Compatibility of type (β) and Common fixed point theorem in intuitionistic fuzzy metric space

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Abstract *- In this paper, we prove a common fixed point theorem by using compatibility of type (β) in intuitionistic fuzzy metric space. Our result extends and generalizes the result of Cho [4].*

Keywords *- Common fixed points, fuzzy metric space, compatible maps, and compatible maps of type (β).*

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1. Introduction:

Zadeh [11] introduced the notion of fuzzy sets. Later on many authors have extensively developed the theory of fuzzy sets and applications. Kramosil and Michalek [5] introduced the concept of fuzzy metric spaces. George and Veeramani [4] modified this concept and defined a Hausdorff topology on fuzzy metric spaces.

Singh and Chouhan [9] introduced the concept of compatible mappings in Fuzzy metric space and proved some common fixed point theorems. Jungck [6] introduced the concept of compatible maps. Jungck et al. [7] introduced the concept of compatible maps of type (A) in metric space and proved the fixed point theorems.

Turkoglu et al. [1] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β).

For the sake of completeness, we recall some definitions and known results in fuzzy metric space,

2. Preliminaries.

Definition 2.1[8] A binary operation *: [0, 1] × [0,1] → [0,1] is a continuous t-norm if * satisfies the following conditions:

(i) ∗ is commutative and associative;
(ii) ∗ is continuous;
(iii) a∗1 = a, for all a ∈ [0,1];
(iv) a∗b ≤ c∗d whenever a ≤ c and b ≤ d, for all a, b, c, d ∈ [0,1].

Examples of t-norm are a*b = ab and a*b = min {a, b}.

Definition 2.2[8] A binary operation ◊: [0, 1] × [0, 1] → [0,1] is a continuous t-conorm if ◊ satisfies the following conditions:

(i) ◊ is commutative and associative;
(ii) ◊ is continuous;
(iii) a ◊ 0 = a, for all a ∈ [0,1];
(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d, for all a, b, c, d ∈ [0,1].

Examples of t-norm are a ◊ b = min{a+1,1} and and a ◊ b = max {a, b}.

Definition 2.2 *[2]* A 5-tuple (X, M, N,*,◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set,* is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

for all x, y, z $\in$ X and s, t > 0.

(IFM-1) M(x, y, t) + N(x, y, t) $\leq$1;

(IFM-2) M(x, y, 0) = 0 ;

(IFM-3) M(x, y, t) = 1 if and only if x = y;

(IFM-4) M(x, y, t) = M(y, x, t);

(IFM-5) M(x, y, t)*M(y, z, s) $\leq$ M(x, z, t +s);

(IFM-6) M(x, y,·): [0,∞) $\rightarrow$ [0,1] is left continuous;

(IFM-7) $\lim_{n\to\infty} M(x, y, t) = 1$;

(IFM-8) N(x, y, 0) = 1;

(IFM-9) N(x, y, t) = 0 if and only if x = y;

(IFM-10) N(x, y, t) = N(y, x, t);

(IFM-11)N(x, y, t) ◊ N(y, z, s) $\geq$ N(x, z, t +s);

(IFM-12) N(x, y,·): [0,∞) $\rightarrow$ [0,1] is right continuous;

(IFM-13) $\lim_{n\to\infty} N(x, y, t) = 0$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.1[1] Every fuzzy metric space (X, M,$\ast$) is an intuitionistic fuzzy metric space of the form (X, M, 1-M,$\ast$, ◊) such that t-norm $\ast$ and t-conorm ◊ are associated, i.e., x◊y = 1-((1-x) $\ast$ (1-y)) for all x, y $\in$ X.
Remark 2.2.[1] In Intuitionistic fuzzy Metric space X, M(x, y, .) is non decreasing and N(x, y, .) is non increasing for all x, y $\in$ X.
Example 2.1. Let (X, d) be a metric space. Define a * b = ab and a ◊ b = min {1, a+b}, for all a, b $\in$ [0, 1] and let M and N be fuzzy sets on X² × [0, ∞) defined as follows: $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all x, y $\in$ X and all t > 0. Then (M, N) is called an intuitionistic fuzzy metric space on X. We call this intuitionistic fuzzy metric induced by a metric d, the standard intuitionistic fuzzy metric.
Definition 2.3 *[2]* Let (X, M, N,*, ◊) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$in X is said to be convergent to a point x $\in$ X if, for all t > 0,
$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$:

Since * and ◊ are continuous, the limit is uniquely determined from (IFM-5) and (IFM-11) of definition 2.2, respectively.

Definition 2.4*[2]* An intuitionistic fuzzy metric space (X, M, N,*, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.5*[2]* Let X be a set, f, g self maps of X, A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.6[2] An intuitionistic fuzzy metric space (X, M, N,*, ◊) is said to be compact if every sequence in X contains a convergent subsequences.

Definition 2.7[10] Let f and g be maps from an intuitionistic fuzzy metric space (X, M, N,*, ◊) into itself. The maps f and g are said to be compatible if, for all t > 0,

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0,$$

Whenever $\{x_n\}$ is a sequence X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some z ∈ X.

Definition 2.9 A pair (f, g) of self maps of an intuitionistic fuzzy metric space (X, M, N,*, ◊) are said to be semi-compatible if

$$\lim_{n\to\infty} M(fgx_n, gx, t) = 1 \text{ and } \lim_{n\to\infty} N(fgx_n, gx, t) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x.$$

Proposition 2.1*[2]* Let A and S be self maps on an intuitionistic fuzzy metric space (X, M, N,*, ◊).If S is continuous, then (A, S) is semi compatible if and only if (A, S) is compatible.

Definition 3*[10]* Let f and g be maps from an intuitionistic fuzzy metric space (X, M, N,*, ◊) into itself. The maps f and g are said to be compatible of type (β) if

$$\lim_{n\to\infty} M(ffx_n, ggx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ffx_n, ggx_n, t) = 0.$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$.

Example 2.2*[1]* Let (X, M, N,*, ◊) be an intuitionistic fuzzy metric space where X = [0, 2] t-norm is defined by a*b = min {a, b} and t co-norm is defined by a◊ b=max {a, b} for all a, b∈ [0,2] and

$$M(x, y, t) = \left[\exp\left(\frac{|x-y|}{t}\right)\right]^{-1} \text{ and}$$

$$N(x, y, t) = \left[\exp\left(\frac{|x-y|}{t}\right) - 1\right]\left[\exp\left(\frac{|x-y|}{t}\right)\right]^{-1}$$

for all x, y∈ X, t > 0.

Clearly (X, M, N,*, ◊) is an intuitionistic fuzzy metric space.

let A and B be defined by

Ax=1 for all x∈ [0,1],Ax=1+x for all x∈ (1, ∞)

and

Bx=1+x for all x∈ [0,1), Bx=1 for all x∈ [1, ∞).

Let $\{x_n\}$ be a sequence in X such that Ax_n, $Sx_n \to z$ as $n \to \infty$ for some z∈ X. by definition of A and B, z∈ {1}and $\lim_{n\to\infty} x_n = 0$.

A and B both are discontinuous at z=1.

Therefore, we have

$$M(AAx_n, BAx_n, t) \to 1, M(AAx_n, BBx_n, t) \to 1$$

$$N(AAx_n, BAx_n, t) \to 0, N(AAx_n, BBx_n, t) \to 1 \text{ as } n \to \infty$$

also, we consider the sequence $\{x_n\}$ in X defined by

$$\{x_n\} = \frac{1}{2^n}, n=1, 2\ldots,$$

Then we have $Ax_n, Bx_n \to 1$ as $n \to \infty$.

further, for t>0, we have

$$M(ABx_n, BAx_n, t) \to \left[\exp\left(\frac{1}{t}\right)\right]^{-1} \neq 1,$$

$$M(ABx_n, BBx_n, t) \to \left[\exp\left(\frac{1}{t}\right)\right]^{-1} \neq 1, \text{ and}$$

$$N(ABx_n, BAx_n, t) \to \left[\exp\left(\frac{1}{t}\right) - 1\right]\left[\exp\left(\frac{1}{t}\right)\right]^{-1} \neq 0,$$

$$N(ABx_n, BBx_n, t) \to \left[\exp\left(\frac{1}{t}\right) - 1\right]\left[\exp\left(\frac{1}{t}\right)\right]^{-1} \neq 0, \text{ as } n \to \infty.$$

therefore, A, B is compatible of type (β) but they are not compatible.

Lemma 2.3 [1] Let (X, M, N,*, ◊) be an intuitionistic fuzzy metric space if there exists $k \in (0, 1)$ such that

$$M(x, y, kt) \geq M(x, y, t)$$
and
$$N(x, y, kt) \leq N(x, y, t)$$

for all $x, y \in X$ then $x = y$.

Lemma 2.4*[1]* Let (X, M, N,*, ◊) be an intuitionistic fuzzy metric space. Then for all $x, y \in X$, M(x, y,·) is non-decreasing and N(x, y,·) is non-increasing.

Lemma 2.5[1] Let(X, M, N,*, ◊) be an intuitionistic fuzzy metric space. If there exists a number $k \in (0,1)$ such that

$$M(y_{n+2}, y_{n+1}, qt) \geq M(y_{n+1}, y_n, t)$$
and
$$N(y_{n+2}, y_{n+1}, qt) \leq N(y_{n+1}, y_n, t) \text{ for all } t>0 \text{ and } n=1, 2\ldots$$

then $\{y_n\}$ is a Cauchy sequence in X.

3 Main Results

Theorem 3.1 Let (X, M, N,∗, ◊) be a complete intuitionistic fuzzy metric space and let A,B,S,T,P and Q be mappings from X into itself such that the following condition are satisfied

1) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$.
2) AB = BA, ST = TS, PB = BP, QT = TQ.
3) Either P or AB is continuous.

4) (P, AB) is compatible of type (β) and (Q, ST) is semi-compatible.
5) There exists $k \in (0,1)$ such that for every $x, y \in X, \alpha \in (0,2)$ and $t>0$

$$M(Px, Qy, kt) \geq \min\{M(ABx,Qy,(2-\alpha)t),M(ABx,STy, t),M(ABx,Px,t),M(STy,Qy,t)\}$$

and

$$N(Px,Qy,kt) \leq \max\{N(ABx,Qy,(2-\alpha)t),N(ABx,STy, t),N(ABx,Px,t),N(STy,Qy,t)\}$$

Then the mapping AB,ST,P and Q have a unique common fixed point in X and also A, B, P, Q, S and T have a unique common fixed point in X.

Proof: Let $x_0 \in X$ then from (1) there exists $x_1, x_2 \in X$

such that $Px_0=STx_1=y_0$ and $Qx_1=ABx_2=y_1$

Inductively we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n}=Px_{2n}=STx_{2n+1} \quad \text{and}$$
$$y_{2n+1}=Qx_{2n+1}=ABx_{2n+2} \quad \text{with } n=0, 1, 2\ldots$$

Put $x=x_{2n+2}$ and $y=x_{2n+1}$ for all $t>0$ and $\alpha = 1 - q$ with $q \in (0,1)$ in (5),we have

$$M(y_{2n+1}, y_{2n+2}, kt) = M(Px_{2n+2}, Qx_{2n+1}, kt)$$

$$\geq \min\{M(ABx_{2n+2},Qx_{2n+1},(2-(1-\alpha))t), M(ABx_{2n+2},STx_{2n+1},t), M(ABx_{2n+2},Px_{2n+2},t),M(STx_{2n+1},Qx_{2n+1},t)\}$$

$$\geq \min\{M(y_{2n+1},y_{2n+1},(1+\alpha))t), M(y_{2n+1},y_{2n},t),M(y_{2n+1},y_{2n+2},t),M(y_{2n}, y_{2n+1},t)\}$$

$$=\min\{1,M(y_{2n},y_{2n+1},t),M(y_{2n+1},y_{2n+2},t)\} \qquad \text{(i)}$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) = N(Px_{2n+2}, Qx_{2n+1}, kt)$$

$$\leq \max\{N(ABx_{2n+2},Qx_{2n+1},(2-(1-\alpha)),t),N(ABx_{2n+2},STx_{2n+1},t), N(ABx_{2n+2}, Px_{2n+2}, t),N(STx_{2n+1},Qx_{2n+1},t)\}$$

$$\leq \max\{N(y_{2n+1},y_{2n+1},(1+\alpha))t), N(y_{2n+1},y_{2n},t),N(y_{2n+1},y_{2n+2},t),N(y_{2n}, y_{2n+1}, t)\}$$

$$=\max\{1, M(y_{2n}, y_{2n+1}, t), M(y_{2n+1},y_{2n+2},t)\} \qquad \text{(ii)}$$

Therefore from lemma 2.3 and 2.5

we get,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n+1},y_{2n},t)$$

and

$$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n+1},y_{2n},t)$$

Similarly, we can also have

$$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1},y_{2n+2},t)$$

and

$$N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1},y_{2n+2},t)$$

Thus, we have

$$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$$

and

$$N(y_{n+1}, y_{n+2}, kt) \leq N(y_n, y_{n+1}, t)$$

for n=1, 2… and so

$$M(y_n, y_{n+1}, t) \geq M(y_n, y_{n-1}, \tfrac{t}{q}) \geq M\left(y_{n-2}, y_{n-1}, \tfrac{t}{q^2}\right) \ldots \geq M(y_1, y_2, \tfrac{t}{q^n}) \rightarrow 1,$$

as $n \rightarrow \infty$

and

$$N(y_n, y_{n+1}, t) \leq N(y_n, y_{n-1}, \tfrac{t}{q}) \leq N\left(y_{n-2}, y_{n-1}, \tfrac{t}{q^2}\right) \ldots \leq N(y_1, y_2, \tfrac{t}{q^n}) \rightarrow 0$$

as $n \rightarrow \infty$, and hence $M(y_n, y_{n+1}, t) \rightarrow 1$ and $N(y_n, y_{n+1}, t) \rightarrow 0$

as $n \rightarrow \infty$, for any $t>0$.

for each $\varepsilon > 0$ and each $t>0$, we can choose $n_0 \in N$, such that

$M(y_{2n}, y_{2n+1}, t) > 1-\varepsilon$ and $N(y_{2n}, y_{2n+1}, t) < \varepsilon$ for all $n > n_0$.

for m, $n \in N$, we suppose $m \geq n$ then we have that

$M(y_n, y_m, t)$

$$\geq M(y_n, y_{n+1}, \tfrac{t}{m-n}) * M(y_{n+1}, y_{n+2}, \tfrac{t}{m-n}) * \ldots * M(y_{m-1}, y_m \tfrac{t}{m-n})$$

$$\geq \{(1-\varepsilon) * (1-\varepsilon) * \ldots (1-\varepsilon)\}(m-n \text{ times})$$

$$\geq (1-\varepsilon)$$

$N(y_n, y_m, t)$

$$\leq N(y_n, y_{n+1}, \tfrac{t}{m-n}) \lozenge N(y_{n+1}, y_{n+2}, \tfrac{t}{m-n}) \lozenge \ldots \lozenge N(y_{m-1}, y_m \tfrac{t}{m-n})$$

$$\leq \{\varepsilon \lozenge \varepsilon \lozenge \ldots \lozenge \varepsilon\}(m-n \text{ times}) < \varepsilon$$

and hence $\{y_n\}$ is a Cauchy sequence in X.

Since $(X, M, N, *, \lozenge)$ is complete, $\{y_n\}$ converges to some point $z \in X$.also its subsequences converges to the same point i.e. $z \in X$

i.e. $\{Qx_{2n+1}\} \rightarrow z$ and $\{STx_{2n+1}\} \rightarrow z$ (8)

$\{Px_{2n}\} \rightarrow z$ and $\{ABx_{2n}\} \rightarrow z$ (9)

Case I Suppose AB is continuous.

Since AB is continuous, we have

$$(AB)^2x_{2n} \to ABz \text{ and } ABPx_{2n} \to ABz.$$

as (P, AB) is compatible of type (β), we have

$\lim_{n\to\infty} M(PPx_{2n}, AB(AB)x_{2n}, t)=1$, for all $t > 0$

This gives $M(Pz,ABz,t) = 1$ or $Pz=ABz$

Also by the semi-compatibility of (P, AB) we get $ABPx_{2n} \to ABz$

Now we will show that Pz=ABz=z.

Step 1: Put $x=ABx_{2n}$ and $y=x_{2n+1}$ with $\alpha = 1$ in equation (5), we get

$$M(PABx_{2n}, Qx_{2n+1}, kt) \geq \min\{M(AB(AB)x_{2n}, Qx_{2n+1},t), M(AB(AB)x_{2n}, STx_{2n+1}, t),$$

$$M(AB(AB)x_{2n}, P(AB)x_{2n}, t), M(STx_{2n+1}, Qx_{2n+1}, t)\}$$

letting $n\to \infty$

$M(Pz,z,kt) \geq \min\{M(ABz,z,t), M(ABz,z,t), M(ABz,Pz,t), M(z,z,t)\}$ $M(ABz,z,kt)$

$M(ABz,z,kt) \geq \min\{M(ABz,z,t), M(ABz,z,t), M(ABz,ABz,t), M(z,z,t)\}$

$M(ABz,z,kt) \geq M(ABz,z,t)$

and similarly

$$N(PABx_{2n}, Qx_{2n+1}, kt) \leq \max\{N(AB(AB)x_{2n}, Qx_{2n+1},t), N(AB(AB)x_{2n}, STx_{2n+1}, t),$$

$$N(AB(AB)x_{2n}, P(AB)x_{2n},t), N(STx_{2n+1}, Q_{2n+1}, t)\}$$

letting $n\to \infty$

$N(Pz,z,kt) \leq \max\{N(ABz,z,t), N(ABz,z,t), N(ABz,Pz,t), N(z,z,t)\}$
$N(ABz,z,kt) \leq \max\{N(ABz,z,t), N(ABz,z,t), N(ABz,ABz,t), N(z,z,t)\}$

$N(ABz,z,kt) \leq N(ABz,z,t)$

Therefore by using lemma 2.3, we get

$Pz = ABz = z.$

Step 2.Put $x=z$, $y=x_{2n+1}$ with $\alpha = 1$ in (5) we have

$M(Pz,Qx_{2n+1},kt) \geq \min\{M(ABz,Qx_{2n+1},t), M(ABz,STx_{2n+1},t),$

$M(ABz,Pz,t), M(STx_{2n+1},Qx_{2n+1},t)\}$

letting $n\to \infty$, we have

$M(Pz,z,kt) \geq \min\{M(Pz,z,t), M(Pz,z,t), M(Pz,Pz,t), M(z,z,t)\}$

$M(Pz,z,kt) \geq M(Pz,z,t)$

and

$N(Pz,Qx_{2n+1},kt) \leq \max\{N(ABz,Qx_{2n+1},t),N(ABz,STx_{2n+1},t),$

$N(ABz,Pz,t),N(STx_{2n+1},Qx_{2n+1},t)\}$

letting $n\rightarrow \infty$, we have

$N(Pz,z,kt) \leq \max\{N(Pz,z,t),N(Pz,z,t),N(Pz,Pz,t),N(z,z,t)\}$

$N(Pz,z,kt) \leq N(Pz,z,t)$

Therefore by lemma 2.3, we get

$Pz=z.$

i.e. $z=Pz=ABz.$

Step 3: Put $x=Bz$ and $y=x_{2n+1}$ with $\alpha = 1$ in equation (5), we have

$M(PBz,Qx_{2n+1},kt) \geq \min\{M(AB(Bz),Qx_{2n+1},t),M(AB(Bz),STx_{2n+1},t),$

$M(AB(Bz),P(Bz),t),M(STx_{2n+1},Qx_{2n+1},t)\}$

since $PB=BP$, $AB=BA$,

so $P(Bz)=B(Pz)=Bz$

and $AB(Bz)=B(AB)z=Bz$

letting $n\rightarrow \infty$, we have

$M(Bz,z,kt) \geq \min\{M(Bz,z,t),M(Bz,z,t),M(Bz,Bz,t),M(z,z,t)\}$

$M(Bz,z,kt) \geq M(Bz,z,t)$

and

$N(PBz,Qx_{2n+1},kt) \leq \max\{N(AB(Bz),Qx_{2n+1},t),N(AB(Bz),STx_{2n+1},t),$

$N(AB(Bz),P(Bz),t),N(STx_{2n+1},Qx_{2n+1},t)\}$

letting $n\rightarrow \infty$, we have

$N(Bz,z,kt) \leq \max\{N(Bz,z,t),N(Bz,z,t),N(Bz,Bz,t),N(z,z,t)\}$

$N(Bz,z,kt) \leq N(Bz,z,t)$

Therefore by using lemma 2.3 and 2.5, we get

$Bz=z.$

Thus $ABz=Pz=z$ further $ABz=Az$

$z=ABz \Rightarrow Az=z.$

Therefore ABz=Az=Bz=Pz=z. (A)

Step 4: As $P(X)\subset ST(X)$, there exists $u\in X$:

$$z=Pz=STu.$$

put $x=x_{2n}$ and y=u with $\alpha = 1$ in equation (5), we get

$$M(Px_{2n},Qu,kt)\geq \min\{M(ABx_{2n},Qu,t),M(ABx_{2n},STu,t),$$

$$M(ABx_{2n},Px_{2n},t),M(STu,Qu,t)\}$$

Taking $n\rightarrow \infty$

$$M(z,Qu,kt)\geq \min\{M(z,Qu,t),M(z,z,t),M(z,z,t),M(z,Qu,t)$$

$$M(z,Qu,kt)\geq M(z,Qu,t)$$

and

$$N(Px_{2n},Qu,kt)\leq \max\{N(ABx_{2n},Qu,t),N(ABx_{2n},STu,t),$$

$$N(ABx_{2n},Px_{2n},t),N(STu,Qu,t)\}$$

Taking $n\rightarrow \infty$

$$N(z,Qu,kt)\leq \max\{N(z,Qu,t),N(z,z,t),N(z,z,t),N(z,Qu,t)$$

$$N(z,Qu,kt)\leq N(z,Qu,t)$$

Therefore by using lemma 2.3 and 2.5

we get Qu=z

Hence STu=z=Qu.

Since (Q,ST) is semi-compatible

$\lim_{n\rightarrow\infty}STQx_{2n}=Qz$

$\lim_{n\rightarrow\infty}STx_{2n}=\lim_{n\rightarrow\infty}Qx_{2n}=z$

$\lim_{n\rightarrow\infty}STQx_{2n}=Qz$

Thus Qz=STz.

Step 5: Put $x=x_{2n}$ and y=z with $\alpha = 1$ in equation(5), we get

$$M(Px_{2n},Qz,kt)\geq \min\{M(ABx_{2n},Qz,t),M(ABx_{2n},STz,t),$$

$$M(ABx_{2n},Px_{2n},t),M(STz,Qz,t)\}$$

taking $n\rightarrow \infty$

$$M(z,Qz,kt)\geq \min\{M(z,Qz,t),M(z,Qz,t),M(z,z,t),M(Qz,Qz,t)$$

$$M(z,Qz,kt)\geq M(z,Qz,t)$$

$N(Px_{2n},Qz,kt) \leq \max\{N(ABx_{2n},Qz,t),N(ABx_{2n},STz,t),$

$N(ABx_{2n},Px_{2n},t),N(STz,Qz,t)\}$

taking $n \to \infty$

$N(z,Qz,kt) \leq \max\{N(z,Qz,t),N(z,Qz,t),N(z,z,t),N(Qz,Qz,t)$

$N(z,Qz,kt) \leq N(z,Qz,t)$

Therefore by using lemma 2.3

we get $Qz=z$

Step 6: Put $x=x_{2n}$, $y=Tz$ with $\alpha = 1$, we get

$M(Px_{2n},QTz,kt) \geq \min\{M(ABx_{2n},STz,t),M(ABx_{2n},ST(Tz),t),$

$M(ABx_{2n},Px_{2n},t),M(ST(Tz),Q(Tz),t)\}$

as $QT=TQ$ and $ST=TS$,

we have

$QTz=TQz=Tz$ and $ST(Tz)=T(STz)=Tz$.

taking $n \to \infty$, we get

$M(z,Tz,kt) \geq \min\{M(z,Tz,t),M(z,Tz,t),M(z,z,t),M(Tz,Tz,t)\}$

$M(z,Tz,kt) \geq M(z,Tz,t)$

and

$N(Px_{2n},QTz,kt) \leq \max\{N(ABx_{2n},Px_{2n},t),N(ABx_{2n},ST(Tz),t),$

$N(ABx_{2n},Px_{2n},t),N(ST(Tz),Q(Tz),t)\}$

taking $n \to \infty$, we get

$N(z,Tz,kt) \leq \max\{N(z,Tz,t),N(z,Tz,t),N(z,z,t),N(Tz,Tz,t)\}$

$N(z,Tz,kt) \leq N(z,Tz,t)$

Therefore by using lemma 2.3, we have

$Tz=z$

now $STz=Tz=z$ this implies $Sz=z$

Hence $Sz=Tz=Qz=z=STz$ (B)

Combining (A) and (B), we get

$ABz=STz=Az=Bz=Pz=Qz=Sz=Tz=z$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case II: Suppose P is continuous.

since P is continuous, we have $(P)^2x_{2n} \to Pz$ and $P(AB)x_{2n} \to Pz$.
as (P, AB) is compatible of type (β),we have

$$\lim_{n\to\infty} M(PPx_{2n},(AB)(AB)x_{2n},t)=1, \text{for all } t>0$$

Or $M(Pz,ABz,t)=1$.

i.e. $Pz=ABz$

Step 7: Put $x=Px_{2n}$ and $y=x_{2n+1}$ with $\alpha = 1$ from equation (5), we get

$$M(P(Px_{2n}),Qx_{2n+1},kt) \geq \min\{M(AB(P)x_{2n},Qx_{2n+1},t),M(AB(P)x_{2n},STx_{2n+1},t),$$

$$M(AB(P)x_{2n},P(P)x_{2n},t),M(STx_{2n+1},Qx_{2n+1},t)\}$$

taking letting $n\to \infty$

$$M(Pz,z,kt)\geq\min\{M(ABz,z,t),M(ABz,z,t),M(ABz,Pz,t),M(z,z,t)\}$$

$$M(Pz,z,kt)\geq\min\{M(Pz,z,t),M(Pz,z,t),M(Pz,Pz,t),M(z,z,t)\}$$

$$M(Pz,z,kt)\geq M(Pz,z,t)$$

and

$$N(P(Px_{2n}),Qx_{2n+1},kt) \leq \max\{N(AB(P)x_{2n},Qx_{2n+1},t),N(AB(P)x_{2n},STx_{2n+1},t),$$

$$N(AB(P)x_{2n},P(P)x_{2n},t),N(STx_{2n+1},Q_{2n+1},t)\}$$

letting $n\to \infty$

$$N(Pz,z,kt)\leq\max\{N(ABz,z,t),N(ABz,z,t),N(ABz,Pz,t),N(z,z,t)\}$$

$$N(Pz,z,kt)\leq\max\{N(Pz,z,t),N(Pz,z,t),N(Pz,Pz,t),N(z,z,t)\}$$

$$N(Pz,z,kt)\leq N(Pz,z,t)$$

Therefore by using lemma 2.3, we get

$Pz=z$.

Hence $ABz=Pz=z$

Similarly we can apply step 4, to get $Bz=z$ and so

$Az=Bz=Pz=z$

further, using steps 5,6 and 7 we get

$Qz=STz=Sz=Tz=z$.

i.e. z is the common fixed point of the six self maps A,B,S,T,P and Q in this case also.

Uniqueness: Let u be another common fixed point of A, B, S, T, P and Q

Then $Au=Bu=Pu=Qu=Su=Tu=u$.

Put x=z and y=u and $\alpha = 1$ in (5), we get

M(Pz,Qu,kt)≥ min{M(ABz,Qu,t),M(ABz,STu,t),M(ABz,Pz,t),M(STu,Qu,t)}

M(z,u,t)≥min{M(z,u,t),M(z,u,t),M(z,z,t),M(u,u,t)}

M(z,u,t)≥M(z,u,t)

and

N(Pz,Qu,kt)≤ max{N(ABz,Qu,t),N(ABz,STu,t),N(ABz,Pz,t),N(STu,Qu,t)}

N(z,u,t)≤max{N(z,u,t),N(z,u,t),N(z,z,t),N(u,u,t)}

N(z,u,t)≤N(z,u,t)

Therefore by lemma 2.3 we get z=u
Hence z is a unique common fixed point of A, B, S, T, P and Q.

Remark 3.1 If we take B=T=I, the identity map on X in theorem 3.1, then condition (2) is satisfied trivially and we get

Corollary 3.1 Let (X, M, N,*, ◊) be a complete intuitionistic fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied

1) P(X)⊂ S(X), Q(X) ⊂A(X).
2) Either P or A is continuous.
3) (P, A) is compatible of type (β) and (Q, S) is semi-compatible.
4) There exists k∈(0,1) such that for every x, y∈X,$\alpha \in (0,2)$ and t>0

M(Px, Qy, kt) ≥min{M(Ax,Qy,(2-α)t),M(Ax,Sy, t),M (Ax,Px,t),M(Sy,Qy,t)}

and

N(Px,Qy,kt) ≤max {N(Ax,Qy,(2-α)t),N(Ax,Sy, t), N (Ax,Px,t),N(Sy,Qy,t)}

Then A, P, Q and S have a unique common fixed point in X.

Conclusion:-In view of remark 3.1,corollary 3.1 is a generalization of the result of Cho[3] in the sense that condition of compatibility of the pairs of self maps in intuitionistic fuzzy metric space has been restricted to compatibility of type $(\beta)$and semi compatibility and only one map of the first pair is needed to be continuous.

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