$

Similarly,

$$\begin{aligned} d(x_0,x_8) &\leq s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_4) \\ &\quad +d(x_4,x_5)+d(x_5,x_6)+d(x_6,x_8)] \\ &\leq s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1)) \\ &\quad +\varphi^3(d(x_0,x_1))+\varphi^4(d(x_0,x_1)) \\ &\quad +\varphi^5(d(x_0,x_1))+\varphi^6(d(x_0,x_2))] \\ &\leq s[\Sigma_{i=0}^{5}\varphi^i(d(x_0,x_1))+\varphi^6(d(x_0,x_2))] \end{aligned}$$

By induction,we obtain for each k=1,2,3....

$$d(x_0,x_{3k+2}) \leq s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1))+\varphi^{3k}(d(x_0,x_2))] \tag{9}$$

Again,by using (2),(4)and b-pentagonal property,we have

$$\begin{aligned} d(x_0,x_6) &\leq s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_6)] \\ &\leq s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1))+\varphi^3(d(x_0,x_3))] \\ &\leq s[\Sigma_{i=0}^{2}\varphi^i(d(x_0,x_1))+\varphi^3(d(x_0,x_3))] \end{aligned}$$

similarly,

$$\begin{aligned} d(x_0,x_9) &\le s[d(x_0,x_1)+d(x_1,x_2)+d(x_2,x_3)+d(x_3,x_4) \\ &+d(x_4,x_5)+d(x_5,x_6)+d(x_6,x_9)] \\ &\le s[d(x_0,x_1)+\varphi(d(x_0,x_1))+\varphi^2(d(x_0,x_1)) \\ &+\varphi^3(d(x_0,x_1))+\varphi^4(d(x_0,x_1)) \\ &+\varphi^5(d(x_0,x_1))+\varphi^6(d(x_0,x_3))] \\ &\le s[\Sigma_{i=0}^{5}\varphi^i(d(x_0,x_1))+\varphi^6(d(x_0,x_3))] \end{aligned}$$

By induction,we obtain for each k=1,2,3...

$$d(x_0,x_{3k+3}) \le s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1))+\varphi^{3k}(d(x_0,x_3))] \tag{10}$$

Using inequality(5)and(8)for k=1,2,3... we have
$d(x_n,x_{n+3k+1}) \le \varphi^n[d(x_0,x_{3k+1})]$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k}\varphi^i(d(x_0,x_1))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]] \tag{11}$$

Similarly for k=1,2,3..., inequalities(6)and(9)implies that
$d(x_n,x_{n+3k+2}) \le \varphi^n(d(x_0,x_{3k+2}))$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1))+\varphi^{3k}(d(x_0,x_2))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))+\varphi^{3k}((d(x_0,x_1)+ d(x_0,x_2))+d(x_0,x_3))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]]$$
$$\le \varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]] \tag{12}$$

Again for k=1,2,3,..., inequalities(7)and(10)implies that
$d(x_n,x_{n+3k+3}) \le \varphi^n(d(x_0,x_{3k+3}))$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1))+\varphi^{3k}(d(x_0,x_3))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k-1}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))+\varphi^{3k}((d(x_0,x_1)+ d(x_0,x_2))+d(x_0,x_3))]]$$

$$\le \varphi^n[s[\Sigma_{i=0}^{3k}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]]$$
$$\le \varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0,x_1)+d(x_0,x_2)+d(x_0,x_3))]] \tag{13}$$

Therefore,by inequality(11),(12)and(13)we have,for each m,

$$d(x_n, x_{n+m}) \leq \varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0, x_1) + d(x_0, x_2) + d(x_0, x_3))]] \qquad (14)$$

Since$[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0, x_1)+d(x_0, x_2)+d(x_0, x_3))]]$ converges(by definition2.12),where $d(x_0, x_1)+d(x_0, x_2)+d(x_0, x_3) \in P\backslash\{0\}$ and P is closed then $[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0, x_1)+ d(x_0, x_2) + d(x_0, x_3))]] \in P\backslash\{0\}$.Hence

$$lim_{n\to\infty}\varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0, x_1) + d(x_0, x_2) + d(x_0, x_3))]] = 0.$$

Then,for given c$\gg$ 0,there is a natural number N_1 such that

$$\varphi^n[s[\Sigma_{i=0}^{\infty}\varphi^i(d(x_0, x_1) + d(x_0, x_2) + d(x_0, x_3))]] \ll c, \forall n \geq N_1 \qquad (15)$$

Thus,from(14)and(15),we have

$$d(x_n, x_{n+m}) \ll c, \forall n \geq N_1$$

Therefore,x_n is a Cauchy sequence in X.Since X is complete,there exists a point z$\in$X such that $lim_{n\to\infty}x_n = lim_{n\to\infty}Tx_{n-1} = z$ as $n \to \infty$.

We show that Tz=z.Given c$\gg$0,we choose a natural numbers N_2,N_3,N_4 such that $d(z, x_n) \ll \frac{c}{4}$ $\forall n \geq N_2$,$d(x_{n+1}, x_n) \ll \frac{c}{4}$ $\forall n \geq N_3$ and $d(x_{n-1}, z) \ll \frac{c}{4}$ $\forall n \geq N_4$.Since $x_n \neq x_m$ for n$\neq$m,therefore by b-pentagonal property,we have

$$\begin{aligned} d(Tz, z) &\leq d(Tz, Tx_n) + d(Tx_n, Tx_{n-1}) + d(Tx_{n-1}, Tx_{n-2}) + d(Tx_{n-2}, z) \\ &\leq \varphi(d(z, x_n) + d(x_{n+1}, x_n + d(x_n, x_{n-1} + d(x_{n-1}, z))) \end{aligned}$$

$$< d(z, x_n) + d(x_{n+1}, x_n) + d(x_n, x_{n-1}) + d(x_{n-1}, z) \qquad (16)$$

Hence,from(16),

$$d(Tz, z) \ll \frac{c}{4} + \frac{c}{4} + \frac{c}{4} + \frac{c}{4} = c \ \forall n \geq N$$

where $N := max\{N_2, N_3, N_4\}$. Since c is arbitrary we have $d(Tz, z) \ll \frac{c}{m}$ $\forall m \in N$.Since $\frac{c}{m} \to 0$ as $m \to \infty$,we conclude $\frac{c}{m}$-d(Tz,z)$\to$-d(Tz,z) as m$\to \infty$.Since P is closed,-d(Tz,z)$\in$P.Hence d(Tz,z)$\in$ P $\cap$-P by definition of cone we get that d(Tz,z)=0 and so Tz = z.Therefore,T has a fixed point that is z in X.
Next we show that z is unique.For suppose z' be another fixed point of T such that Tz' = z'.Therefore,

d(z,z')=d(Tz,Sz')$\leq$ φ(d(z,z'))<d(z,z').

Hence z = z'.This completes the proof of the theorem.

□

Corollary 3.2. *Let (X,d)be a complete cone b-pentagonal metric space.Suppose the mapping $T : X \to X$ satisfy the following:*

$$d(T^m x, T^m y) \leq \varphi d(x, y), \tag{17}$$

for every x,y $\in$ X,where $\varphi \in \Phi$.Then T has a unique fixed point in X.

Proof. From Theorem 3.1,we conclude that T^m has a fixed point say z.Hence

$$Tz = T(T^m z) = T^{m+1} z = T^m(Tz). \tag{18}$$

Then Tz is also a fixed point to T^m.By uniqueness of z,we have Tz=z. □

Corollary 3.3. *(see[4])Let (X,d)be a complete cone b-pentagonal metric space. Suppose the mapping $T : X \to X$ Satisfy the following:*

$$d(Tx, Ty) \leq \lambda d(x, y), \tag{19}$$

for every x,y $\in$ X, where $\lambda \in$ [0,1).Then T has a unique fixed point in X.

Proof. Define $\Phi : P \to P$by φ(t)=λt.Then it is clear that φ satisfies the conditions in defintions2.12.Hence the results follows from theorem3.1 □

References

[1] M.Abbas and G.Jungck, *Common fixed point results for non commuting mappings without continuity in cone metric spaces,Journal of Mathematical Analysis and Applications* 341(2008),416-420.

[2] A.Auwalu, *A new general Iterative method for an infinite family of Nonexpansive Mappings in Hilbert spaces,International Journal of Modern Mathematical Sciences,*4(1)(2012),1-20.

[3] A.Azam,M.Arshad and I.Beg, *Banach Contraction Principle on Cone Rectangular Metric Space,Applicable Analysis and Discrete Mathematics,*3(2)(2009),236-241.

[4] M.Garg and S.Agarwal, *Banach Contraction Principle on Cone Pentagonal Metric Space,Journal of Advanced Studied in Topology,*3(1)(2012),12-18.

[5] D.Hie and V.Rakocevic, *Common fixed Points for Maps on Cone Metric Space,Journal Of Mathematical Analysis and Applications,*341(2008),876-882.

[6] L.G.Huang and X.Zhang, *Cone metric spaces and fixed point theorems of contractive mappings,Journal Of Mathematical Analysis and Applications*332(2)(2007),1468-1476.

[7] R.A.Rashwan and S.M.saleh, *Some fixed point theorems in cone rectangular metric spaces,Mathematica Aeterna,*2(6)(2012),573-587.

[8] S.Rezapour and R.Hamlbarani, *Some notes on paper cone metric spaces and fixed point theorems of contractive mappings,Journal of Mathematical Analysis And Applications,*347(2)(2008),719-724.

[9] Bakhtin,I.A, *The contraction mapping principle in almost metric spaces,Functional Analysis;30,Unianowsk,Gos.Ped.Inst;*(1989),26-37.

[10] Czerwik,S; *Contraction mappings in b-metric spaces,Act Mathematica et Informatica Universitatis Ostraviensis* 1(1993),5-11.