Importance of Quasi-Concavity and Quasi Convexity in Consumer Optimization Problems

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Abstract. *Concavity and convexity play a vital role in Mathematics. Through this paper we want to check how Concavity and Convexity plays a role in Solving Consumer Problems. We want to see if a Quasi Concave function will lead to Consumer Maximization problem or it will minimize the consumer Utility.*

Keywords - *Quasi-Concavity, Quasi-Convexity, Consumers Problem.*

I. INTRODUCTION

A function is said to be Quasi concave if upper level set is the convex set. i.e. in **R^2+** for each real number "a", define the set , P_a by

$$P_a = \{(x, y) \in S : f(x, y) \geq a\},$$

then P_a should be a convex set.

We will use Lagrange to reach our result.

II. OBJECTIVE

To solve Consumers Problem using Mathematical concepts, and check how to maximize or minimize Consumers utility using **Lagrange optimisation.**

III. RESEARCH METHODOLOGY

Basic Mathematical concepts were used including the Graphs to find the solution to Consumers Problem.

IV. ANALYSIS

Claim

If "Utility function" of the consumer is **"Quasi-concave"** and **"twice-differentiable",** then **"Lagrange-Optimization"** will solve maxima, i.e.

$$L = U(x, y) + \lambda[M - P_x x - P_y y]$$

$$\frac{\partial L}{\partial x} = 0, \frac{\partial L}{\partial y} = 0, \frac{\partial L}{\partial \lambda} = 0$$

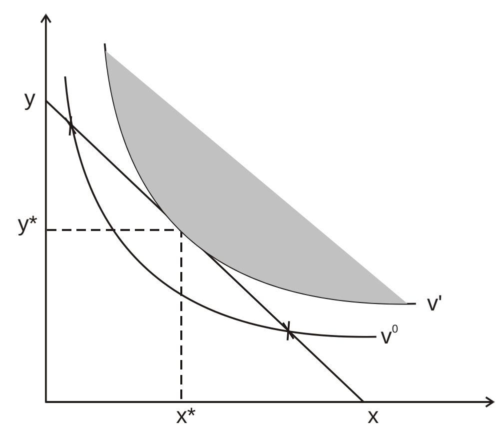
will give (x^*, y^*) which will maximize $U(x^*, y^*)$.

$$U(x, y) = xy \qquad \forall\ x > 0,\ y > 0$$

Indifference curve is:

UPPER LEVEL set is **CONVEX SET.**

Hence Lagrange will solve maximization problem



$$L = xy + \lambda[M - P_x x - P_y y]$$

$$\frac{\partial L}{\partial x} = y - \lambda P_x = 0\,, \tag{1}$$

$$\frac{\partial L}{\partial y} = x - \lambda P_y = 0 \tag{2}$$

$$\frac{\partial L}{\partial y} = M - P_x x - P_y y = 0$$

$$\lambda = \frac{y}{P_x} \tag{1'}$$

$$\lambda = \frac{x}{P_y} \tag{2'}$$

$$\frac{y}{P_x} = \frac{x}{P_y} \Rightarrow P_y y = x P_x$$

$$M = P_x x + x P_x = 2 P_x x$$

$$x^* = \frac{M}{2P_x}, \qquad y^* = \frac{M}{2P_y}$$

which maximizes the above utility function.

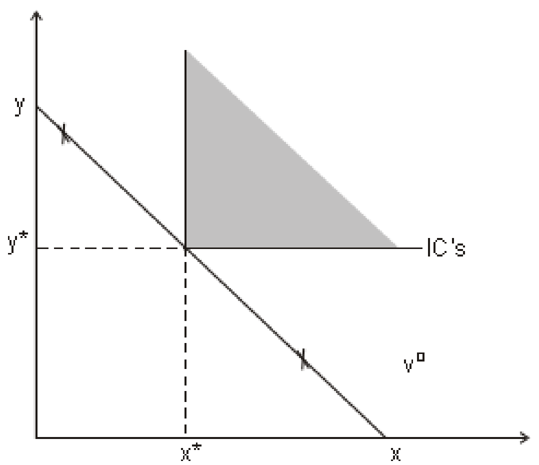
V. AIM

Suppose a utility function is "non-differentiable" Quasi-concave utility function, then also objective function will solve for "maximization problem".

Example

$$U(x, y) = \min(x, y)$$

subject to $M = P_x x + P_y y$



Since the upper level set is the convex set, hence it will solve maximization problem, which happens at tangency point i.e. $x = y$

$$P_x x + P_y x = M \Rightarrow x = \frac{M}{P_x + P_y}$$

$$x^* + y^* = \frac{M}{P_x + P_y}$$

Claim

If "utility function" of the consumer is **"Quasi-convex"** and **"twice-differentiable"**, then **"Langrange optimization"** will solve for **minima.**

"Quasi-convex" in R_+^2 is

For each real number "a", define the set P_a, by

$$P_a = \{(x, y) \in S : f(x, y) \leq a\},$$

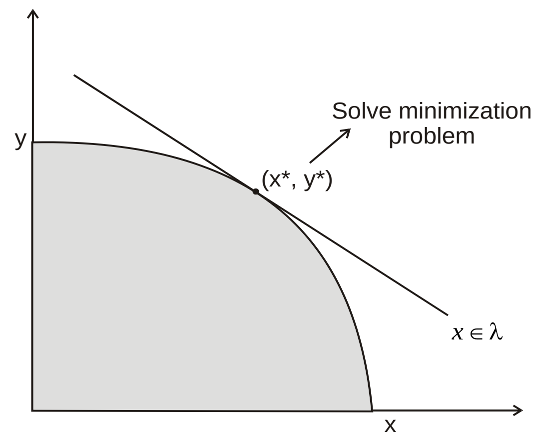
i.e. lower level set, should be a convex set.

Example

$$U = x^2 + y^2$$

subject to $M = P_x x + P_y y$

Here lower level set is convex set. Hence the above function is "Quasi-convex"



$$L = x^2 + y^2 + \lambda[M - P_x x - P_y y]$$

$$\frac{\partial L}{\partial x} = 2x - \lambda P_x = 0 \qquad (4)$$

$$\frac{\partial L}{\partial y} = 2y - \lambda P_y = 0 \qquad (5)$$

$$\frac{\partial L}{\partial \lambda} = M - P_x x - P_y y = 0 \qquad (6)$$

$$\lambda = \frac{2x}{P_x} \qquad \lambda = \frac{2y}{P_y} \qquad (4')$$

$$M = P_x\left(\frac{yP_x}{P_y}\right) + P_y y$$

$$M = \left(\frac{P_x^2}{P_y}\right) y + P_y y$$

$$M.P_y = (P_x^2 + P_y^2) y$$

$$y = \frac{M.P_y}{P_x^2 + P_y^2}, \qquad x = y\frac{P_x}{P_y} = \left(\frac{M.P_y}{P_x^2 + P_y^2}\right)\left(\frac{P_x}{P_y}\right)$$

$$y^* = \frac{M.P_y}{P_x^2 + P_y^2} \qquad x^* = \frac{M.P_x}{P_x^2 + P_y^2}$$

is the minimization solution of the above problem.

In MAXIMIZATION problem

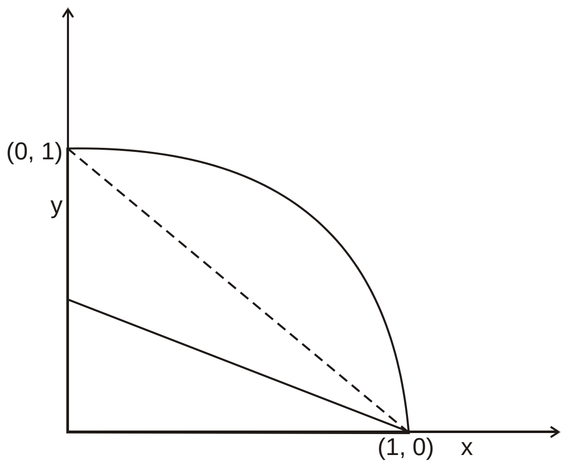
$U = x^2 + y^2$ **subject** $P_x x + P_y y = M$

Maximization Problem records a "Corner solution" that could be turned out from **"KHUN TUCKER"** Methodology

CASE I

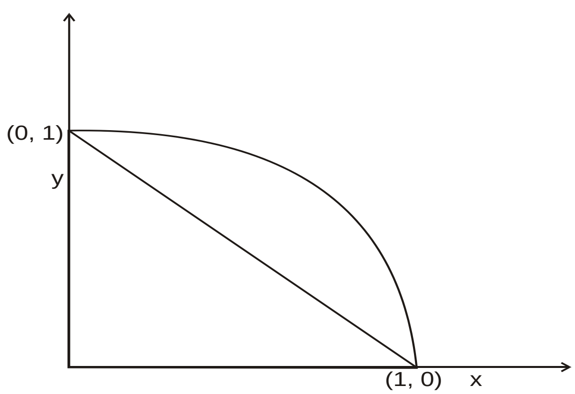
$$\frac{P_x}{P_y} < 1, \qquad x^* = \frac{M}{P_x}, y^* = 0$$

is the solution to above problem.



CASE II

$$\frac{P_x}{P_y} = 1, \quad x^* = \left\{0, \frac{M}{P_x}\right\} \text{ and } y^* = \frac{M - P_x x^*}{P_y}$$



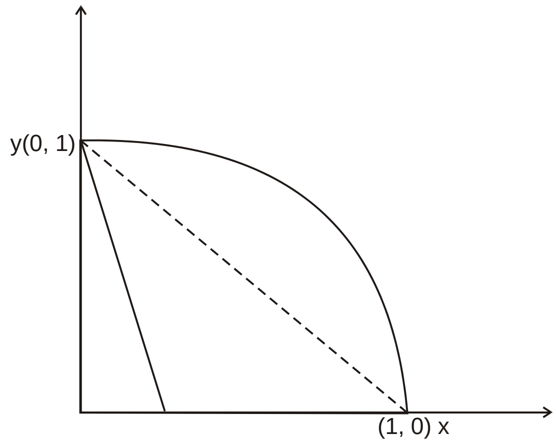
(CII) $\frac{P_x}{P_y} = 1, \quad x^* = \left\{0, \frac{M}{P_x}\right\}$ and $y^* = \frac{M - P_x x^*}{P_y}$

Is the **optimal solution** of above problem.

CASE III

$$\frac{P_x}{P_y} > 1, \qquad x^* = 0, \qquad y^* = \frac{M}{P_y}$$

Is the **optimal solution** of the above problem



Try the same exercise for **U(x , y) = max (x , y)** which is also a **quasi convex but non differentiable utility function.**

CONCLUSIONS

1. A Quasi Concave Utility function will always lead to Consumer Maximization Problem, if we apply **LANGRANGE.**
2. A Quasi Convex Utility function on the other hand will Minimize Consumer Utility on applying Lagrange. Hence we have to apply for corner solutions by applying **" KHUN TUCKER "**

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