Peak Load Pricing: An Illustration through Kuhn-Tucker

Harmeet Singh Gulati, Deepinder Kaur

Assistant Professor, Economics Department, Delhi University
Assistant Professor, Mathematics Department, Delhi University

I. ABSTRACT

"Peak Load Pricing" is a pricing strategy wherein the high price is charged for the goods and services during times when demand is at peak.

This type of price discrimination is based on the efficiency i.e. a Firm discriminates on the basic of high usage, high-traffic, high demand times and low demand times. The consumer who purchases the commodity during the demand period has to pay more as compared to the one who buys during the low demand periods.

II. KEYWORDS

Peak Load Pricing, Real Life Relation, Kuhn Tucker, Profit Maximization

III. INTRODUCTION WITH REAL LIFE EXAMPLES

(1) Theatres that offered shown in the evening (peak) charges higher price of the movie compared to shown offered in matinees (off peak).
(2) Charges made by health and sports club higher at (weekend and in evening) which is considered to be a peak period.
(3) Telephone call charges are higher during business hours compared to non business hours (evening and weekend).
(4) Cabs apply peak/ surge charges during peak hours.

IV. MODEL

Consider a profit-maximizing company that faces two demand curves

$P_1 = P^1(Q_1)$ in the day times (peak period)

$P_2 = P^2(Q_2)$ in the night time (off peak period)

Let "K" denote the total of capacity in each period i.e. both "peak period and "non peak period" which in measured in terms of Q.

To operate firm must pay "C" per unit of variable cost whether in day or night.

Furthermore, the firm must purchase capacity at a cost of "r" per unit if capacity.

* Now who should be charge for the capacity costs, peak, off peak, or both sets of customer's?

Ans. It depends on wether the "off-peak" constraint in binding or nonbinding,

(a) If "off-peak" constraint is non binding i.e. $(Q_1 = K > Q_2)$ then whole capacity per unit cost in borne by "peak period" consumers.
(b) If "off-peak'' constraint is binding i.e. $(Q_1 = Q_2 = K)$ then "capacity cost" would be shared between "peak" period "consumers and off-peak" period consumers.

Firm's maximization problem becomes

$$\max \pi_1 = P^1(Q_1)Q_1 + P^2(Q_2)Q_2 - c(Q_1+Q_2) - rk$$

subject to
$$Q_1 \le K$$
$$Q_2 \le K$$
$$P_1 = P^1(Q_1)$$
$$P_2 = P^2(Q_2)$$
and $Q_1, Q_2, K \ge 0$

Now $P^1(Q_1)Q_1 = R_1(Q_1), P^2(Q_2)Q_2 = R_2(Q_2)$

maximize $\pi = R_1(Q_1) + R_2(Q_2) - c(Q_1+Q_2)K$

subject to
$$Q_1 \le K$$
$$Q_2 \le K$$
and $Q_1, Q_2, K \ge 0$

I have applied "Kuhn-Tucker" because "off-period" demand could be either "Binding" or "Non Binding" i.e. "$Q_2 = K$" or "$Q_2 < K$" respectively applying while language "optimization would lead to "Binding" results i.e. "$Q_2 = K$" which is not valid in every situation.

Hence we will apply "Kuhn-Tucker" for optimization.

The langrangian function is

$$f = R_1(Q_1) + R_2(Q_2) - c(Q_1+Q_2) - rk + \lambda_1[K - Q_1] + \lambda_2[K - Q_2]$$

"Kuhn-Tucker" conditions are:

(1)
$$\frac{\partial f}{\partial Q_1} = MR_1 - c - \lambda_1 \le 0, Q_1 \ge 0, Q_1 \frac{\partial f}{\partial Q_1} = 0$$

(2)
$$\frac{\partial f}{\partial Q_2} = MR_2 - c - \lambda_2 \le 0, Q_2 \ge 0, Q_2 \frac{\partial f}{\partial Q_2} = 0$$

(3) $$\frac{\partial f}{\partial K} = -r + \lambda_1 + \lambda_2 \le 0,\ K \ge 0,\ K\frac{\partial f}{\partial K} = 0$$

(4) $$\frac{\partial f}{\partial \lambda_1} = K - Q_1 \ge 0,\ \lambda_1 \ge 0,\ \lambda_1 \frac{\partial f}{\partial \lambda_1} = 0$$

(5) $\frac{\partial f}{\partial \lambda_2} = K - Q_2 \geq 0,\ \lambda_2 \geq 0,\ \lambda_2 \frac{\partial f}{\partial \lambda_2} = 0$

Now we assume that $Q_1 < 0,\ Q_2 > 0,\ K > 0$.

So by (1), (2) and (3)

$$\frac{\partial f}{\partial Q_1} = MR_1 - c - \lambda_1 = 0 \Rightarrow \boxed{MR_1 = c + \lambda_1} \qquad (6)$$

$$\frac{\partial f}{\partial Q_1} = MR_2 - c - \lambda_2 = 0 \Rightarrow \boxed{MR_2 = c + \lambda_2} \qquad (7)$$

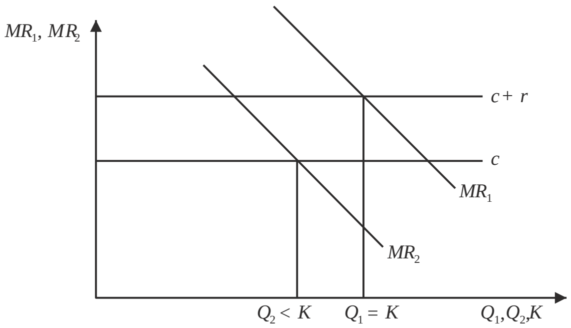
$$\frac{\partial f}{\partial K} = -r + \lambda_1 + \lambda_2 = 0 \Rightarrow \boxed{\lambda_1 + \lambda_2 = r} \qquad (8)$$

Case I. "Off peak" demand is "Non-Binding" i.e. "$Q_2 < K$" and "$Q_1 = K$"

Now "$Q_2 < K$" from (5) implies $\lambda_2 = 0$.

So putting values of $\lambda_2 = 0$ in (6), (7) and (8).

So $MR_2 = c,\ \lambda_1 = r,\ MR_1 = c + r$.



In the above case all "capacity cost" of "*r*" per unit is borne by "peak" period customers.

Which is given by "$MR_1 = c_1 + r$".

Case II. "off-peak" constraint is binding i.e .

$Q_1 = K$ and $Q_1 = K$ implies $Q_1 = Q_2 = K$.

$$\boxed{MR_1 = C + \lambda_1} \qquad (6)$$

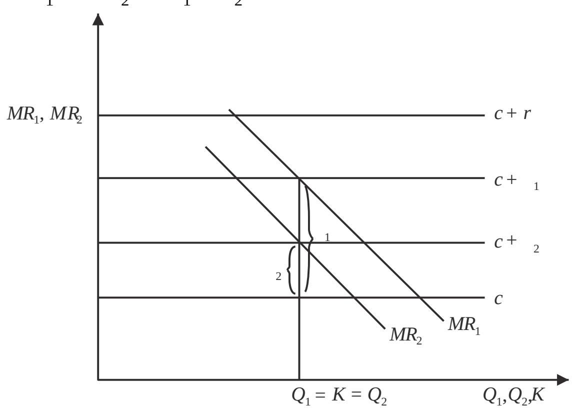
$$\boxed{MR_2 = C + \lambda_2} \qquad (7)$$

$$\boxed{r = \lambda_1 + \lambda_2} \qquad (8)$$

Now we know that "peak" demand curve in higher than "off peak" demand curve

So

$MR_1 > MR_2 \Rightarrow \lambda_1 > \lambda_2$



From above diagram at "$MR_2 = C$", $Q_2 > K$ which implies a "binding constraint".

Hence solution to above problem would be $Q_1 = Q_2 = K$

and "$\lambda_2 > 0$" because of " binding constraint"

at "$\lambda_2 = 0$", $Q_2 > K$ which in not possible

So $\lambda_2 > 0$ implies $\Rightarrow Q_2 = K$ by (5) and since $\lambda_1 > \lambda_2 > 0$ implies $Q_1 = K$ by (4).

Hence solution is $\boxed{Q_1 = Q_2 = K}$.

Here "λ_1" in the proportion out of "*r*" per unit capacity cost which in borne by "peak period" consumers, and

"λ_2" is the proportion out of "*r*" per unit capacity cost which is borne by "off peak period" consumers.

(Illustration) by examples.

Q. Suppose an electric company has a plan to setup a power plan in a country and it has to plan its capacity. Peak-period demand curve is given by $P_1 = 400 - Q_1$. Off peak-period demand curve is given by $P_1 = 380 - Q_2$.

Variable cost is 20 per unit (paid in both markets) and capacity costs given by 10 per unit which is paid only once and is used is both periods.

Solution: $C = 20, r = 10$

$$MR_1 = C + \lambda_1$$

$$MR_2 = C + \lambda_2$$

$$\lambda_1 + \lambda_2 = r$$

Case I. "off peak" constraint non binding

i.e. $[Q_2 < K \Rightarrow \lambda_2 = 0]$ and $Q_1 = K$

Hence

$$MR_1 = c + r,\ MR_2 = c$$

$$P_1 = 400 - Q_1 \Rightarrow TR_1 = 400Q_1 - Q_1^2$$

$$MR_1 = 400 - 2Q_1$$

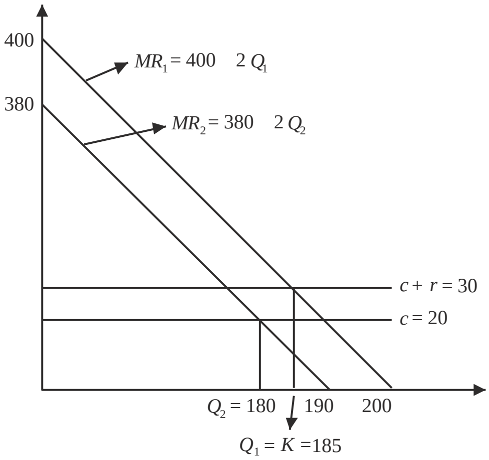
$$MR_1 = c + r \Rightarrow 400 - 2Q_1 = 30 \Rightarrow \boxed{Q_1 = 185}$$

$$P_2 = 380 - Q_2 \Rightarrow TR_2 = 380Q_2 - Q_2^2$$

$$MR_2 = c \Rightarrow 380 - 2Q_2 = 20 \Rightarrow Q_1 = 180$$

$$\boxed{Q_1 = K = 185},\ \boxed{Q_2 = 180 < K = 185}$$

which is the only solution.



Now suppose "capacity cost" has increased to "30 per unit". Then how will the quantities change?

Solution. $C = 20,\ r = 30$

$$MR_1 = c + \lambda_1$$
$$MR_2 = c + \lambda_2$$
$$\lambda_1 + \lambda_2 = r$$

Case I. "off peak" constraint is non binding i.e.

$Q_2 < K \Rightarrow \lambda_2 = 0]$ and $Q_1 = K$

$MR_1 = c + r,\ MR_2 = c$

$400 - 2Q = 50 \qquad 380 - 2Q_2 = 20$

$2Q_1 = 350 \qquad 2Q_2 = 360$

$Q_1 = 175 \qquad Q_2 = 180$

$Q_1 = 175 = K$

$Q_2 = 180 > K = 175$

Hence it is the case of "off-peak constraint's Binding.

So we wil proceed to (Case II)

(CaseII) " off-peak "constraint is a Binding constraint ie $Q_2 = K \,\&\, Q_1 = K \Rightarrow Q_1 = Q_2 = K$

$$MR_1 = c + \lambda_1$$
$$MR_2 = c + \lambda_2$$
$$\lambda_1 + \lambda_2 = r$$

Now "$\lambda_2 > 0$" since at "$\lambda_2 = 0$" $\Rightarrow$ "$Q_2 > K$" which is not possible and since $MR_1 > MR_2 \Rightarrow \lambda_1 > \lambda_2 > 0$

So $Q_1 = Q_2 = K$ has to be a solution by "Kuhn-tucker" conditions.

$$\boxed{MR_1 = c + \lambda_1}$$

$$400 - 2Q_1 = 20 + \lambda_1 \Rightarrow \boxed{\lambda_1 = 380 - 2K}$$

$$\boxed{MR_2 = c + \lambda_2} \Rightarrow 380 - 2Q_2 = 20 + \lambda_2$$

$$\Rightarrow \boxed{\lambda_2 = 360 - 2K}$$

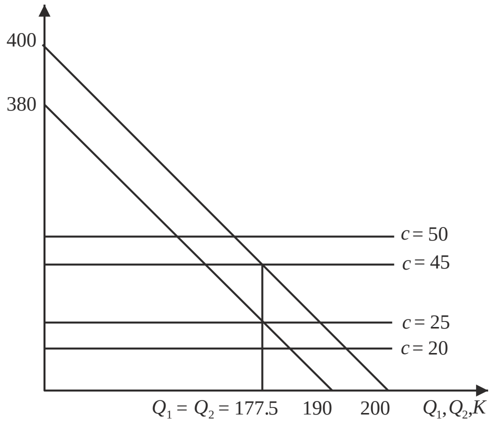
$$\lambda_1 + \lambda_2 = r \Rightarrow 380 - 2K + 360 - 2K = 30$$

$$\boxed{K = 1775} \qquad 710 = 4K$$

$$\boxed{\lambda_1 = 380 - 2(177.5) = 380 - 355 = 25}$$

$$\boxed{\lambda_2 = 360 - 2(177.5) = 360 - 355 = 5}$$

Hence out of "$r = 30$" per unit of capacity cost "$\lambda_1 = 25$" per unit is paid by "peak period" consumers and "$\lambda_2 = 5$" per cent capacity cost is paid by "off peak period" consumer.



CONCLUSIONS

1. In Case if off peak constraint is non binding then all burden of capacity cost is borne by peak period consumers.
2. If off peak constraint is binding, then capacity cost burden is shared with higher propotion shared by peak period consumers since they have higher willingness to pay(We have showed this Model using Kuhn Tucker Methodology)

REFRENCES

[1] Intermediate Micro Economics,A Modern Approach,HAL R.VARIAN,8th Edition.

[2] Linear Partial Differential Equations,Tyn Myint-U,Lokenath Debnath,4th Edition.

[3] Optimal Control Modelsi in Finance,Applied optimization,Ping Chen,Sardar M.N.Islam,Springers.

[4] Mathematics for Economics Analysis, KnutSydsaeter, Peter .J.Hammond, Pearson.