On New Types of $T_{1/2}$ Spaces

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ABSTRACT

In this paper we introduce new types of topological ordered spaces with the help of g-closed, g*-closed sets and sg-closed sets. We define $giT_{1/2}$, $gdT_{1/2}$, $gbT_{1/2}$, $g{*}iT_{1/2}$, $g{*}dT_{1/2}$, $g{*}bT_{1/2}$, $sgiT_{1/2}$, $sgdT_{1/2}$, $sgbT_{1/2}$ spaces and study the relations between these spaces.

MATHEMATICS SUBJECT CLASSIFICATION: 54A05

KEYWORDS:

g-closed set, sg-closed set, g*-closed set, $T_{1/2}$ space, increasing set, decreasing set, balanced set, topological ordered space.

1. INTRODUCTION

A topological ordered space is a topological space in which a partial order is available. Using order relation one can think of increasing, decreasing and balanced sets. N.Levine in 1970 defined generalized closed (briefly g-closed) set by slightly weakening the notion of closedness. They are not only the natural generalizations of closed sets but they can suggest more properties of topological spaces. In recent years, some authors have introduced notions which uses both topological and order structure, for example generalized increasing sets. Using these notions we define some new type of separation axioms. L.Nachbin **[1]** initiated the study of Topological ordered spaces (TOS). A TOS is a triple (X, τ, ≤) where "τ" is a topology on X and "≤" is a partial order on X. For any $x \in X$, the sets $[x, \rightarrow]$ and $[\leftarrow, x]$ are defined as $[x, \rightarrow] = \{y \in X / x \leq y\}$ and $[\leftarrow, x] = \{y \in X / y \leq x\}$. A subset A of a TOS (X, τ, ≤) is said to be increasing if $A = i[A]$ and decreasing if $A = d[A]$ where $i[A] = \bigcup_{a \in A}[a, \rightarrow]$ and $d[A] = \bigcup_{a \in A}[\leftarrow, a]$.The complement of an increasing set is a decreasing set and vice versa. A subset of a TOS (X, τ, ≤) is said to be a balanced set if it is both increasing and decreasing.M.K.R.S.Veera Kumar **[2]** introduced the study of i-closed, d-closed and b-closed sets in 2001.Norman Levine **[4]** introduced semi open sets. The complement of a semi open set is called semi-closed. The author **[5]** also introduced generalized closed (briefly sg-closed) sets in 1970.Bhattacharya & Lahiri **[6]** introduced and studied sg-closed sets. g*-closed sets were introduced and studied in **[3]**.

2. PRELIMINARIES

Throughout this paper (X, τ) represent non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), the closure and the interior of A respectively are denoted by cl (A) and int(A).

DEFINITION 2.1. A subset A of a topological space (X, τ) is called a semi-open set **[4]** if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if $\text{int}(cl(A)) \subseteq A$.

The intersection of all closed sets containing a subset A of (X, τ) is called closure of A denoted by cl(A) and the intersection of semi-closed sets containing a subset A of (X, τ) is called the semi-closure of A and is denoted by scl(A).

DEFINITION 2.2. A subset A of a topological space (X, τ) is called

(1) a *generalized closed set* (briefly g-closed)**[5]** if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).The complement of a g-closed set is a g-open set.

(2) a *semi-generalized closed* set (briefly sg-closed) **[6]** if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).The complement of a sg-closed set is a sg-open set.

(3) a g*- *closed* set **[3]** if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).

We have taken a topological ordered space (X, τ, ≤) in order to introduce the following concepts.

DEFINITION 2.3[7]: A subset A of a topological space (X, τ, ≤) is called

(1) a generalized increasing closed set (briefly gi-closed) if A is a g-closed set and an increasing set.
(2) a semi generalized increasing closed set (briefly sgi-closed) if A is a sg-closed set and an increasing set.
(3) a g*i- closed set if A is a g*-closed set and an increasing set.

In a similar way we can introduce decreasing, balanced type closed sets.

3. $T_{1/2}$ SPACES USING g-CLOSED TYPE SETS.

In this section we introduce new types of $T_{1/2}$ spaces using generalized increasing, decreasing and balanced closed sets.

We introduce the following spaces.

DEFINITION 3.1: [8] A topological ordered space (X, τ, ≤) is called

(1) a ${}_{gi}T_{1/2}$ space if every gi-closed set is closed.

(2) a ${}_{gd}T_{1/2}$ space if every gd-closed set is closed.

(3) a ${}_{gb}T_{1/2}$ space if every gb-closed set is closed.

In view of the above definitions, we observe that every gb-closed set is both gi-closed and gd-closed. So, we have the following theorems.

THEOREM 3.1: Every ${}_{gi}T_{1/2}$ space is a ${}_{gb}T_{1/2}$ space.

THEOREM 3.2: Every ${}_{gd}T_{1/2}$ space is a ${}_{gb}T_{1/2}$ space.

REMARK 3.3: A ${}_{gb}T_{1/2}$ space need not be a ${}_{gi}T_{1/2}$ space. This can be seen in the following example.

EXAMPLE 3.4: Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)}.Then, (X, τ_2, $\leq_1$) is a topological ordered space. The closed sets in this space are ϕ, X, {b, c}.The balanced sets are ϕ, X and the g-closed sets are ϕ, X, {b}, {c}, {a, b}, {b, c}, {c, a}.Then, the gb-closed sets are ϕ, X. The increasing sets are ϕ, X, {c}, {b, c}.So, the gi-closed sets are ϕ, X, {b, c}, {c}.Clearly, every gb-closed set is a closed set. Hence, the space (X, τ_2, $\leq_1$) is a $_{gb}T_{1/2}$ space. The subset {c} is a gi-closed set but not a closed set. Hence, the space (X, τ_2, $\leq_1$) is not a $_{gi}T_{1/2}$ space.

REMARK 3.5: A $_{gb}T_{1/2}$ space need not be a $_{gd}T_{1/2}$ space. This can be seen in the following example.

EXAMPLE 3.6: Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)}.Then, (X, τ_2, $\leq_1$) is a topological ordered space. The balanced sets in this space are ϕ, X and g-closed sets are ϕ, X, {b}, {c}, {a, b}, {b, c}, {c, a}.Then, the gb-closed sets are ϕ, X. The closed sets in this space are ϕ, X, {b, c}.The decreasing sets are ϕ, X, {a, b}, {a}. So, the gd-closed sets are ϕ, X, {a, b}.It is clear that, every gb-closed set in X is a closed set. So, the topological ordered space (X, τ_2, $\leq_1$) is a $_{gb}T_{1/2}$ space. The subset {a, b} is a gd-closed set but not a closed set. Hence, the space (X, τ_2, $\leq_1$) is not a $_{gd}T_{1/2}$ space.

We say that two notions P and Q are independent if there are examples satisfying notion P but not Q and vice versa. In view of this we have the following independency.

THEOREM 3.7: The notions $_{gi}T_{1/2}$ space and $_{gd}T_{1/2}$ space are independent notions.

PROOF: This follows from the following examples.

EXAMPLE 3.8: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Then, (X, τ_{11}, $\leq_2$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {b, a}.The increasing sets are ϕ, X, {a, b}, {b, c}, {b} and g-closed sets are ϕ, X, {a}, {a, b}, {a, c}.Then, the gi-closed sets are ϕ, X, {b, a}.Clearly, every gi-closed set is a closed set. So, the space (X, τ_{11}, $\leq_2$) is a $_{gi}T_{1/2}$ space. Decreasing sets in this space are ϕ, X, {a}, {c}, {a, c}.So, the gd-closed sets are ϕ, X, {a}, {c, a}.The subset {a, c} is a gd-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_2$) is not a $_{gd}T_{1/2}$ space.

EXAMPLE 3.9: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_5$ = {(a, a), (b, b), (c, c), (b, c), (a, c)}.Then, (X, τ_{11}, $\leq_5$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {a, b}. The decreasing sets are ϕ, X, {a}, {b}, {a, b} and g-closed sets are ϕ, X, {a}, {a, b}, {a, c}.Then, the gd-closed sets are ϕ, X, {a}, {a, b}.We observe that, every gd-closed set in X is a closed set. Hence, the space (X, τ_{11}, $\leq_5$) is a $_{gd}T_{1/2}$ space. The increasing sets in this space are ϕ, X, {c}, {c, b}, {a, c}.Then, the gi-closed sets are ϕ, X, {a, c}.The subset {a, c} is a gi-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_5$) is not a $_{gi}T_{1/2}$ space.

The following **figure 1** indicates the relationships between the spaces discussed above. Here, A ⟶ B (A ⟷̸ B) indicates A implies B but not conversely (A and B are independent notions).

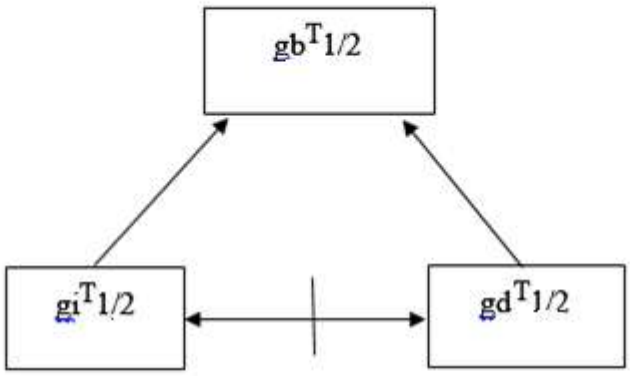

Fig. 1

4. $T_{1/2}$ SPACES USING g*-CLOSED TYPE SETS

In this section we introduce new types of $T_{1/2}$ spaces using g*-increasing, g*-decreasing and g*-balanced closed sets.

We introduce the following definitions:

DEFINITION 4.1: [8] A topological ordered space (X, τ, ≤) is called

(1) a $g*i^{T}{}_{1/2}$ space if every g*i-closed set is a closed set.

(2) a $g*d^{T}{}_{1/2}$ space if every g*d-closed set is a closed set.

(3) a $g*b^{T}{}_{1/2}$ space if every g*b-closed set is a closed set.

In view of the above definitions, we observe that every g*b-closed set is both g*i-closed and g*d-closed. So, we have the following theorems.

THEOREM 4.2: Every $g*i^{T}{}_{1/2}$ space is a $g*b^{T}{}_{1/2}$ space.

THEOREM 4.3: Every $g*d^{T}{}_{1/2}$ space is a $g*b^{T}{}_{1/2}$ space.

REMARK 4.4: A $g*b^{T}{}_{1/2}$ space need not be a $g*i^{T}{}_{1/2}$ space. This can be seen in the following example.

EXAMPLE 4.5: Let X = {a, b, c}, τ_4 = {ϕ, X, {a}, {a, c}} and $\leq_4$ = {(a, a), (b, b), (c, c), (a, b), (c, b), (c, a)}.Then, (X, τ_4, $\leq_4$) is a topological ordered space. The closed sets in this space are ϕ, X, {b, c}, {b}. The balanced sets are ϕ, X and g*-closed sets are ϕ, X, {b}, {a, b}, {a, c}.Then, g*b-closed sets in this space are ϕ, X. Clearly, every g*b-closed set is a closed set. Hence, the space (X, τ_4, $\leq_4$) is a $g^*b T_{1/2}$ space. The increasing sets in this space are ϕ, X, {b}, {a, b}.The g*i-closed sets are ϕ, X, {a, b}, {b}.The subset {a, b} is a g*i-closed set but not a closed set. Hence, the space (X, τ_4, $\leq_4$) is not a $g^*i T_{1/2}$ space.

REMARK 4.6: A $g^*b T_{1/2}$ space need not be a $g^*d T_{1/2}$ space. This can be seen in the following example.

EXAMPLE 4.7: Let X = {a, b, c},}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Then, (X, τ_{11}, $\leq_2$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {a, b}.The balanced sets in this space are ϕ, X and g*-closed sets are ϕ, X, {a}, {a, b}, {a, c}.Then, g*b-closed sets in this space are ϕ, X. Every g*b-closed set is a closed set. So, the space (X, τ_{11}, $\leq_2$) is a $g^*b T_{1/2}$ space. The decreasing sets in this space are ϕ, X, {a}, {c}, {a, c}. The g*d-closed sets are ϕ, X, {a}, {a, c}.The subset {a, c} is a g*d-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_2$) is not a $g^*d T_{1/2}$ space.

THEOREM 4.8: The notions $g^*i T_{1/2}$ space and $g^*d T_{1/2}$ space are independent notions.

PROOF: This follows from the following examples.

EXAMPLE 4.9: Let X = {a, b, c},}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Then, (X, τ_{11}, $\leq_2$) is a topological ordered space. The closed sets in this space are ϕ, X,{a}, {a, b}.The increasing sets are ϕ, X, {b}, {a, b}, {b, c} and g*-closed sets are ϕ, X, {a}, {a, b}, {a, c}.Then, g*i-closed sets in this space are ϕ, X, {a, b}.Clearly, every g*i-closed set is a closed set. So, the space (X, τ_{11}, $\leq_2$) is a $g^*i T_{1/2}$ space. The decreasing sets are ϕ, X, {a}, {c}, {a, c}.The g*d-closed sets are ϕ, X, {a, c}, {a}.The subset {a, c} is a g*d-closed set but not a closed set. Hence, the topological space (X, τ_{11}, $\leq_2$) is not a $g^*d T_{1/2}$ space.

EXAMPLE 4.10: Let X = {a, b, c}, τ_{10} = {ϕ, X, {a}, {b, a}} and $\leq_5$ = {(a, a), (b, b), (c, c), (a, c), (b, c)}.Then, (X, τ_{10}, $\leq_5$) is a topological ordered space. The closed sets in this space are ϕ, X, {c}, {c, b}.Decreasing sets in this space are ϕ, X, {a}, {b}, {a, b} and g*-closed sets are ϕ, X, {c}, {c, b}, {a, c}. Then, g*d-closed sets are ϕ, X. Clearly, every g*d-closed set is a closed set. So, the space (X, τ_{10}, $\leq_5$) is a $g^*d T_{1/2}$ space. Increasing sets in this space are ϕ, X, {c}, {b, c}, {a, c}.The g*i-closed sets are ϕ, X, {c}, {b, c}, {a, c}.The subset {a, c} is a g*i-closed set but not a closed set. Hence, the topological ordered space(X, τ_{10}, $\leq_5$) is not a $g^*i T_{1/2}$ space.

The following **figure 2 indicates** the relationships between the spaces discussed above. Here, A ⟶ B (A ⟵|⟶ B) indicates A implies B but not conversely (A and B are independent notions).

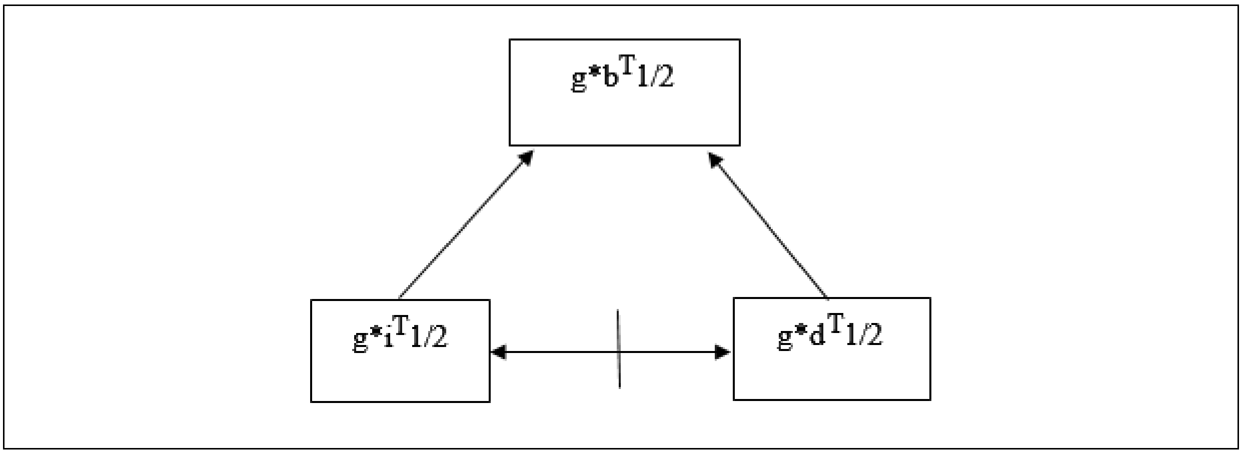

Fig. 2

5. $T_{1/2}$ SPACES USING sg-CLOSED TYPE SETS

In this section we introduce new types of $T_{1/2}$ spaces using sgi-closed, sgd-closed and sgb- closed sets.

We introduce the following definitions.

DEFINITION 5.1: [8] A topological ordered space (X, τ,≤) is called

1. a $sgiT_{1/2}$ space if every sgi-closed set is a closed set.
2. a $sgdT_{1/2}$ space if every sgd-closed set is a closed set.
3. a $sgbT_{1/2}$ space if every sgb-closed set is a closed set.

In view of the above definitions, it is observed that every sgb-closed set is both sgi-closed and sgd-closed. So, we have the following theorems.

THEOREM 5.2: Every $sgiT_{1/2}$ space is a $sgbT_{1/2}$ space.

THEOREM 5.3: Every $sgdT_{1/2}$ space is a $sgbT_{1/2}$ space.

REMARK 5.4: A $sgbT_{1/2}$ space need not be a $sgiT_{1/2}$ space. This can be seen in the following example.

EXAMPLE 5.5: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Then, (X, τ_{11}, $\leq_2$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {a, b}.Balanced sets in this space are ϕ, X and sg-closed sets are ϕ, X, {a}, {b}, {a, b}.Then, sgb-closed sets are ϕ, X. Increasing sets in this space are ϕ, X, {b}, {a, b}, {b, c}.So, sgi-closed sets are ϕ, X, {b}, {a, b}.Clealry, sgb-closed set in X is a closed set. So, the space (X, τ_{11}, $\leq_2$) is a $_{sgb}T_{1/2}$ space. The subset {b} is a sgi-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_2$) is not a $_{sgi}T_{1/2}$ space.

REMARK 5.6: A $_{sgb}T_{1/2}$ space need not be a $_{sgd}T_{1/2}$ space. This can be seen in the following example.

EXAMPLE 5.7: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_5$ = {(a, a), (b, b), (c, c), (a, c), (b, c)}.Then, (X, τ_{11}, $\leq_5$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {a, b}.Balanced sets in this space are ϕ, X and sg-closed sets are ϕ, X, {a}, {b}, {a, b}.Then, sgb-closed sets in this space are ϕ, X. Decreasing sets in this space are ϕ, X, {b}, {a, b} and sgd-closed sets are ϕ, X, {b}, {a, b}.Clearly, every sgb-closed set in X is a closed set. So, the space (X, τ_{11}, $\leq_5$) is a $_{sgb}T_{1/2}$ space. The subset {b} is a sgd-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_5$) is not a $_{sgd}T_{1/2}$ space.

THEOREM 5.8: The notions $_{sgi}T_{1/2}$ space and $_{sgd}T_{1/2}$ space are independent notions.

PROOF: This follows from the following examples.

EXAMPLE 5.9: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Then, (X, τ_{11}, $\leq_2$) is a topological ordered space. The closed sets in this space are ϕ, X, {a}, {a, b}.Decreasing sets in this space are ϕ, X, {a}, {c}, {a, c} and sg-closed sets are ϕ, X, {a}, {b}, {a, b}.Then, sgd-closed sets are ϕ, X, {a}.The increasing sets are ϕ, X, {b}, {a, b}, {b, c}.So, sgi-closed sets in this space are ϕ, X, {b},{a, b}.Clearly, every sgd-closed set in X is a closed set. So, the topological ordered space (X, τ_{11}, $\leq_2$) is a $_{sgd}T_{1/2}$ space. The subset {b} is a sgi-closed set but not a closed set. Hence, the space (X, τ_{11}, $\leq_2$) is not a $_{sgi}T_{1/2}$ space.

EXAMPLE 5.10: Let X = {a, b, c}, τ_{11} = {ϕ, X, {c}, {b, c}} and $\leq_6$ = {(a, a), (b, b), (c, c), (b, a), (a, c), (b, c)}.Then, (X, τ_{11}, $\leq_6$) is a topological ordered space. Closed sets in this space are ϕ, X, {a}, {a, b}.Decreasing sets in this space are ϕ, X, {b}, {a, b} and sg-closed sets are ϕ, X, {a}, {b}, {a, b}.Then, the sgd-closed sets in this space are ϕ, X, {b}, {a, b}.Increasing sets are ϕ, X, {c}, {a, c}.So, the sgi-closed sets are ϕ, X. Clearly, every sgi-closed set in X is a closed set. So, the space (X, τ_{11}, $\leq_6$) is a $_{sgi}T_{1/2}$ space. The subset {b} is a sgd-closed set but, it is not a closed set. Hence, the space (X, τ_{11}, $\leq_6$) is not a $_{sgd}T_{1/2}$ space.

The following **figure 3 indicates** the relationships between the spaces discussed above. Here, A ⟶ B (A ←|→ B) indicates A implies B but not conversely (A and B are independent notions).

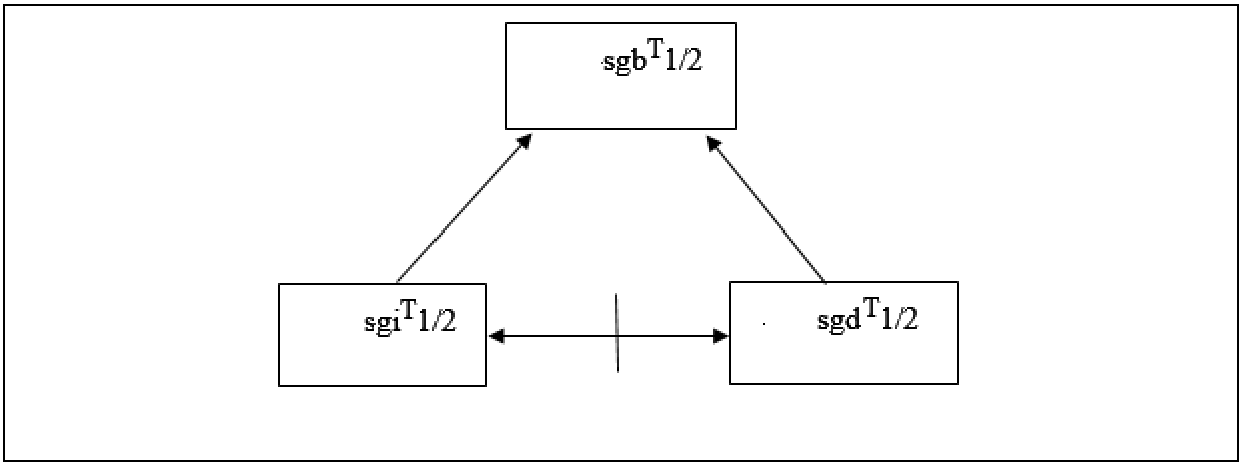

Fig. 3

6. CONCLUSION:

In this paper we defined new types of $T_{1/2}$ spaces using g-closed type, g*-closed type and sg-closed type sets. We studied the relations between these spaces and provided examples for the independency between some notions.

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