Radio Mean Square Labeling of Some Graphs

Dr.D.S.T.Ramesh[1], Dr.A.Subramanian[2], K.Sunitha[3]

1 . Associate Professor , Department of mathematics, Nazareth Margoschis College , Nasarath.
2. Associate Professor , Department of mathematics, M.D.T.Hindu College, Tirunelveli.
3. Assistant Professor, Department of mathematics, Women's Christian College, Nagercoil

Abstract

A radio labeling of a graph G is an injective function $f : V(G) \rightarrow N \cup \{o\}$ such that for every $u, v \in V$, $|f(u) - f(v)| \geq \text{diam}(G) + 1 - d(u,v)$. In this paper we investigate Radio mean square labeling of some graphs such as incentric Subdivision of spoke wheel graph and Bi-wheel graph **.**

Keywords : Radio labeling, Radio number, Radio Mean labeling , Radio mean square labeling.
AMS Subject Classification : 05C78

I. Introduction

By a graph G = (V(G), E(G)) with P vertices and q edges we mean a simple, connected and undirected graph. The distance between two vertices x and y of G is denoted by d(x,y) and diam(G) indicate the diameter of G. Radio labeling is motivated by the channel assignment problem introduced by Hale(1980). Chartrand et. al. investigated the upper bound for the radio number of path P_n. The exact value for the radio number of path was given by Liu and Zhu[4]. R. Ponraj and S. Kala[3] have introduced the notion of Radio Mean labeling of graphs. In this paper we investigate Radio mean square labeling of some graphs such as incentric Subdivision of spoke wheel graph and Bi-wheel graph.

Definition 1.1 [3] A Radio mean labeling is a one to one mapping $f : V(G) \rightarrow N$ satisfying the condition $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \text{diam}(G)$, for every $u,v \in V(G)$.

Definition1.2 A radio mean square labeling is a one to one mapping f from V(G) to N Satisfying the condition $d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{2} \right\rceil \geq 1 + \text{diam}(G)$, for every $u, v \in V(G)$. The radio mean square number of G is denoted by rmsn(G)

2.RadioMean Square Labeling of incentric Subdivision of spoke wheel graphs

Definition 2.1 Incentric subdivision is the graph obtained by inserting a vertex of degree two into every edge inside the original graph. If every edge inside a wheel graph is subdivided then the resulting graph is called incentric subdivision of spoke wheel graph.

Definition 2.2 The graph $SS(W_n)$ is obtained from the wheel W_n by subdividing each spokes by a vertex. Let $W_n = C_n + K_1$ where $C_n = v_1 v_2 . . . v_n v_1$ and $V(K_1) = \{u\}$ and the spokes are subdivided by the vertices $u_i (1 \leq i \leq n)$.

Note that the diameter of $SS(W_n)$ is 4.

Theorem 2.1 $\text{rmsn}(SS(W_n)) = 2n + 1,\ n \geq 8$

Proof Let u be the centre vertex. The cyclic vertices are $v_1, v_2, . . . v_n$ and the vertices of subdivision of spokes are $u_1, u_2, . . . u_n$.

Clearly $SS(W_n)$ has 2n+1 vertices and diam $(SS(W_n)) = 4$

Define the radio mean labeling $f : V \to N$ satisfying the condition

$d(u, v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{2} \right\rceil \geq 1 + \text{diam}\,(SS(W_n)) = 5$ for every pair of distinct vertices u and v.

Consider the rim vertices v_i $(1 \leq i \leq n)$

Define the radio mean labeling $f : V \to N$ by

$$f(v_i) = \begin{cases} 1 & \text{if} \quad i = 1 \\ n - i & \text{if} \quad i = 2 \\ n - i + 2 & \text{if} \quad i = 3 \\ 2 & \text{if} \quad i = 4 \\ n & \text{if} \quad i = 5 \\ i - 3 & \text{if} \quad i = 6, 7, \ldots n \end{cases}$$

$f(u_i) = n + i,\ i = 1, 2, \ldots, n$ and $f(u) = 2n + 1$

Now we check the radio mean square condition for the above labeling f.

Case 1 Consider the pair (u_i, u_j)

Subcase a Verify the pair (v_1, v_i) .

It is easy to verify that the pairs (v_1, v_i), $i = 4,\ 6,\ 7,\ 8$

For $i \notin \{4,\ 6,\ 7,\ 8\}$ we have

$$d(v_1, v_i) + \left\lceil \frac{(f(v_1))^2 + (f(v_i))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{1 + 36}{2} \right\rceil \geq 20.$$

Subcase b Verify the pair (v_4, v_i)

The pairs (v_4, v_6), (v_4, v_7) satisfy the radio mean square condition.

For $i \neq 1,\ 6,\ 7$, $d(v_4, v_i) + \left\lceil \frac{(f(v_4))^2 + (f(v_i))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{4 + 25}{2} \right\rceil \geq 16.$

Subcase c Check the pair (v_i, v_j), $i,\ j \notin \{1,\ 4\}$

$$d(v_i, v_j) + \left\lceil \frac{(f(v_i))^2 + (f(v_j))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{9 + 16}{2} \right\rceil \geq 14$$

Case 2 Check the pair (u_i, u_j)

$$d(u_i, u_j) + \left\lceil \frac{(f(u_i))^2 + (f(u_j))^2}{2} \right\rceil \geq 2 + \left\lceil \frac{(n+1)^2 + (n+2)^2}{2} \right\rceil \geq 93$$

Case 3 Check the pair (v_i, u_j)

$$d(v_i,u_j) + \left\lceil \frac{(f(v_i))^2 + (f(u_j))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{1 + (n+1)^2}{2} \right\rceil \geq 42$$

Case 4 Examine the pair (u, u_i)

$$d(u,u_i) + \left\lceil \frac{(f(u))^2 + (f(u_i))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{(2n+1)^2 + (n+1)^2}{2} \right\rceil \geq 186$$

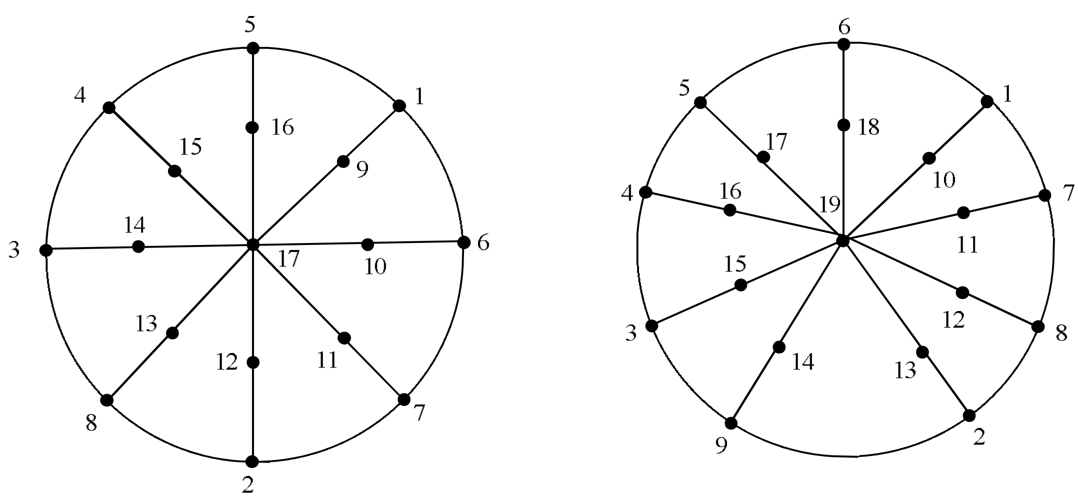
These cases establish the radio mean square condition.

Hence $rmsn(SS(Wn)) \leq 2n + 1$.

Since the number of vertices is 2n+1 and the labels are unique, it follows that $rmsn(SS(Wn)) \geq 2n + 1$.

Hence $rmsn(SS(Wn)) = 2n + 1$

Example 2.1 The radio mean square number of some incentric subdivision of spoke wheel graphs are given in Fig1



(a) $rmsn(SS(W_8)) = 17$ (b) $rmsn(SS(W_9)) = 19$

Figure 1

3. Radio Mean square Labeling of Bi-wheel graph

Definition 3.1 The graph $W_{m,n}$ is obtained from the wheels W_m and W_n by joining the rim vertices of the two wheels with an edge. Let $W_m = C_m + K_1$ and $W_n = C_n + K_1$ where C_m is the cycle $u_1 u_2 \ldots u_m u_1$ and C_n is the cycle

$v_1 v_2 \ldots v_n v_1$ .Let $V(W_m) = V(C_m) \cup \{u_1\}$, $V(W_n) = V(C_n) \cup \{v_1\}$ and

$E(W_{m,n}) = E(W_m) \cup E(W_n) \cup \{u_1 v_1\}$.

Definition 3.2 Bi-wheel graph $BW_{m,n}$ [6] consist of two disjoint copies of wheel which are joined by an edge between two rim vertices.

Theorem 3.1 $rmsn(W_{n,n}) = 2n + 2, \; n > 3$

Proof: Clearly there are 2n+2 vertices and $diam(BW_{n,n}) = 5$.

Define $f : V \rightarrow N$ and satisfies the radio mean square condition

$$d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{2} \right\rceil \geq 1 + diam(BW_{n,n}) = 6$$

Let v be the central vertex of second wheel W_n.

The vertices adjacent to v are labeled sequentially by $\{v_1, v_2, \ldots, v_n\}$ in counter clockwise direction.

Let u be the central vertex of first wheel W_n.

The vertices adjacent to u are labeled sequentially by $\{u_1, u_2, \ldots, u_n\}$in clockwise direction respectively.

First consider the vertices of W_n. Define $f : V \rightarrow N$ by

$$f(v_i) = \begin{cases} 3 & \text{if} \quad i = 1 \\ n + i & \text{if} \quad i = 2 \\ 2 & \text{if} \quad i = 3 \\ i + 1 & \text{if} \quad 4 \leq i \leq n \end{cases} \qquad , \; f(v) = n + 3$$

$$f(u_i) = \begin{cases} 4 & \text{if} \quad i = 1 \\ 2n + i - 1 & \text{if} \quad i = 2 \\ 1 & \text{if} \quad i = 3 \\ n + i & \text{if} \quad 4 \leq i \leq n \end{cases} \qquad , \; f(u) = 2n + 2$$

We check the radio mean square condition $d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{2} \right\rceil \geq 6$ holds for every pairs (u, v) with $u \neq v$.

Case a Examine the pair (u, u_i),

$$d(u, u_i) + \left\lceil \frac{(f(u))^2 + (f(u_i))^2}{2} \right\rceil \geq 1 + \left\lceil \frac{(n+4)^2 + 1}{2} \right\rceil \geq 34$$

Case b Examine the pair (u_i, u_j), Clearly (u_3, u_1) satisfies the radio mean square condition.

Subcase 1 Examine the pair (u_3, u_i), $i \neq 1$

$$d(u_3,u_i)+\left\lceil \frac{(f(u_3))^2+(f(u_i))^2}{2} \right\rceil \geq 1+\left\lceil \frac{1+(n+4)^2}{2} \right\rceil \geq 34$$

Subcase 2 Examine the pair (u_1, u_i), $i \neq 3$

$$d(u_1,u_i)+\left\lceil \frac{(f(u_1))^2+(f(u_i))^2}{2} \right\rceil \geq 1+\left\lceil \frac{16+(n+4)^2}{2} \right\rceil \geq 41$$

Subcase 3 Check the pair (u_i,u_j),$i, j \notin \{1, 3\}$

$$d(u_i,u_j)+\left\lceil \frac{(f(u_i))^2+(f(u_j))^2}{2} \right\rceil \geq 1+\left\lceil \frac{(n+4)^2+(n+5)^2}{2} \right\rceil \geq 74$$

Case c Consider the pair (u_i,v_j)

Subcase 1 Verify the pair (u_3,v_3) satisfies the radio mean square condition. So take $j \neq 3$.

$$d(u_3,v_j)+\left\lceil \frac{(f(u_3))^2+(f(v_j))^2}{2} \right\rceil \geq 3+\left\lceil \frac{1+9}{2} \right\rceil \geq 8$$

Subcase 2 Examine the pair (u_1, v_j)

Since $d(u_1,v_3) = 3$, (u_1,v_3) satisfies the radio mean square condition.

Consider (u_1,v_j) with $j \neq 3$

$$d(u_1,v_j)+\left\lceil \frac{(f(u_1))^2+(f(v_j))^2}{2} \right\rceil \geq 1+\left\lceil \frac{16+9}{2} \right\rceil \geq 13$$

Subcase 3 Check the pair (u_i,v_j), $i \neq 1,3$

$$d(u_i, v_j)+\left\lceil \frac{(f(u_i))^2+(f(v_j))^2}{2} \right\rceil \geq 2+\left\lceil \frac{(n+4)^2+9}{2} \right\rceil \geq 39$$

Case d Consider the pair (u,v_j),

Similar to subcase 3 of case c

Case e Consider the pair (u_i, v)

$$d(u_i,v)+\left\lceil \frac{(f(u_i))^2+(f(v))^2}{2} \right\rceil \geq 2+\left\lceil \frac{1+(n+3)^2}{2} \right\rceil \geq 27$$

Case f Check the pair (u,v)

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{2} \right\rceil \geq 3+\left\lceil \frac{(2n+2)^2+(n+3)^2}{2} \right\rceil \geq 78$$

Case g Verify the pair (v, v_i)

$$d(v,v_i)+\left\lceil \frac{(f(v))^2+(f(v_i))^2}{2} \right\rceil \geq 1+\left\lceil \frac{(n+3)^2+4}{2} \right\rceil \geq 28$$

Case h Examine the pair (v_i, v_j)

Subcase 1 Consider to check that the pair (v_1, v_3)

It is easy to check that the pair (v_1, v_3) satisfies the radio mean square condition.

Subcase 2 Verify the pair (v_3, v_j), $j \neq 1$

$$d(v_3,v_j)+\left\lceil \frac{(f(v_3))^2+(f(v_j))^2}{2} \right\rceil \geq 1+\left\lceil \frac{4+25}{2} \right\rceil \geq 16$$

Subcase 3 Examine the pair (v_1, v_j), $j \neq 3$

$$d(v_1,v_j)+\left\lceil \frac{(f(v_1))^2+(f(v_j))^2}{2} \right\rceil \geq 1+\left\lceil \frac{9+25}{2} \right\rceil \geq 18$$

Subcase 4 Check the pair (v_i, v_j), $i \neq 1, 3$ and $j \neq 1, 3$

$$d(v_i,v_j)+\left\lceil \frac{(f(v_i))^2+(f(v_j))^2}{2} \right\rceil \geq 1+\left\lceil \frac{25+36}{2} \right\rceil \geq 32$$

Hence these cases establishes the radio mean square condition.

Therefore $rmsn(BW_{n,n}) \leq 2n+2$

Since $diam(BW_{n,n}) = 5$, it follows that 1 and 2 cannot be labels of the same wheel.

This implies $rmsn(BW_{n,n}) > 2n+1$

But $rmsn(BW_{n,n}) \leq 2n+2$

Hence $rmsn(BW_{n,n}) = 2n+2$

Example 3.1 For the graph $BW_{11,11}$ in Fig 2 , $rmsn(BW_{11,11}) = 24$

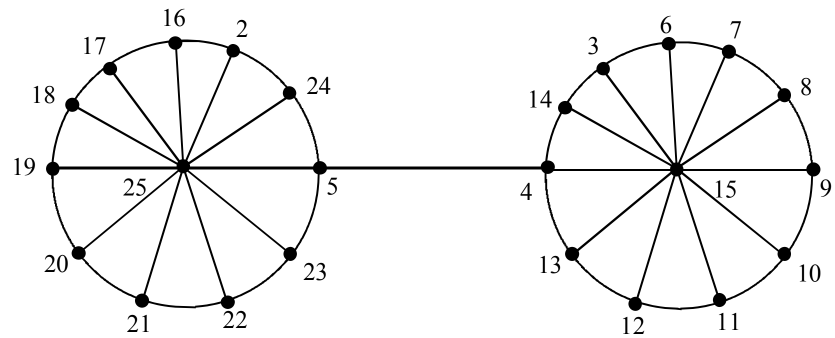


Fig 2

Observation $rmsn(BW_{n,n}) = rmsn(BW_{n-1,n-1}) + 2$

References

[1] J.A. Gallian, A dynamic survey of labeling, The Electronics Journal of Combinatorics (2014)

[2] J.A. Bondy and U.S.R Murthy, Graph Theory and Applications, Newyor (1976).

[3] R.Ponraj, S.Sathish Narayanan and R.Kala,Radio mean number of some wheel related graphs, Jordan Journal of mathematics and statistics (JJMS) 7(4),2014,PP.273-286.

[4] D. Liu and X. Zhu, Multilevel distance labelings for paths and cycles, SIAM Journal on discrete mathematics, Vol. 19, no.3, pp 281- 293, 2005.

[5] D.S.T.Ramesh ,A.Subramanian and K.Sunitha ,Radio odd mean and even mean labeling of some graphs of diameter 4, Communicated to International Journal of Mathematics and Soft computing .

[6] A.A.Bhatti,Aster Nisar and Maria Kanwal,Radio number of wheel like graphs, International journal of graph theory in wireless ad hoc networks and sensor networks (GRAPH-HOC) Vol.3, No.4, December 2011

[7] D.S.T.Ramesh and K.Sunitha ,Radio labeling for Barycentric Subdivision of Path and Incentric Subdivision of Spokewheel graphs,International Journal of Mathematical Archive -6(6),2015,187-196