Application of Generalized Kudryashov Method to the Burger Equation

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Abstract- *The paper considers the Burger equation to search new solutions win the help of generalized Kudryashov method. As a result we obtained exponential type solution involving kink soliton, singular kink soliton and multi soliton solutions with some free parameters. It has been shown that the method provides a powerful mathematical tool for solving non-linear wave equations in mathematical physics and engineering problems.*

Keywords: *The Burger equation; generalized Kudryashov method; travelling wave solution; non-linear evolution equations*

I. INTRODUCTION

Non-Linear Evolution Equations (NLEEs) are widely used to describe many important phenomena and dynamical processes in mathematical physics and engineering. The investigation of exact solutions of NLEEs plays an important role in the study of nonlinear physical phenomena. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Hirota's bilinear transformation method [1, 2], the tanh-function method [3], the extended tanh-method [4], the Exp-function method [5], the Adomian decomposition method [6], the auxiliary equation method [7], the Jacobi elliptic function method [8], the (G'/G)-expansion method [9], the Modified simple equation method [10] and so on.

In this paper, we pay attention to an analytical method named generalized Kudryashov method [11] for finding the exact solutions to the nonlinear Burger equation.

II. THE GENERALIZED KUDRYASHOV METHOD

In this section, we will describe the algorithm of the generalized Kudryashov method for finding travelling wave solutions of non-linear evolution equations. Consider a nonlinear equation, say in two independent variables x and t is given by:

$$P(u,u_t,u_x,u_{xx},u_{tt},u_{xt},\Lambda \quad \Lambda)=0. \qquad (1)$$

where $u(x,t)$ is an unknown function, P is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and non-linear terms are involved. In the following, we give the main steps of this method [11]:

Step 1: Combining the independent variables x and t into one variable

$$u(x,t)=u(\xi); \quad \xi = x-\omega t, \qquad (2)$$

where ω be the speed of the wave.

The travelling wave transformation equation (2) permits us to transform equation (1) to the following ordinary differential equation (ODE):

$$Q(u,u',u'',\Lambda \quad \Lambda)=0. \qquad (3)$$

where Q is a polynomial in $u(\xi)$ and its derivatives with respect to ξ.

***Step 2*:** We suppose that equation (3) has the formal solution

$$u=\frac{\sum_{i=0}^{M} A_i\varphi^i(\xi)}{\sum_{j=0}^{N} B_j\varphi^j(\xi)}, \qquad (4)$$

where $A_i(0\le i\le M)$ and $B_j(0\le j\le N)$ are constants to be determined, such that $A_M\neq 0$, $B_N\neq 0$ and $\varphi(\xi)$ satisfies the following ODE:

$$\varphi'(\xi)=\varphi^2(\xi)-\varphi(\xi). \qquad (5)$$

Equation (5) gives the following solution:

$$\varphi(\xi)=\frac{1}{1+C\exp(\xi)}. \qquad (6)$$

where C is an arbitrary constant.

The positive integer M, N can be determined by considering the homogeneous balance between the highest order derivatives and the non-linear terms appearing in equation (3).

Step 3: Inserting equation (4) into (3) and then we account the function $\varphi(\xi)$. As a result of this substitution, we get a polynomial of $\varphi(\xi)$. We equate all the coefficients of same power of $\varphi(\xi)$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_i(0\le i\le M)$, $B_j(0\le j\le N)$ and ω.

Substituting the values of $A_i(0 \le i \le M)$, $B_j(0 \le j \le N)$ and ω into equation (4) along with general solutions of equation (5) completes the determination of the solution of equation (1).

III. THE BURGER EQUATION

In this section, we will apply the generalized Kudryashov method to find the exact solutions `of Burger equation:

$$u_t + uu_x - \nu u_{xx} = 0 \qquad (7)$$

You might wonder why (7) is called the "viscous" rather than the "diffusive" Burgers equation. The answer is that in the simplest model of fluid flow, viscosity arises from diffusion between adjacent fluid elements. Thus the term u_{xx} can be thought of as representing diffusion or viscosity.

Now, using the traveling wave transformation Eq.(2) into the Eq.(7) convert to ODE:

$$-\omega u' + uu' - \nu u'' = 0. \qquad (8)$$

Integrating once we actually reach to

$$-\omega u + u^2/2 - \nu u' + k = 0. \qquad (9)$$

Taking homogeneous balance between the highest order derivative term u' and highest nonlinear term u^2 in the Eq.(9), we get the relation

$$M = N + 1.$$

When $N = 1$, then $M = 2$.

Thus the solution for the equation Eq.(9) takes the form

$$u = \frac{A_0 + A_1\varphi(\xi) + A_2\varphi^2(\xi)}{B_0 + B_1\varphi(\xi)}, \qquad (10)$$

where A_i and $B_i, i = 0,1$ are constants to be determined such that $A_2 \ne 0, B_1 \ne 0$.

Substituting equation Eq.(10) into equation Eq.(9) and then equating the coefficients of $\varphi^j(\xi)$ to zero, where $j \ge 0$, we obtain the following algebraic equations:

$$A_2^2 + 2\nu A_2 B_1 = 0 \qquad (11)$$

$$-2A_1A_2 + 2\omega A_2 B_1 - 2\nu A_2 B_1 + 4\nu A_2 B_0 = 0 \qquad (12)$$

$$-A_1^2 - 4\nu A_2 B_0 - 2kB_1^2 + 2\omega A_2 B_0 - 2A_0A_2 - 2\nu A_0 B_1 + 2\nu A_1 B_0 + 2\omega A_1 B_1 = 0 \qquad (13)$$

$$2\omega A_0 B_1 - 4kB_0B_1 + 2\omega A_1 B_0 - 2\nu A_1 B_0 - 2A_1A_0 + 2\nu A_0 B_1 = 0 \qquad (14)$$

$$2\omega A_0 B_0 - A_0^2 - 2kB_0^2 = 0 \qquad (15)$$

Solving equation (11)–(15) by using Maple, we find that solution of equation (9) exists only in the following two cases:

Set-1:

$$k = \frac{A_1^2 - 4\nu B_0 B_1 - 4\nu A_1 B_0 + 4\nu^2 B_0^2 + 2\nu A_1 B_1}{2B_1^2},$$

$$\omega = \frac{A_1 + \nu B_1 - 2\nu B_0}{B_1},\; A_0 = \frac{B_0(A_1 - 2\nu B_0)}{B_1},$$

$$A_1 = const.,\; A_2 = 2\nu B_1,\; B_0 = const., B_1 = const.$$

and

Set-2: $k = \dfrac{A_1^2 - 8\nu A_1 B_0}{8B_0^2},\; \omega = \dfrac{4\nu B_0 - A_1}{2B_0},$

$$A_0 = \frac{-A_1}{2}, A_1 = const.,\; A_2 = -4\nu B_0,$$

$$B_0 = const., B_1 = -2B_0.$$

Case 1: Exact travelling wave solutions of equation (9) which corresponds to Set 1 is given by

$$u = \frac{B_0(A_1 - 2\nu B_0)(1 + C\exp(\xi))^2 + A_1B_1(1 + C\exp(\xi)) + 2\nu B_1^2}{B_1\{B_1 + B_0(1 + C\exp(\xi))\}(1 + C\exp(\xi))}, \qquad (16)$$

(16)

where $\xi = x - \dfrac{A_1 + \nu B_1 - 2\nu B_0}{B_1}t$.

Case 2: Exact travelling wave solutions of equation (9) which corresponds to Set 2 is given by

$$u = \frac{2A_1(1 + C\exp(\xi)) - A_1(1 + C\exp(\xi))^2 - 8\nu B_0}{2B_0(C\exp(\xi) - 1)(1 + C\exp(\xi))}, \qquad (17)$$

where $\xi = x - \dfrac{4\nu B_0 - A_1}{2B_0}t$.

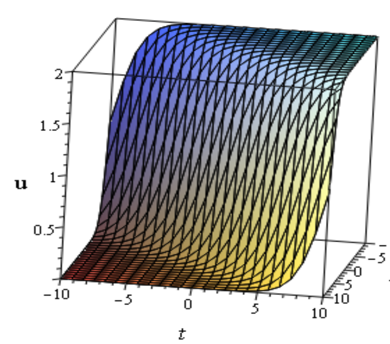

Fig-1: 3d Profile of Eq.(16) for the parameters $A_1 = B_1 = 2, B_0 = C = \nu = 1.$

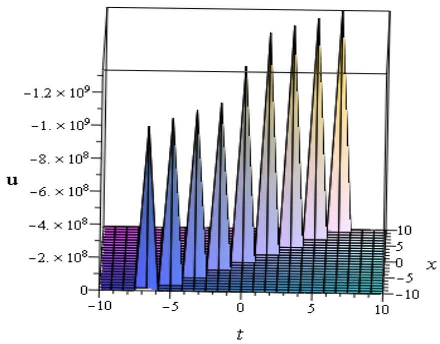

Fig-2: 3d Profile of Eq.(17) for the parameters $A_1 = B_0 = C = \nu = 1.$

IV. CONCLUSION

In this article, we successfully applied the generalized Kudryashov method to the Burgers equation. As a result, some new exact traveling wave solutions, so called kink soliton, singular kink soliton and multi soliton solutions with some free parameters are obtained. The method which we have used in this letter is standard, direct and computerized method which allows us to do complicated and tedious algebraic calculation. It is shown that the algorithm used in this paper can be also applied to other NLEEs in mathematical physics.

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