Super Fibonacci Graceful Labeling Of Some Cycle Related Graphs

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ABSTRACT

The function $f:V(G) \longrightarrow \{0, F_1, F_2, \dots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Super Fibonacci graceful if the induced edge labeling $f^* : E(G) \longrightarrow \{F_1, F_2, F_3, \dots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective. In this paper an analysis is made on Friendship graph, Butterfly graph $B_{3,n}$, The graph $(K_4 - e)^n$and the graph G obtained by attaching a triangle at each pendant vertex of a Star graph,(3,n)-kite graph under Super Fibonacci graceful labeling .

AMS Classification 05C78

KEYWORDS

Friendship graph, Butterfly graph $B_{3,n}$, The graph $(K_4 - e)^n$, The graph G , Fibonacci graceful labeling and Super Fibonacci graceful labeling.

1. INTRODUCTION

In graph labeling vertices or edges or both assigned values subject to the conditions. The "graceful labeling" introduced by Rosa (1967)[7] . J.A. Gallian [4] studied a complete survey on graph labeling.David.W. and Anthony. E. Baraaukas [2] have investigated the cycle structure of Fibonacci graceful mapping. A Fibonacci graceful labeling and Super Fibonacci graceful labeling have been introduced by Kathiresan and Amutha [5] in 2006.S. Meena and P. Kavitha [6] proved that the butterfly graphs are Prime graphs. R. Uma and D.Amuthavalli [8] proved that some Star- related graphs under Fibonacci graceful. Dushyant Tanna [4] has proved Friendship graphs are Harmonious. Dr. M. Subbiah [9] was proved that windmill graphs are strong edge graceful.

2. DEFINITIONS

Definition 2.1.

If the vertices or edges or both assigned values subject to certain condition(s) then it is known as graph labeling.

Definition 2.2.

The function f is called a graceful labeling of agraph G if $f : V \longrightarrow \{0, 1, 2, \dots . . q\}$ is injective and the induced function $f^* : E \longrightarrow \{1, 2, 3, \dots q\}$ defined as $f^*(e = uv) = |f(u) - f(v)|$ is bijective.

Definition 2.3.

The function $f:V(G) \longrightarrow \{0,1,2 \dots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Fibonacci graceful if the induced edge labeling $f^* : E(G) \longrightarrow \{F_1, F_2, F_3, \dots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective.

Definition 2.4.

The function $f:V(G) \longrightarrow \{0, F_1, F_2, \dots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Super Fibonacci graceful if the induced edge labeling $f^* : E(G) \longrightarrow \{F_1, F_2, F_3, \dots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective.

Definition 2.4

A Friendship graph F_n is a graph which consists of n triangles with a common vertex.

Definition 2.5

Two cycles of the same order n sharing a common vertex with an arbitrary number m of pendant edges attached at the common vertex called butterfly graph $B_{n,m}$ where n, m are two positive integers.

Definition 2.6

The wind mill graphs $(K_m)^n$ to be the family of graphs consisting of n copies of K_m with a vertex in common.

Definition 2.7

The graph $(K_4 - e)^n$ is obtained from the windmill graph $(K_4)^n$, removing an edge in each K_4 .

Definition 2.8

A graph is obtained by attaching a triangle at each pendant vertex of the Star graphs.

Definition 2.9

The kite (m, n) graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge.

3.RESULTS

3.1 Theorem :

Friendship graphs are Super Fibonacci graceful

Proof:

Let F_n be the friendship graph. The order of F_n is $p = 2n + 1$ and the size of F_n is $q = 3n$. By the definition of F_n , the vertex set $V = \{u_1, u_2, \ldots, u_n, v_0, v_1, v_2, \ldots, v_n\}$. Let $u_1, u_2, \ldots, u_n$ be the second vertices of the triangles and let $v_1, v_2, \ldots, v_n$ be the third vertices of the triangles, and let the apex vertex v_0 be the first vertex of all the triangles. The edge set $E = \{e_i, e_{ii}, e^*\}$ where $e_i = (v_0, v_i)$, $e_{ii} = (v_i, u_i)$ and $e_i^* = (v_0, u_i)$.

Now let us define the function

$f: V \longrightarrow \{0, F_1, F_2, \ldots, F_q\}$ as follows

$$f(v_0) = 0$$

$$f(u_i) = F_{3i-1} \quad for\ i = 1,2,3, \ldots, n$$

$$f(v_i) = F_{3i} \quad for\ i = 1,2,3, \ldots, n$$

Then the above defined function f admits Super Fibonacci graceful labeling. Hence Friendship graphs are Super Fibonacci graceful. The Friendship graph F_n are shown in figure1.

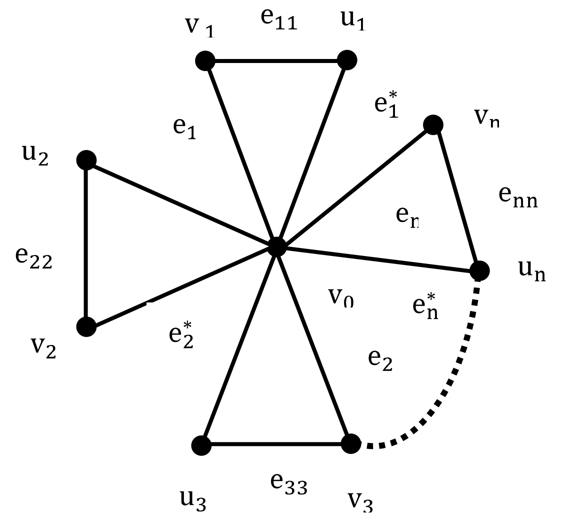


Figure 1. The Friendship graph F_n

3.2 Example:

The Friendship graph F_4 is shown in figure 2. In F_4 the order is $p = 9$ and the size is $q = 12$.

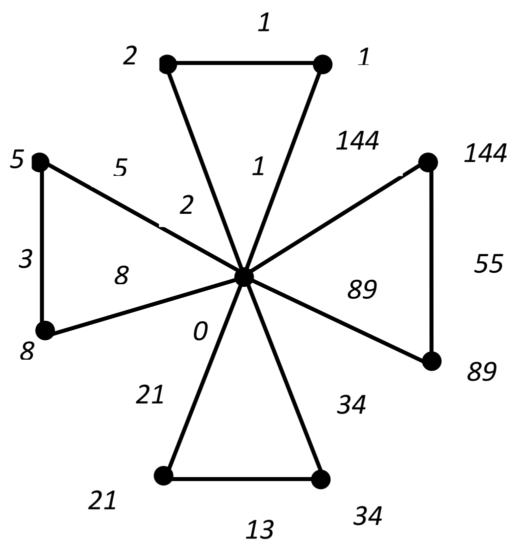


Figure 2. The Friendship graph F_4

3.3 Theorem :

The Butterfly graphs $B_{3,m}$ are Super Fibonacci graceful graphs.

Proof :

Let $B_{3,m}$ be the Butterfly graph. The order of the Butterfly graph is $p = 5 + m$ and the size of the Butterfly graph is $q = 6 + m$. By the definition of the Butterfly graph, the vertex set $V(G) = \{u_1, u_2, u_3, u_4, u_5, v_1, v_2, \dots, v_m\}$. Let u_1, u_2, u_3, u_4, u_5 be the vertices of the two cycles C_3 and u_1 be the apex vertex of the two cycles C_3. The edge set $E = \{e_i, e_{ij}\}$ where $e^* = (u_1, v_i)$and $e_{ij} = (u_i, u_j)$.

Now let us define the function

$f: V \longrightarrow \{0, F_1, F_2, \dots F_q\}$ as follows

$$f(u_1) = 0$$

$$f(u_i) = F_i \quad for\ i = 2,3$$

$$f(u_i) = F_{i+1} \quad for\ i = 4,5$$

$$f(v_i) = F_{i+6}\ for\ i = 1,2,\dots,m$$

Then the above defined function f admits Super Fibonacci graceful labeling.

Hence the Butterfly graphs are Super Fibonacci graceful .

The generalized graph $B_{3,m}$ is shown in figure 3.

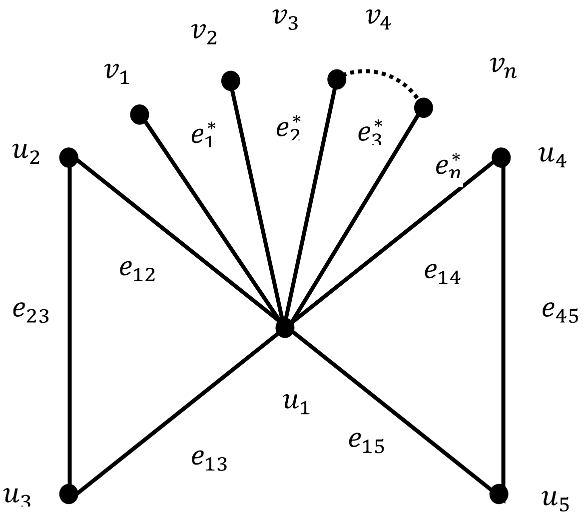


Figure 3. The Butterfly graph $B_{3,m}$

3.4 Example:

The Butterfly graph $B_{3,5}$ is shown in figure 4. The order and size of the graph is $p = 10$ and $q = 11$ respectively.

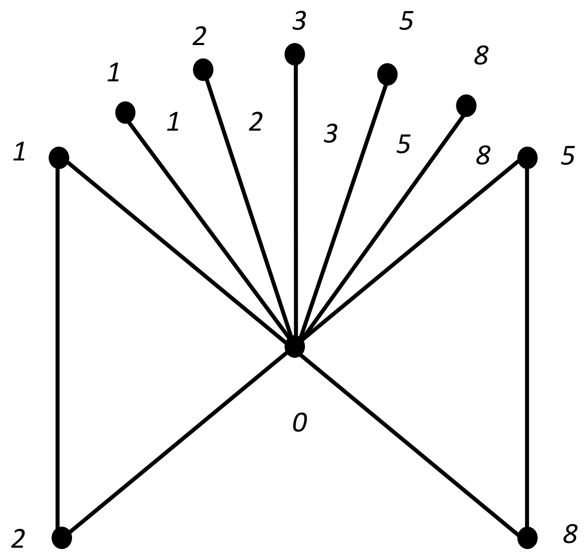


Figure 4. The Butterfly graph $B_{3,5}$

3.5 Theorem :

The graphs $(K_4 - e)^n$ are Super Fibonacci graceful.

Proof:

Let $G = (K_4 - e)^n$ be a graph . The order of the graph G is $p = 3n + 1$ and the size of the graph G is $q = 5n$. Thenthe vertex set $V = \{x, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$. Let $u_1, u_2, \dots u_n$ be the second vertices of the $(K_4 - e)^n$, let$v_1, v_2, \dots, v_n$ be the third vertices of the $(K_4 - e)^n$, let $w_1, w_2, \dots, w_n$ be the fourth vertices of the $(K_4 - e)^n$.And let the central vertex x be the first vertex of all the $(K_4 - e)^n$. The edge set $E = \{e_i, g_i, h_i, e_{ii}, e_{ii}^*\}$ where $e_i = (x, u_i)$, $e_{ii} = (u_i, v_i)$, $g_i = (x, v_i)$, $h_i = (x, w_i)$, $e_{ii}^* = (v_i, w_i)$.

Now let us defined the function

$f: V \longrightarrow \{0, F_1, F_2, \dots F_q\}$ as follows

$$f(x) = 0$$

$$f(u_{i+1}) = F_{5i+1} \quad for\ i = 0,1,2,3, \dots..n-1$$

$$f(v_i) = F_{5i-2} \quad for\ i = 1,2,3 \dots..n$$

$$f(v_i) = F_{5i} \quad for\ i = 1,2,3 \dots..n$$

Then the above defined function f admits Super Fibonacci graceful labeling.

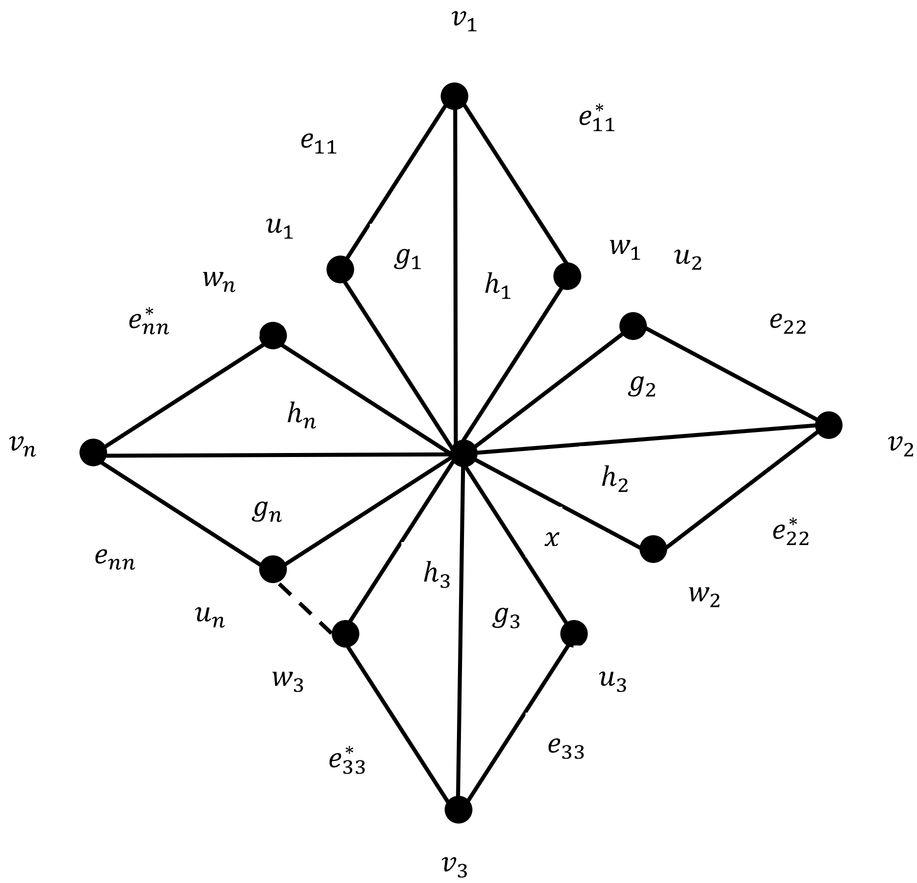


Figure 5. The graph $(K_4 - e)^n$

3.6 Example :

Consider the graph $(K_4 - e)^4$ is shown in figure 6 .The order and size of the graph $(K_4 - e)^4$ is $p = 13$ and $q = 20$ respectively.

Hence the graphs $(K_4 - e)^n$ are Super Fibonacci graceful.

The generalized graph $(K_4 - e)^n$is shown in figure5

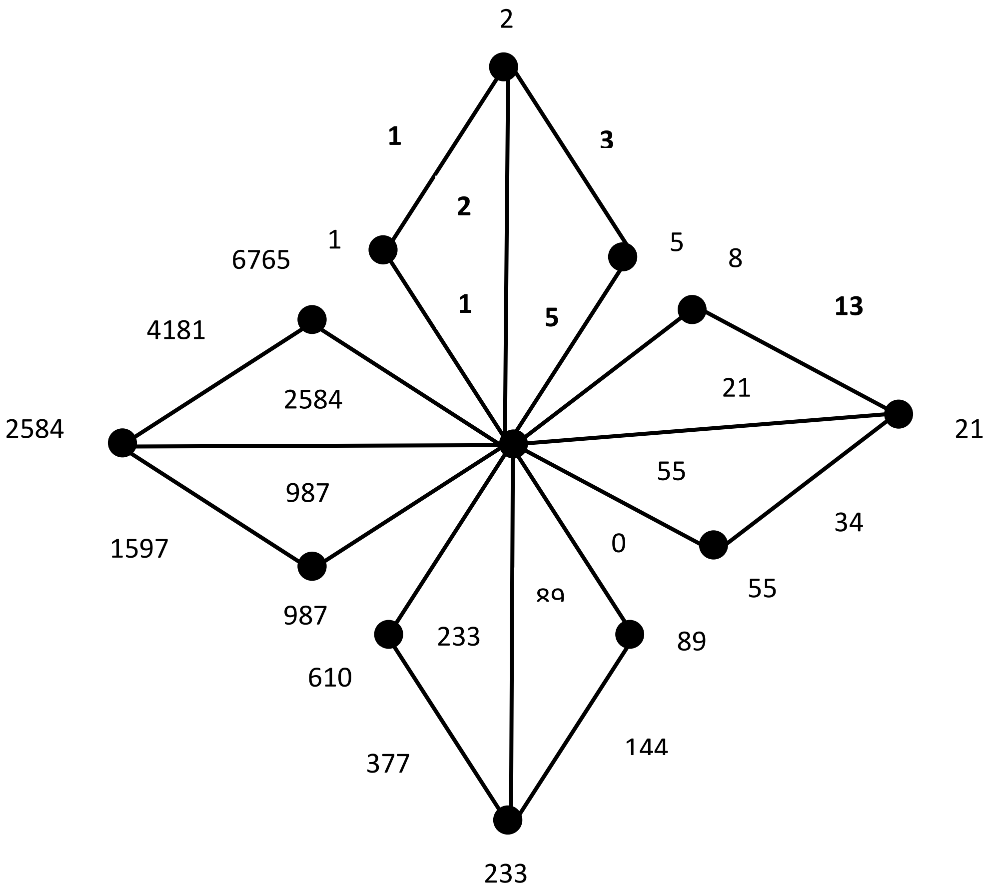


Figure 6. The graph $(K_4 - e)^3$

3.7 Theorem :

A graph G obtained by attaching a triangle at each pendant vertex of a Star graph is Super Fibonacci graceful labeling .

Proof :

Let $G = (V, E, f)$ be a graph . The order of the graph G is $p = 3n + 1$ and the size of the graph G is $q = 4n$. By the definition of the graph the vertex set $V = \{x, u_1, u_2, \ldots, u_{n,} v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$. Let $u_1, u_2, \ldots, u_n$ be the pendant vertices of the star and these are the first vertices of the triangles, let $v_1, v_2, \ldots, v_n$ be the second vertices of the triangles, Let$w_1, w_2, \ldots, w_n$ be the third vertices of the triangle. And let the central vertex x be the apex vertex of the star .

The edge set $E = \{e_i, e_{ii}, e_{ii}^*, e_{ii}^+\}$ where $e_i = (x, u_i)$, $e_{ii} = (u_i, v_i)$, $e_{ii}^+ = (v_i, w_i)$, $e_{ii}^* = (u_i, w_i)$.

Now let us defined the function

$f: V \longrightarrow \{0, F_1, F_2, \ldots F_q\}$ as follows

$$f(x) = 0$$

$$f(u_i) = F_{4i} \qquad for\ i = 1,2,3, \ldots, n$$

$$f(v_i) = F_{4i-1} \qquad for\ i = 1,2,3 \ldots n$$

$$f(w_i) = F_{4i-2} \qquad for\ i = 1,2,3, \ldots n$$

Then the above defined function f admits Super Fibonacci graceful labeling.

Hence the graphs G are Super Fibonacci graceful.

The generalized graph G is shown in figure 7.

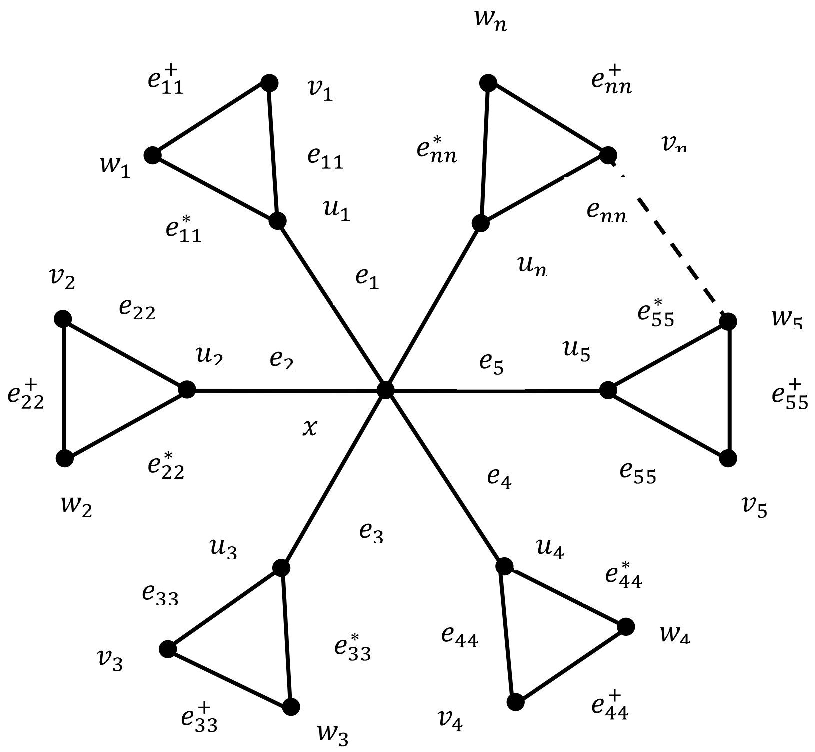


Figure 7 . The graph G

3.8 Example :

Consider the graph G is shown in figure 8.The order and size of the graph is $p = 13$ and $q = 16$ respectively.

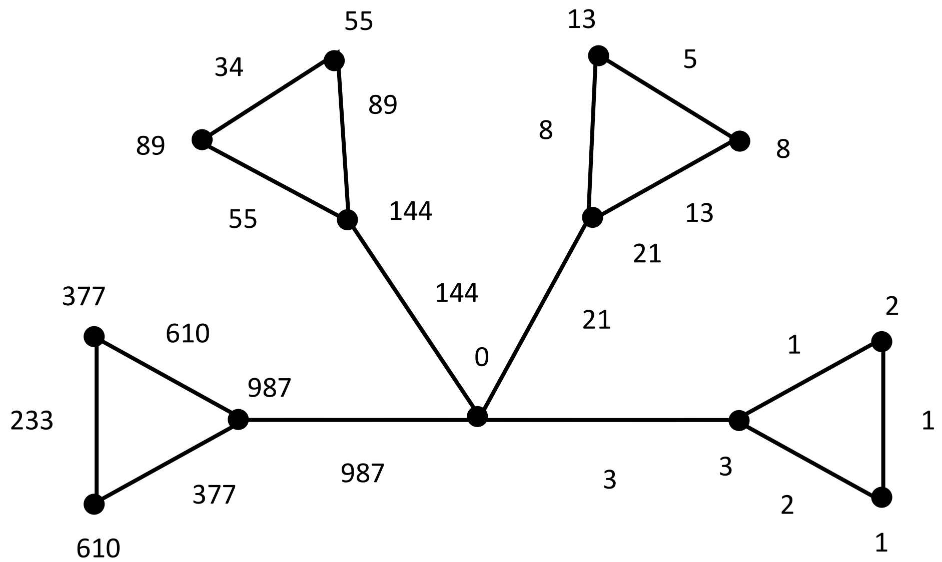


Figure 8 . The graph G

3.9 Theorem :

The Kite graphs $(3, n)$ are Super Fibonacci graceful graphs.

Proof:

Let G be a graph, the order of a graph G is $p = n + 2$ and the size $q = n + 3$. The vertex set $V(G) = \{u_1, u_2, u_3, v_1, v_2, \ldots v_n\}$ and the edge set $E(G) = \{e_1, e_2, \ldots e_{n+3}\}$ Let u_1, u_2, u_3 be the vertices of cycle C_3. And let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n.The edge set $E(G) = \{e_{ij}^*, e_{ij}, e_k\}$ where $e_{ij} = (v_i v_j)$ and $e_{ij}^* = (u_i, u_j)$ $e_k = (u_3 v_1)$.

Now let us define the function

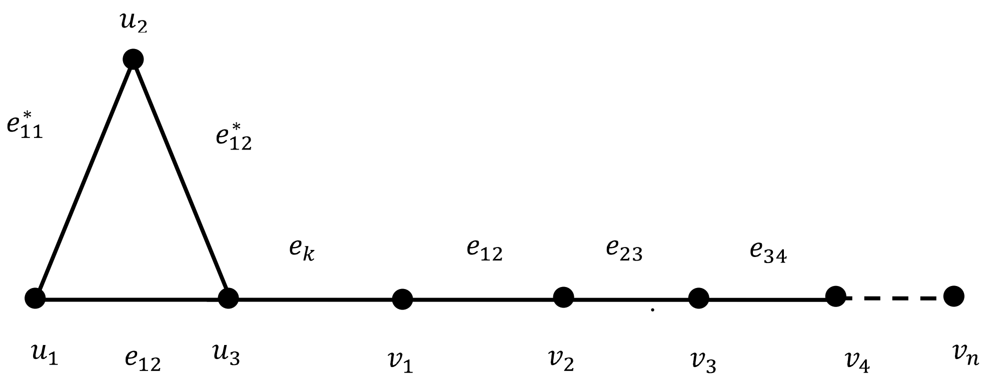
$f: V \longrightarrow \{0,1,2 \ldots\ldots F_q\}$ as follows

$f(u_1) = 0,$

$f(u_{i+1}) = F_{q-(i-1)}$ $\qquad i = 1,2$

$f(v_i) = F_{q-2-i}$ $\qquad i = 1,2, \ldots, n$

Then the above defined function f admits Super Fibonacci graceful labeling.
Hence $(3, n) - kite$ graphs are Super Fibonacci graceful graphs



3.8 Example :
Consider the graph $(3, n) - kite$ is shown in figure 10. The order and size of the graph is $p = 5$ and $q = 5$ respectively.

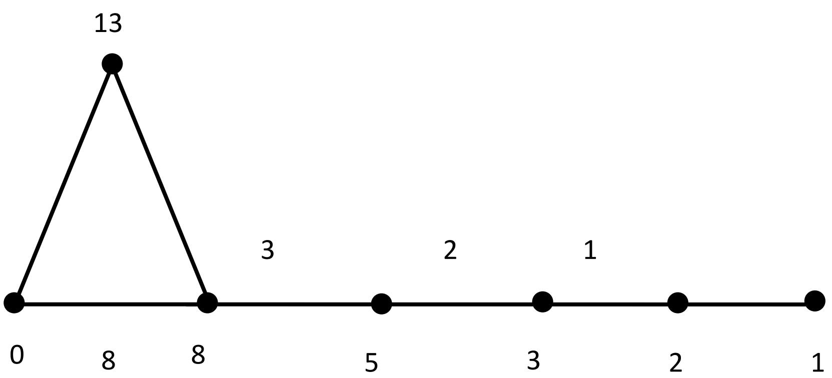


Figure 10

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