Cordial and total magic cordial labelings for the Extended duplicate graph of Arrow Graph

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Abstract: *In this paper , we prove that the extended duplicate graph of arrow graph is cordial, total cordial and total magic cordial.*

AMS Subject Classification **:** 05C78

Keywords: *Graph labeling, Cordial labeling, Total magic cordial labeling, Arrow graph.*

1.INTRODUCTION:

Graph theory is the fast growing area of combinatorics. Graph labeling is one of the major research area in graph theory. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labeling used in many applications like coding theory, radar circuit design, communication network and X ray etc.,. Various papers based on graph labeling have been studied in over 2100 papers [1]. Several researchers refer to Rosa's [2] work

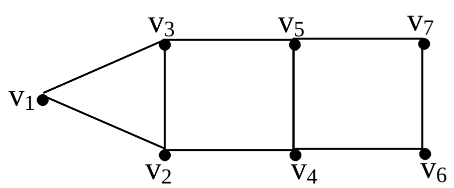
One of the most famous labeling of graph theory is cordial labeling. This labeling was introduced by Cahit in the year 1987 [3]. Cahit has introduced the concept of total magic cordial graph.The concept of duplicate graph was introduced by E. Sampath kumar and he proved many results on it [4] .K.Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of a path graph P_m is cordial [5]. V.J.Kaneria, M.M.Jariya and H.M.Makadia have proved the gracefulness of arrow graphs and double arrow graphs [7]. N.Deepa and R.Sridevi have proved the total 3-sum cordial labeling of arrow graphs and double arrow graphs [8]. K.Thirusangu, B.Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial[6]. B.Selvam and K.Thirusangu have proved that the extended duplicate graph of twig graphs is total magic cordial [9].

2. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. Let G=G(V,E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers).

Definition 2.1 ARROW GRAPH: An arrow graph A_m^n with width 'n' and length 'm' is obtained by joining a vertex 'v' with superior vertices of $P_t \times P_m$ by ' t' new edges from one end. Clearly it has 2m+1 vertices and 3m edges.

Illustration 1: ARROW GRAPH (A_3^2 ***)***



Definition 2.2 DUPLICATE GRAPH: Let G (V,E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1) ,where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge

set E_1 of DG is defined as the edge *ab* is in E if and only if both *ab′* and *a′b* are edges in E_1.

Definition2.3 EXTENDED DUPLICATE GRAPH OF ARROW GRAPH: Let DG = (V_1,E_1) be a duplicate graph of the arrow graph G(V,E). Extended duplicate graph of arrow graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG (A_m^2). Clearly it has 4m+2 vertices and 6m+1 edges, where 'm' is the number of length.

Illustration 2: EXTENDED DUPLICATE GRAPH OF ARROW GRAPH (A_3^2)

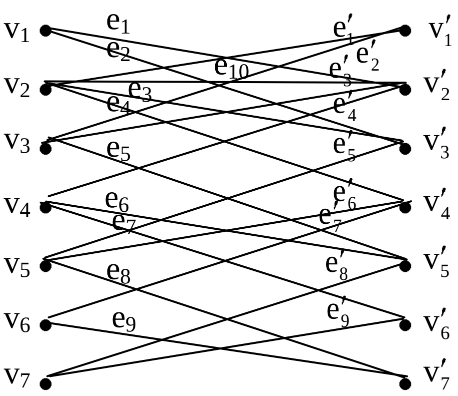


Fig : EDG (A_3^2)

Definition 2.4 CORDIAL LABELING:

A function $f : V \to \{0,1\}$ such that each edge *uv* receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one, and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

Definition 2.5 TOTAL CORDIAL LABELING:

A function $f : V \to \{0,1\}$ such that each edge uv receives the label $| f(u) - f(v)|$ is said to be total cordial labeling if the number of vertices and edges labeled '0' and the number of vertices and edges labeled '1' differ by at most one.

Definition 2.6 TOTAL MAGIC CORDIAL LABELING: A graph G(V,E) is said to admit total total magic cordial labeling if $f : V \cup E \to \{0, 1\}$ such that

(i) $\{f(x) + f(y) + f(xy)\}$ (mod 2) is constant for all edges $xy \in E$.

(ii) $|f(0) - f(1)| \leq 1$, where

a) $f(0)$ denotes the sum of the number of the vertices labeled with ' 0' and the number of edges labeled with ' 0' .

b) $f(1)$ denotes the sum of the number of the vertices labeled with '1' and the number of edges labeled with '1' .

3. MAIN RESULTS

3.1 CORDIAL LABELING

In this section, we present an algorithm and prove the existence of cordial labeling for the extended duplicate graph of arrow graph A_m^2 , $m \geq 2$.

ALGORITHM: 3.1

Procedure[Cordial labeling for EDG (A_m^2),m≥2]

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v_{2m+1}, v'_1, v'_2, \ldots, v'_{2m}, v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m}, e_{3m+1}, e_1', e_2', \ldots, e_{3m}'\}$

$v_1 \leftarrow 1$, $v'_1 \leftarrow 0$

For i = 0 to (m-1) **do**

$v_{2+2i} \leftarrow 1$

$v_{3+2i} \leftarrow 0$

end for

for i=0 to [(m-1)/2] **do**

$v'_{2+4i} \leftarrow 0$

$v'_{3+4i} \leftarrow 1$

end for

for i = 0 to [(m-2)/2] **do**

$v'_{4+4i} \leftarrow 1$

$v'_{5+4i} \leftarrow 0$

end for

end procedure

THEOREM 3.1 **:** The extended duplicate graph of arrow graph A^2_m , $m \geq 2$ is cordial labeling.

Proof: Let A^2_m , $m \geq 2$ be a arrow graph . In order to label the vertices, define a function $f : V \rightarrow \{0,1\}$ as given in algorithm 3.1. The vertices v_1 and v'_1 receive label '1' and ' 0' respectively ; the vertices v_{2+2i} receive label '1' and the vertices v_{3+2i} receive label '0' for $0 \leq i \leq (m-1)$; the vertices v'_{2+4i} receive label '0' and the vertices v'_{3+4i} receive label '1' for $0 \leq i \leq [(m-1)/2]$; the vertices v'_{4+4i} receive label '1' and the vertices v'_{5+4i} receive label '0' for $0 \leq i \leq [(m-2)/2]$. Thus all the 4m+2 vertices namely the vertices v_1 and v'_1 receive label '1' and ' 0' respectively ; the vertices $v_2, v_4, v_6, v_8, v_{10}, \ldots, v_{2m}$ receive label '1'; the vertices $v_3, v_5, v_7, v_9, v_{11}, v_{13}, \ldots, v_{2m+1}$ receive label '0' ;

when m is odd , the vertices $v'_2, v'_6, v'_{10}, v'_{14}, v'_{18}, \ldots, v'_{2m}$ receive label '0' ; the vertices $v'_3, v'_7, v'_{11}, v'_{15}, v'_{19}, \ldots, v'_{2m+1}$ receive label '1' ; the vertices $v'_4, v'_8, v'_{12}, v'_{16}, v'_{20}, \ldots, v'_{2m-2}$ receive label '1' and the vertices $v'_5, v'_9, v'_{13}, v'_{17}, \ldots, v'_{2m-1}$ receive label '0';

when 'm' is even , $v'_2, v'_6, v'_{10}, v'_{14}, v'_{18}, \ldots, v'_{2m-2}$ receive label '0' ; the vertices $v'_3, v'_7, v'_{11}, v'_{15}, v'_{19}, \ldots, v'_{2m-1}$ receive label '1' ; the vertices $v'_4, v'_8, v'_{12}, v'_{16}, v'_{20}, \ldots, v'_{2m}$ receive label '1' and the vertices $v'_5, v'_9, v'_{13}, v'_{17}, \ldots, v'_{2m+1}$ receive label '0'.

Hence the entire 4m+2 vertices are labeled such that the number of vertices labeled '0' is 2m+1 and the number of vertices labeled '1' is 2m+1 , which differ by atmost one and satisfies the required condition.

To obtain the labels for edges , we define the induced function $f^* : E \rightarrow \{0, 1\}$ such that

$$f^*(v_i v_j) = | f(v_i) - f(v_j) | \text{ where } v_i, v_j \in V$$

The induced function yields the label '1' for the edges e_1, e'_1 and e_{3m+1} **;** label '0' for the edges e_2, e_3, e'_2 and e'_3 **;** label '0' for the edges e_{4+j+6i} for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$ **;** label '1' for the edges e_{6+6i} for $0 \leq i \leq [(m-2)/2]$ **;** label '1' for the edges e_{7+j+6i} for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$ **;** label '0' for the edges e_{9+6i} for $0 \leq i \leq [(m-3)/2]$**;** label '1' for the edges e'_{4+j+6i} for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 2$ **;** label '0' for the edges e'_{7+j+6i} for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 2$.

Thus the entire 6m+1 edges are labeled in such a way that when 'm' is odd , 3m+1 edges namely edges e_2, e'_2 and e'_3 receive label '0' ; edges $e_3, e_9, e_{15}, e_{21}, \ldots, e_{3m}$ receive label '0' ; edges $e_4, e_5, e_{10}, e_{11}, e_{16}, e_{17}, \ldots, e_{3m-5}, e_{3m-4}$ receive label '0' ; edges $e'_7, e'_8, e'_9, e'_{13}, e'_{14}, e'_{15}, \ldots, e'_{3m-2}, e'_{3m-1}, e'_{3m}$ receive label '0' and 3m edges namely edges e_1, e'_1 and e_{3m+4} receive label '1'; edges $e_6, e_{12}, e_{18}, \ldots, e_{3m-3}$ receive label '1'; edges $e_7, e_8, e_{13}, e_{14}, \ldots, e_{3m-2}, e_{3m-1}$ receive label '1'; edges $e'_4, e'_5, e'_6, e'_{10}, e'_{11}, e'_{12}, \ldots, e'_{3m-5}, e'_{3m-4}, e'_{3m-3}$ receive label '1' .

when 'm' is even , 3m edges namely edges e_2, e'_2 and e'_3 receive label '0' ; edges $e_3, e_9, e_{15}, e_{21}, \ldots, e_{3m-3}$ receive label '0' ; edges $e_4, e_5, e_{10}, e_{11}, e_{16}, e_{17}, \ldots, e_{3m-2}, e_{3m-1}$ receive label '0' ; edges $e'_7, e'_8, e'_9, e'_{13}, e'_{14}, e'_{15}, \ldots, e'_{3m-5}, e'_{3m-4}, e'_{3m-3}$ receive label '0' and 3m+1 edges namely edges e_1, e'_1 and e_{3m+4} receive label '1'; edges $e_6, e_{12}, e_{18}, \ldots, e_{3m}$ receive label '1'; edges $e_7, e_8, e_{13}, e_{14}, \ldots, e_{3m-5}, e_{3m-4}$ receive label '1'; edges e'_4,

$e'_5, e'_6, e'_{10}, e'_{11}, e'_{12}, e'_{16}, e'_{17}, e'_{18}, \ldots, e'_{3m-2}, e'_{3m-1}, e'_{3m}$ receive label '1' .

Hence all the 6m+1 edges are labeled such that when 'm' is odd , 3m+1 edges receive label '0' and 3m edges receive label '1' and when 'm' is even , 3m edges receive label '0' and 3m+1 edges receive label '1' which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of arrow graph A_m^2 , $m \geq 2$ is cordial labeling.

THEOREM 3.2 **:**The extended duplicate graph of arrow graph A_m^2 , $m \geq 2$ is total cordial.

Proof: In theorem 3.1, (2m+1) vertices were assigned the label '0 ', (2m+1) vertices were assigned the label '1' and it has been proved that when 'm' is odd, the number of edges labeled '0' is 3m and the number of edges labeled '1' is (3m+1) and when 'm' is even, the number of edges labeled '0' is (3m+1) and the number of edges labeled '1' is 3m. From this we conclude that, when 'm' is odd, the number of vertices and edges labeled '0' is (2m+1) + 3m = 5m+1 and the number of vertices and edges labeled '1' is (2m+1) + (3m+1) = 5m+2 and when 'm' is even , the number of vertices and edges labeled '0' is (2m+1) + (3m+1) = 5m+2 and the number of vertices and edges labeled '1' is (2m+1) + 3m = 5m+1, which differ by atmost one and satisfies the required condition.

Hence the extended duplicate graph of arrow graph A_m^2 , $m \geq 2$ is total cordial.

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Illustration 3: Cordial labeling for the graphs **EDG(** A_5^2 **)** and **EDG(** A_6^2 **)**

CORDIAL LABELING FOR THE EXTENDED DUPLICATE OF ARROW GRAPH

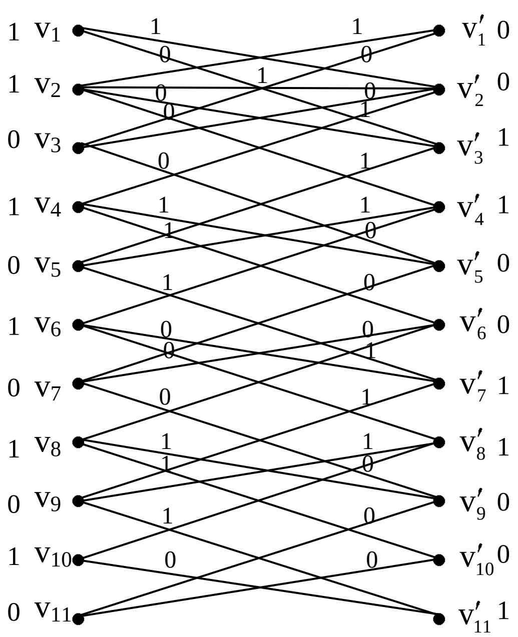

Fig: EDG (A_5^2 ***)***

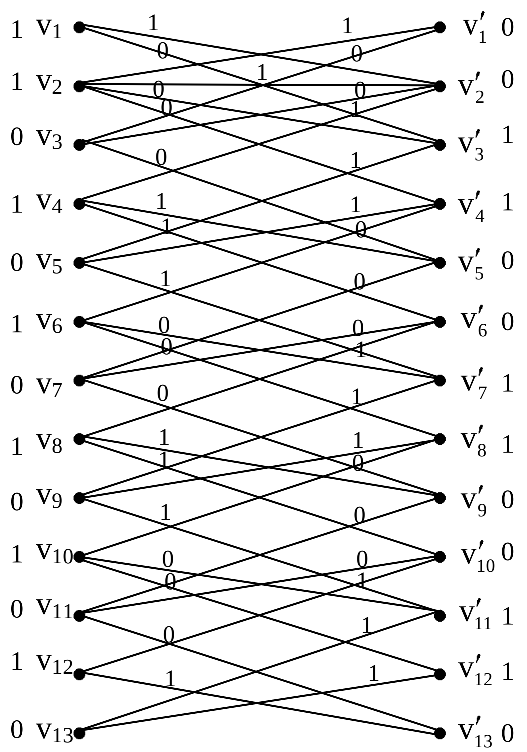

Fig: EDG (A_6^2 ***)***

3.2 TOTAL MAGIC CORDIAL LABELING

ALGORITHM: 3.2

PROCEDURE (Total magic cordial labeling for EDG (A_m^2), m ≥ 2)

V ← { $v_1, v_2, \ldots, v_{2m}, v_{2m+1}, v'_1, v'_2, \ldots, v'_{2m}, v'_{2m+1}$ }

E ← { $e_1, e_2, \ldots, e_{3m}, e_{3m+1}, e'_1, e'_2, \ldots, e'_{3m}$ }

$v_1 \leftarrow 0$, $v_2 \leftarrow 0$, $v_3 \leftarrow 1$, $v'_1 \leftarrow 1$,

$v'_2 \leftarrow 1$, $v'_3 \leftarrow 0$

$e_1 \leftarrow 1$, $e_2 \leftarrow 0$, $e_3 \leftarrow 0$, $e_{3m+1} \leftarrow 1$,

$e'_1 \leftarrow 1$, $e'_2 \leftarrow 0$, $e'_3 \leftarrow 0$

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for i=0 to [(m-1)/2]  do
        v_{4+4i}  ← 1
        v_{5+4i}  ← 0
end for

for i = 0 to [(m-2)/2]  do
        v_{6+4i}  ← 0
        v_{7+4i}  ← 1
end for

for i=0 to (m-1)  do
        v'_{4+2i}  ← 1
        v'_{5+2i}  ← 0
end for

for i=0 to [(m-1)/2]  do
        for  j=0 to 2  do
            e_{4+j+6i}  ← 1
        end for
end for

for i=0 to [(m-2)/2] do
        for  j=0 to 2  do
            e_{7+j+6i}  ← 1
        end for
end for

for i=0 to [(m-1)/2] do
        for  j=0 to1  do
            e'_{4+j+6i}  ← 1
        end for
end for

for i=0 to [(m-1)/2] do
            e'_{6+6i}  ← 1
end for

for i=0 to [(m-2)/2] do
        for  j=0 to 1  do
            e'_{7+j+6i}  ← 1
        end for
end for

for i=0 to [(m-2)/2] do
            e'_{9+6i}  ← 1
        end for

end for
end procedure
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THEOREM 3.3 : The extended duplicate graph of arrow graph A_m^2 , m ≥ 2 is total magic cordial .

Proof: Let A_m^2 , m ≥ 2 be a arrow graph . In order to label the vertices and edges , define a function f : VUE→{0,1} as given in algorithm 3.2.

The vertices v_1 , v_2 and v'_3 receive label '0' ; the vertices v_3 , v'_1 and v'_2 receive label '1' ; the vertices v_{4+4i} receive label '1' and the vertices v_{5+4i} receive label '0' for 0 ≤ i ≤ [(m-1)/2] ; the vertices v_{6+4i} receive label '0' and the vertices v_{7+4i} receive label '1' for 0 ≤ i ≤ [(m-2)/2] ; the vertices v'_{4+2i} receive label '1' and the vertices v'_{5+2i} receive label '0' for 0 ≤ i ≤ (m-1) .

Thus when 'm' is odd , the vertices $v_1, v_2, v_6, v_{10}, v_{14}, \ldots, v_{2m}$ receive label '0' ; the vertices $v_5, v_9, v_{13}, v_{17}, v_{21}, \ldots, v_{2m-1}$ receive label '0' ; the vertices $v'_3, v'_7, v'_{11}, v'_{15}, v'_{19}, \ldots, v'_{2m+1}$ receive label '0' ; the vertices $v'_5, v'_9, v'_{13}, v'_{17}, v'_{21}, \ldots, v'_{2m-1}$ receive label '0' ; the vertices $v_3, v_7, v_{11}, v_{15}, v_{19}, \ldots, v_{2m+1}$ receive label '1' ; the vertices $v_4, v_8, v_{12}, v_{16}, v_{20}, \ldots, v_{2m-2}$ receive label '1' ; the vertices $v'_1, v'_2, v'_6, v'_{10}, v'_{14}, \ldots, v'_{2m}$ receive label '1' ; the vertices $v'_4, v'_8, v'_{12}, v'_{16}, \ldots, v'_{2m-2}$ receive label '1' and

when 'm' is even , the vertices $v_1, v_2, v_6, v_{10}, v_{14}, \ldots, v_{2m-2}$ receive label '0' ; the vertices $v_5, v_9, v_{13}, v_{17}, v_{21}, \ldots, v_{2m+1}$ receive label '0' ; the vertices $v'_3, v'_7, v'_{11}, v'_{15}, v'_{19}, \ldots, v'_{2m-1}$ receive label '0' ; the vertices $v'_5, v'_9, v'_{13}, v'_{17}, v'_{21}, \ldots, v'_{2m+1}$ receive label '0' ; the vertices $v_3, v_7, v_{11}, v_{15}, v_{19}, \ldots, v_{2m-1}$ receive label '1' ; the vertices $v_4, v_8, v_{12}, v_{16}, v_{20}, \ldots, v_{2m}$ receive label '1' ; the vertices $v'_1, v'_2, v'_6, v'_{10}, v'_{14}, \ldots, v'_{2m-2}$ receive label '1' ; the vertices $v'_4, v'_8, v'_{12}, v'_{16}, \ldots, v'_{2m}$ receive label '1' .

The edges e_1, e'_1 and e_{3m+1} receive label '1' ; the edges e_2 , e_3 , e'_2 and e'_3 receive label '0' ; the edges e_{4+j+6i} receive label '1' for $0 \le i \le [(m-1)/2]$ and $0 \le j \le 2$; the edges e_{7+j+6i} receive label '1' for $0 \le i \le [(m-2)/2]$ and $0 \le j \le 2$; the edges e'_{4+j+6i} receive label '0' for $0 \le i \le [(m-1)/2]$ and $0 \le j \le 1$; the edges e'_{6+6i} receive label '1' for $0 \le i \le [(m-1)/2]$; the edges e'_{7+j+6i} receive label '1' for $0 \le i \le [(m-2)/2]$ and $0 \le j \le 1$; the edges e'_{9+6i} receive label '0' for $0 \le i \le [(m-2)/2]$.

Thus when 'm' is odd , the edges $e_1, e_4, e_5, e_6, e_{10}, e_{11}, e_{12}, \ldots, e_{3m-5}, e_{3m-4}, e_{3m-3}$ receive label '1' ; the edges $e'_1, e'_6, e'_{12}, e'_{18}, \ldots, e'_{3m-3}$, receive label '1' ; the edges $e'_2, e'_3, e'_4, e'_5, e'_{10}, e'_{11}, e'_{16}, e'_{17}, \ldots, e'_{3m-5}, e'_{3m-4}$ receive label '0' ; the edges $e_2, e_3, e_7, e_8, e_9, e_{13}, e_{14}, e_{15}, \ldots, e_{3m-2}, e_{3m-1}, e_{3m}$ receive label '0' ; the edges $e'_7, e'_8, e'_{13}, e'_{14}, \ldots, e'_{3m-2}, e'_{3m-1}$ receive label '1' ; the edges $e'_9, e'_{15}, e'_{21}, \ldots, e'_{3m}$ receive label '0'.

when 'm' is even , the edges $e_1, e_4, e_5, e_6, e_{10}, e_{11}, e_{12}, \ldots, e_{3m-2}, e_{3m-1}, e_{3m}$ receive label '1' ; the edges $e'_1, e'_6, e'_{12}, e'_{18}, \ldots, e'_{3m}$ receive label '1' ; the edges $e'_2, e'_3, e'_4, e'_5, e'_{10}, e'_{11}, e'_{16}, e'_{17}, \ldots, e'_{3m-2}, e'_{3m-1}$ receive label '0' ; the edges $e_2, e_3, e_7, e_8, e_9, e_{13}, e_{14}, e_{15}, \ldots, e_{3m-5}, e_{3m-4}, e_{3m-3}$ receive label '0' ; the edges $e'_7, e'_8, e'_{13}, e'_{14}, \ldots, e'_{3m-5}, e'_{3m-4}$ receive label '1' ; the edges $e'_9, e'_{15}, e'_{21}, \ldots, e'_{3m-3}$ receive label '0'.

Thus the entire 4m+2 vertices and 6m+1 edges are labeled in such a way that the number of vertices labeled '0' and the number of vertices labeled '1' are same as 2m+1 , when 'm' is odd , the number of edges labeled '0' is 3m and the number of edges labeled '1' is 3m+1 and when 'm' is even , the number of edges labeled '0' is 3m+1 and the number of edges labeled '1' is 3m , which differ by atmost one and satisfies the required condition.

The induced function $f^* : V \cup E \rightarrow \{0,1\}$ is defined as

$f^*(v_i v_j) = \{f(v_i) + f(v_j) + f(v_i v_j)\} (\text{mod } 2)$; $v_i v_j \in E$

Thus the induced function yields the total magic cordial constant '0'.

Hence the extended duplicate graph of arrow graph A^2_m , $m \ge 2$ is total magic cordial.

Illustration 4: Total magic cordial labeling for the graphs **EDG(** A_5^2 **)** and **EDG(** A_6^2 **)**

TOTAL MAGIC CORDIAL LABELING FOR THE EXTENED DUPLICATE OF ARROW GRAPH

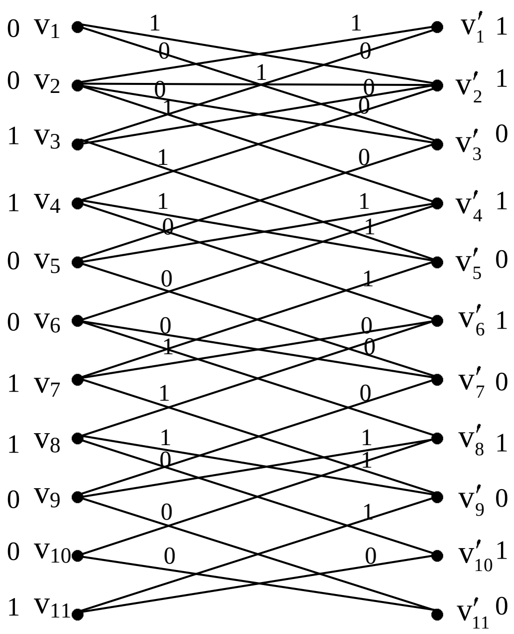

Fig: EDG (A_5^2 ***)***

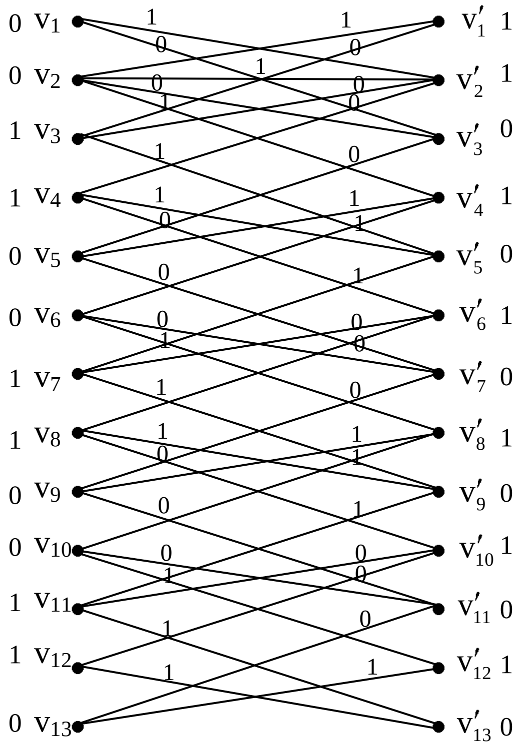

Fig: EDG (A_6^2 ***)***

4. CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of arrow graph A_m^2, $m \geq 2$ is cordial, total cordial and total magic cordial labeling.

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