Analysis of Successive Occurrence of Digit 1 in Prime Numbers till 1 Trillion

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Abstract — *Primes less than 1 trillion are probed for successive occurrence of digit 1 in them. Multiple successive occurrences of 1's are analyzed. The first and last prime numbers containing successive repetitions of various number of 1's are determined within initial 12 ranges of increasing powers of 10.*

Keywords — *Prime numbers, digit 1, successive occurrences.*

Mathematics Subject Classification 2010 — *11Y35, 11Y60, 11Y99.*

I. INTRODUCTION

Owing to seemingly irregular spread of prime numbers amongst integers, relentless efforts are being taken to understand their distribution both theoretically [1] and within huge ranges [4].

The trends of general and successive occurrences of digit 1 in all natural numbers are extensively considered [11], [12], [13]. The presence of digit 1 in prime numbers is also examined [14]. General, successive and non-successive occurrences of digit 0 in in natural numbers [5], [6], [7] and in primes [8], [9], [10] are known.

Continuing the tradition of working in increasing ranges as high as till 1 trillion, in present work, successive 1's in digits of primes are studied.

II. OCCURRENCE OF SINGLE SUCCESSIVE DIGIT 1 IN PRIME NUMBERS

Single occurrence of 1 is considered trivially as successive. So, values determined in [14] for occurrence of single 1 in prime numbers are also of those of as occurrence of single successive 1 in them!

TABLE I : NUMBER OF PRIME NUMBERS IN VARIOUS RANGES WITH SINGLE 1 IN THEIR DIGITS

Sr. No.	Range	Number of Primes with Single Successive(!) 1
1.	$1 - 10^1$	0
2.	$1 - 10^2$	7
3.	$1 - 10^3$	56
4.	$1 - 10^4$	446
5.	$1 - 10^5$	3,650
6.	$1 - 10^6$	30,883
7.	$1 - 10^7$	265,086
8.	$1 - 10^8$	2,297,167
9.	$1 - 10^9$	20,051,508
10.	$1 - 10^{10}$	176,128,174
11.	$1 - 10^{11}$	1,553,607,785
12.	$1 - 10^{12}$	13,750,086,182

15 computers running a special programme for long durations have made these determinations possible using efficient algorithms [3].

III. OCCURRENCE OF MULTIPLE SUCCESSIVE 1'S IN PRIME NUMBERS

Successivity makes a sense when more terms are under consideration. Count of all numbers containing double, triple and higher number of successive 1's in them within the ranges $1 - 10^n$, $1 \le n \le 12$ has been determined [12]. Here the number of primes in these ranges containing multiple number of successive digit 1's is presented.

TABLE II : NUMBER OF PRIMES IN VARIOUS RANGES WITH MULTIPLE SUCCESSIVE 1'S IN THEIR DIGITS

Sr. No.	Number Range <	Number of Primes with				
		2 Successive 1's	3 Successive 1's	4 Successive 1's	5 Successive 1's	6 Successive 1's
1.	10^1	0	0	0	0	0
2.	10^2	1	0	0	0	0
3.	10^3	6	0	0	0	0
4.	10^4	42	4	0	0	0
5.	10^5	363	32	3	0	0
6.	10^6	3,036	299	20	5	0
7.	10^7	26,274	2,608	259	22	0
8.	10^8	230,728	22,873	2,226	224	11
9.	10^9	2,031,652	203,147	20,159	1,994	191
10.	10^{10}	17,975,560	1,818,731	181,910	17,958	1,777
11.	10^{11}	159,596,008	16,266,077	1,643,142	164,771	16,300
12.	10^{12}	1,420,502,386	145,766,730	14,849,571	1,501,908	150,477

TABLE II : CONTINUED ...

Sr. No.	Number Range <	Number of Primes with				
		7 Successive 1's	8 Successive 1's	9 Successive 1's	10 Successive 1's	11 Successive 1's
1.	10^1	0	0	0	0	0
2.	10^2	0	0	0	0	0
3.	10^3	0	0	0	0	0
4.	10^4	0	0	0	0	0
5.	10^5	0	0	0	0	0
6.	10^6	0	0	0	0	0
7.	10^7	0	0	0	0	0
8.	10^8	4	0	0	0	0
9.	10^9	18	1	0	0	0
10.	10^{10}	164	14	0	0	0
11.	10^{11}	1,535	141	12	2	0
12.	10^{12}	14,562	1,457	130	16	1

The number of primes with multiple successive 1's in these ranges of powers of 10 is graphically plotted with vertical axis on logarithmic scale and it shows parallel-like exponential curves.
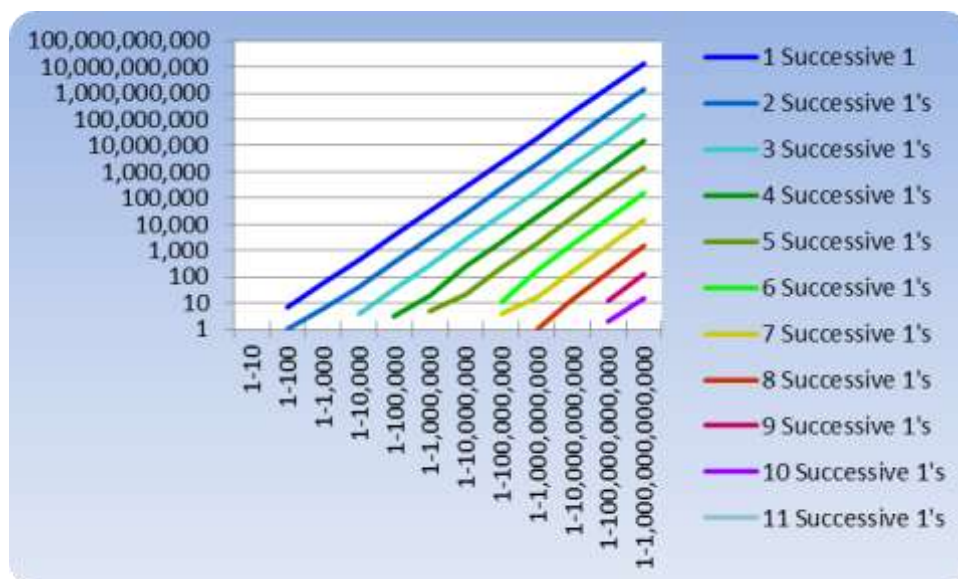

FIG. 1 : NUMBER OF PRIMES IN VARIOUS RANGES WITH MULTIPLE SUCCESSIVE 1'S IN THEIR DIGITS

Their percentage with respect to number of all positive integers with equal number of successive 1's in respective ranges is as follows.
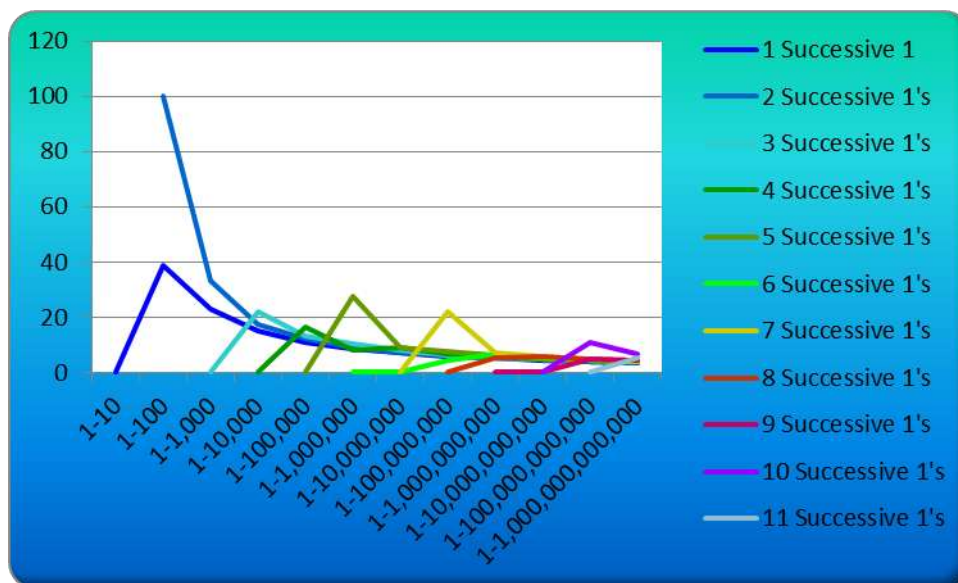
FIG. 2 : PERCENTAGE OF PRIMES IN VARIOUS RANGES WITH MULTIPLE SUCCESSIVE 1'S IN THEIR DIGITS WITH RESPECT TO ALL SUCH INTEGERS IN RESPECTIVE RANGES

Here are the graphs of differences of number of primes with successive 1's and number of primes with equal number of successive 0's in our ranges.
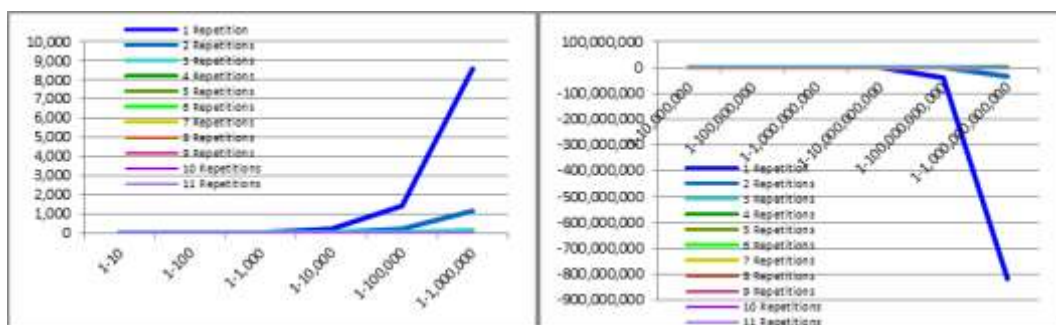

FIG. 3 : THE DIFFERENCE OF NUMBER OF PRIMES WITH MULTIPLE SUCCESSIVE 1'S THAN THOSE WITH MULTIPLE SUCCESSIVE 0'S

IV. FIRST OCCURRENCE OF SUCCESSIVE DIGIT 1'S IN PRIME NUMBERS

The first natural number containing 1 is clearly 1 itself. For higher ranges, first occurrence of 2 1's is in 11, that of 3 is in 111 and so on. In fact, the very first occurrence of multiple 1's happens to be that of successive 1's as given in

Formula 1 [12] : If n and r are natural numbers, then the first occurrence of r number of successive 1's in numbers in range $1 \leq m < 10^n$ is

$$ f = \begin{cases} - & , \text{ if } r > n \\ \sum_{j=0}^{r-1} 1 \times 10^j & , \text{ if } r \leq n \end{cases} $$

There is no formula available till now for such occurrences in prime numbers. These have been determined to be as follows.

TABLE III : FIRST PRIME NUMBERS IN VARIOUS RANGES WITH MULTIPLE SUCCESSIVE 1'S IN THEIR DIGITS

Sr. No.	Range	First Prime Number in Range with					
		1 Successive 1	2 Successive 1's	3 Successive 1's	4 Successive 1's	5 Successive 1's	6 Successive 1's
1.	$1 - 10^1$	-	-	-	-	-	-
2.	$1 - 10^2$	13	11	-	-	-	-
3.	$1 - 10^3$	13	11	-	-	-	-

Sr. No.	Range	First Prime Number in Range with					
		1 Successive 1	2 Successive 1's	3 Successive 1's	4 Successive 1's	5 Successive 1's	6 Successive 1's
4.	$1 - 10^4$	13	11	1,117	-	-	-
5.	$1 - 10^5$	13	11	1,117	11,113	-	-
6.	$1 - 10^6$	13	11	1,117	11,113	111,119	-
7.	$1 - 10^7$	13	11	1,117	11,113	111,119	-
8.	$1 - 10^8$	13	11	1,117	11,113	111,119	11,111,173
9.	$1 - 10^9$	13	11	1,117	11,113	111,119	11,111,173
10.	$1 - 10^{10}$	13	11	1,117	11,113	111,119	11,111,173
11.	$1 - 10^{11}$	13	11	1,117	11,113	111,119	11,111,173
12.	$1 - 10^{12}$	13	11	1,117	11,113	111,119	11,111,173

TABLE III : CONTINUED ...

Sr. No.	Range	First Prime Number in Range with				
		7 Successive 1's	8 Successive 1's	9 Successive 1's	10 Successive 1's	11 Successive 1's
1.	$1 - 10^1$	-	-	-	-	-
2.	$1 - 10^2$	-	-	-	-	-
3.	$1 - 10^3$	-	-	-	-	-
4.	$1 - 10^4$	-	-	-	-	-
5.	$1 - 10^5$	-	-	-	-	-
6.	$1 - 10^6$	-	-	-	-	-
7.	$1 - 10^7$	-	-	-	-	-
8.	$1 - 10^8$	11,111,117	-	-	-	-
9.	$1 - 10^9$	11,111,117	111,111,113	-	-	-
10.	$1 - 10^{10}$	11,111,117	111,111,113	-	-	-
11.	$1 - 10^{11}$	11,111,117	111,111,113	11,111,111,129	11,111,111,113	-
12.	$1 - 10^{12}$	11,111,117	111,111,113	11,111,111,129	11,111,111,113	311,111,111,111

This is notable that many of these occurrences are first occurrences of 1's as well as first occurrences of successive 1's in primes.

V. LAST OCCURRENCE OF SUCCESSIVE DIGIT 1'S IN PRIME NUMBERS

Formula for last occurrence of *r* number of successive 1's in natural numbers in ranges $1 - 10^n$, $1 \leq n \leq 12$, is

Formula 2 [12] : If *n* and *r* are natural numbers, then the last occurrence of *r* successive 1's in numbers in range $1 \leq m < 10^n$ is

$$l = \begin{cases} - & , \text{if } r > n \\ \sum_{j=0}^{r-1} 1 \times 10^j \ + \begin{cases} 0 & , \text{if } r = n \\ \sum_{j=r}^{n-1} 9 \times 10^j & , \text{if } r < n \end{cases} \end{cases}$$

The last occurrences of prime numbers with *r* number of successive 1's in them in these ranges have been determined to be as follows.

TABLE IV : LAST PRIME NUMBERS IN VARIOUS RANGES WITH MULTIPLE SUCCESSIVE 1'S IN THEIR DIGITS

Sr. No.	Number of Successive 1's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	-	71	991	9,941	99,991	999,961	9,999,991	99,999,971
2.	2	-	11	911	9,811	99,611	999,611	9,999,511	99,999,611
3.	3	-	-	-	8,111	95,111	998,111	9,986,111	99,998,111
4.	4	-	-	-	-	11,119	971,111	9,881,111	99,941,111
5.	5	-	-	-	-	-	911,111	9,511,111	99,111,119

Sr. No.	Number of Successive 1's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
6.	6	-	-	-	-	-	-	-	94,111,111
7.	7	-	-	-	-	-	-	-	71,111,111
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

TABLE IV : CONTINUED ...

Sr. No.	Number of Successive 1's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,761	9,999,999,881	99,999,999,871
2.	2	999,999,113	9,999,999,511	99,999,999,119
3.	3	999,995,111	9,999,994,111	99,999,979,111
4.	4	999,821,111	9,999,931,111	99,999,991,111
5.	5	999,911,111	9,998,311,111	99,999,611,111
6.	6	998,111,111	9,974,111,111	99,997,111,111
7.	7	991,111,111	9,991,111,111	99,961,111,111
8.	8	111,111,113	9,511,111,111	99,811,111,111
9.	9	-	-	95,111,111,111
10.	10	-	-	31,111,111,111
11.	11	-	-	-

The remark for general occurrences of 1's also applies to successive occurrences of 1's.

Remark : The maximum number of successive 1's in any prime number in the range 1 - 10^n, for $n > 1$, is at most n.

The numbers coming in all sections here form integer sequences and merit independent treatment.

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