

Anti Q-fuzzy PMS- ideals in PMS-algebras

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Abstract: In this paper, we introduce the concept of Anti Q-fuzzy PMS-ideals of PMS-algebras, lower level cuts of a fuzzy set and proved some results. We discussed few results of anti Q-fuzzy PMS-ideals of PMS-algebras in homomorphism and Cartesian product.

Keywords: PMS-algebra, fuzzy PMS- ideal, Anti Q-fuzzy PMS-ideal, lower level cuts, homomorphism, Cartesian product.

1. Introduction

The concept of fuzzy set was introduced by L.A.Zadeh in 1965 [19]. Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. K.Iseki and S.Tanaka [2] introduced the concept of BCK-algebras in 1978 and K.Iseki [3] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK –algebras is a proper subclass of the class of BCI algebras in 2002. P.M.Sithar Selvam and K.T.Nagalakshmi [5,6] introduced the concept of PMS-algebras , which is a generalization of BCK / BCI / TM / KUS / PS algebras in 2015.R.Biswas[1] introduced the concept of Anti fuzzy subgroups of groups. Modifying his idea, in this paper we applied the idea in PMS-algebras. We introduced the notion of Anti Q- fuzzy PMS-ideals of PMS-algebras and investigate how to deal with the homomorphic, Anti homomorphic and inverse image of Anti Q-fuzzy PMS-ideals of PMS-algebras.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the development of this paper.

Definition 2.1 :[5,6] A nonempty set X with a constant 0 and a binary operation ‘ * ’ is called PMS – algebra if it satisfies the following axioms.

1. $0 * x = x$
2. $(y * x) * (z * x) = z * y, \forall x, y, z \in X.$

In X, we define a binary relation \leq by : $x \leq y$ if and only if $x * y = 0$.

Definition 2.2: [5,6] Let X be a PMS - algebra and I be a subset of X, then I is called a PMS - ideal of X if it satisfies the following conditions:

1. $0 \in I$
2. $z * y \in I$ and $z * x \in I \Rightarrow y * x \in I$ for all $x, y, z \in X$.

Example 2.3: [5,6] Let $X = \{0, a, b, c\}$ be the set with the following table.

*	0	a	b	c
0	0	a	b	c
a	b	0	a	b
b	a	b	0	c
c	c	c	a	0

Then $(X, *, 0)$ is a PMS – algebra and $I = \{0,a,b\}$ is a PMS-ideal.

Definition 2.4: [7,8] : Let S be a non empty subset of a PMS -algebra X , then S is called a PMS-sub algebra of X if $x * y \in S$,for all $x, y \in S$.

Definition 2.5 [17,19] : Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping

$$\mu : X \rightarrow [0, 1].$$

Definition 2.6 [11,12,13] : Let Q and G be any two sets. A mapping $\beta: G \times Q \rightarrow [0, 1]$ is called a Q –fuzzy set in G.

MAIN RESULTS

3. ANTI Q-FUZZY PMS – IDEALS OF PMS ALGEBRAS

Definition 3.1 : A Q- fuzzy set μ in X is called a Q-fuzzy PMS- ideal of X if

- (i) $\mu(0, q) \geq \mu(x, q)$
- (ii) $\mu(y * x, q) \geq \min \{ \mu(z * y, q), \mu(z * x, q) \}$, for all $x, y, z \in X$ and $q \in Q$.

Definition 3.2 : A Q-fuzzy set μ of X is called an anti Q-fuzzy PMS-ideal of X if

- (i) $\mu(0, q) \leq \mu(x, q)$
- (ii) $\mu(y * x, q) \leq \max \{ \mu(z * y, q), \mu(z * x, q) \}$, for all $x, y, z \in X$ and $q \in Q$.

Theorem 3.1 : Every Anti Q - Fuzzy PMS- ideal μ of a PMS-algebra X is order preserving.

Proof : Let μ be an anti Q-Fuzzy PMS- ideal of a PMS-algebra X and let $x, y \in X$ and $q \in Q$ be such that $x \leq y$, then $x * y = 0$.

$$\begin{aligned} \text{Now } \mu(x, q) &= \max \{ \mu(0 * x, q) \} \\ &\leq \max \{ \mu(z * 0, q), \mu((x * y) * (z * y), q) \} \\ &= \max \{ \mu(z * 0, q), \mu(0 * (z * y), q) \} \\ &= \max \{ \mu(z * 0, q), \mu(z * y, q) \} \\ &= \max \{ \mu(0 * 0, q), \mu(0 * y, q) \} \text{ (Taking } z = 0) \\ &= \max \{ \mu(0, q), \mu(y, q) \} \\ &= \mu(y, q) \end{aligned}$$

$$\mu(x, q) \leq \mu(y, q).$$

Theorem 3.2 : μ is a Q-fuzzy PMS-ideal of a PMS-algebra X iff μ^c is an anti Q-fuzzy PMS-ideal of X.

Proof: Let μ be a Q-Fuzzy PMS- ideal of X and let $x, y, z \in X$ and $q \in Q$.

$$\begin{aligned} \mu(0, q) &\geq \mu(x, q) \\ 1 - \mu^c(0, q) &\geq 1 - \mu^c(x, q) \\ \mu^c(0, q) &\leq \mu^c(x, q) \end{aligned}$$

$$\begin{aligned} \text{and } \mu^c(y * x, q) &= 1 - \mu(y * x, q) \\ &\leq 1 - \min \{ \mu(z * y, q), \mu(z * x, q) \} \\ &= 1 - \min \{ 1 - \mu^c(z * y, q), 1 - \mu^c(z * x, q) \} \\ &= \max \{ \mu^c(z * y, q), \mu^c(z * x, q) \} \end{aligned}$$

Thus μ^c is an anti Q-fuzzy PMS-ideal of X. The converse also can be proved similarly.

Theorem 3.3: Let X be a PMS-algebra. For any anti Q- fuzzy PMS-ideal μ of X, $N_\mu = \{ x \in X \text{ and } q \in Q / \mu(x, q) = \mu(0, q) \}$ is a PMS-ideal of X.

Proof: Let $z * y, z * x \in N_\mu$. Then $\mu(z * y, q) = \mu(z * x, q) = \mu(0, q)$

Since μ is an anti Q-fuzzy PMS-ideal of X,

$$\begin{aligned} \mu(y * x, q) &\leq \max \{ \mu(z * x, q), \mu(z * y, q) \} \\ &= \max \{ \mu(0, q), \mu(0, q) \} \\ &= \mu(0, q) \end{aligned}$$

Hence $y * x \in N_\mu$. Therefore N_μ is a PMS-ideal of X.

Theorem 3.4 : If λ and μ are anti Q-fuzzy PMS ideals of a PMS-algebra X, then $\lambda \cap \mu$ is also an anti Q-fuzzy PMS-ideal of X.

Proof : Let $x, y, z \in X$ and $q \in Q$. Then

$$\begin{aligned} (\lambda \cap \mu)(0, q) &= \min \{ \lambda(0, q), \mu(0, q) \} \\ &\leq \min \{ \lambda(x, q), \mu(x, q) \} \end{aligned}$$

$$\begin{aligned}
 &= (\lambda \cap \mu) (x, q) \\
 (\lambda \cap \mu) (y * x, q) &= \min \{ \lambda (y * x, q), \mu (y * x, q) \} \\
 &\leq \min \{ \max \{ \lambda (z * x, q), \lambda (z * y, q) \}, \max \{ \mu (z * x, q), \mu (z * y, q) \} \} \\
 &= \min \{ \max \{ \lambda (z * x, q), \mu (z * x, q) \}, \max \{ \lambda (z * y, q), \mu (z * y, q) \} \} \\
 &\leq \max \{ \min \{ \lambda (z * x, q), \mu (z * x, q) \}, \min \{ \lambda (z * y, q), \mu (z * y, q) \} \} \\
 &= \max \{ (\lambda \cap \mu) (z * x, q), (\lambda \cap \mu) (z * y, q) \}.
 \end{aligned}$$

$$\Rightarrow (\lambda \cap \mu) (y * x, q) \leq \max \{ (\lambda \cap \mu) (z * y, q), (\lambda \cap \mu) (z * x, q) \}.$$

$\Rightarrow (\lambda \cap \mu)$ is also an anti Q-fuzzy PMS ideal of X.

Theorem 3.5: The union of any set of anti Q-fuzzy PMS-ideals in PMS-algebra X is also an anti Q-fuzzy PMS-ideal.

Proof : Let $\{ \mu_i \}$ be a family of anti Q-fuzzy PMS-ideals of PMS-algebras X.

Then for any $x, y, z \in X$ and $q \in Q$.

$$\begin{aligned}
 (\cup \mu_i) (0, q) &= \sup (\mu_i (0, q)) \\
 &\leq \sup (\mu_i (x, q)) \\
 &= (\cup \mu_i) (x, q) \\
 (\cup \mu_i) (y * x, q) &= \sup (\mu_i (y * x, q)) \\
 &\leq \sup \{ \max \{ \mu_i (z * y, q), \mu_i (z * x, q) \} \} \\
 &= \max \{ \sup (\mu_i (z * y, q)), \sup (\mu_i (z * x, q)) \} \\
 &= \max \{ (\cup \mu_i) (z * y, q), (\cup \mu_i) (z * x, q) \}
 \end{aligned}$$

This completes the proof.

Definition 3.6: Let μ be a Q-fuzzy set of X. For a fixed $t \in [0, 1]$, the set $\mu_t = \{x \in X \mid \mu(x, q) \leq t \text{ for all } q \in Q\}$ is called the lower level subset of μ . Clearly $\mu^{t_1} \cup \mu^{t_2} = X$ for $t \in [0, 1]$ if $t_1 < t_2$, then $\mu_{t_1} \subseteq \mu_{t_2}$.

Theorem 3.7 : If μ is an anti Q-fuzzy PMS-ideal of PMS-algebra X, then μ_t is a PMS-ideal of X for every $t \in [0, 1]$.

Proof : Let μ be an anti Q-fuzzy PMS-ideal of PMS-algebra X.

Clearly $0 \in \mu_t$.

Let $z * x \in \mu_t$ and $z * y \in \mu_t$, for all $x, y \in X$ and $q \in Q$.

$$\Rightarrow \mu (z * x, q) \leq t \text{ and } \mu (z * y, q) \leq t.$$

$$\mu (y * x, q) \leq \max \{ \mu (z * y, q), \mu (z * x, q) \} \leq \max \{ t, t \} = t.$$

$\Rightarrow y * x \in \mu_t$.

Hence μ_t is an PMS-ideal of X for every $t \in [0, 1]$.

Theorem 3.8 : Let μ be a Q-fuzzy set of PMS- algebra X. If for each $t \in [0, 1]$, the lower level cut μ_t is a PMS-ideal of X, then μ is an anti Q- fuzzy PMS-ideal of X.

Proof : Let μ_t be a PMS-ideal of X.

If $\mu(0, q) > \mu(x, q)$ for some $x \in X$ and $q \in Q$, then $\mu(0, q) > t_0 > \mu(x, q)$ by taking

$$t_0 = \frac{1}{2} \{ \mu(0, q) + \mu(x, q) \}.$$

Hence $0 \notin \mu_{t_0}$ and $x \in \mu_{t_0}$, which is a contradiction.

Therefore, $\mu(0, q) \leq \mu(x, q)$.

Let $x, y, z \in X$ and $q \in Q$ be such that $\mu (y * x, q) > \max \{ \mu (z * y, q), \mu (z * x, q) \}$.

$$\text{Taking } t_1 = \frac{1}{2} \{ \mu(y * x, q) + \max \{ \mu (z * y, q), \mu (z * x, q) \} \}$$

$$\Rightarrow \mu (y * x, q) > t_1 > \max \{ \mu (z * y, q), \mu (z * x, q) \}.$$

It follows that $z * y, z * x \in \mu_{t_1}$ and $y * x \notin \mu_{t_1}$. This is a contradiction.

Hence $\mu(y * x, q) \leq \max \{ \mu (z * y, q), \mu (z * x, q) \}$

Therefore μ is an anti Q-fuzzy PMS-ideal of X.

4. HOMOMORPHISM AND ANTI HOMOMORPHISM ON ANTI Q-FUZZY PMS- ALGEBRAS

In this section, we discussed about ideals in PMS-algebra under homomorphism and anti homomorphism and some of its properties.

Definition 4.1 : Let $(X, *, 0)$ and $(Y, \Delta, 0)$ be PMS– algebras. A mapping $f: X \rightarrow Y$ is said to be a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

Definition 4.2 : Let $(X, *, 0)$ and $(Y, \Delta, 0)$ be PMS–algebras. A mapping $f: X \rightarrow Y$ is said to be an anti homomorphism if $f(x * y) = f(y) \Delta f(x)$ for all $x, y \in X$.

Definition 4.3 : Let $f: X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all x in X.

Theorem 4.4 : Let f be an endomorphism of a PMS- algebra X. If μ is an anti Q- fuzzy PMS-ideal of X, then so is μ_f .

Proof: Let μ be an anti Q-fuzzy PMS-ideal of X.

$$\begin{aligned} \text{Now, } \mu_f(0, q) &= \mu(f(0, q)) \\ &\leq \mu(f(x, q)) = \mu_f(x, q), \text{ for all } x, y \in X \text{ and } q \in Q. \end{aligned}$$

Let $x, y, z \in X$ and $q \in Q$.

$$\begin{aligned} \text{Then } \mu_f(y * x, q) &= \mu(f(y * x, q)) \\ &= \mu(f(y, q) * f(x, q)) \\ &\leq \max \{ \mu(f(z, q) * f(y, q)), \mu(f(z, q) * f(x, q)) \} \\ &= \max \{ \mu(f(z * y, q)), \mu(f(z * x, q)) \} \\ &= \max \{ \mu_f(z * y, q), \mu_f(z * x, q) \} \\ \therefore \mu_f(y * x, q) &\leq \max \{ \mu_f(z * y, q), \mu_f(z * x, q) \} \end{aligned}$$

Hence μ_f is an anti Q- fuzzy PMS-ideal of X.

Theorem 4.5 : Let $f: X \rightarrow Y$ be an epimorphism of PMS- algebra. If μ_f is an anti Q-fuzzy PMS-ideal of X, then μ is an anti Q-fuzzy PMS-ideal of Y.

Proof: Let μ_f be an anti Q-fuzzy PMS-ideal of X.

Let $y \in Y$ and $q \in Q$. Then there exists $x \in X$ such that $f(x, q) = (y, q)$.

$$\begin{aligned} \text{Now, } \mu(0, q) &= \mu(f(0, q)) \\ &= \mu_f(0, q) \\ &\leq \mu_f(x, q) = \mu(f(x, q)) = \mu(y, q) \end{aligned}$$

$$\therefore \mu(0, q) \leq \mu(y, q)$$

Let $y_1, y_2, y_3 \in Y$.

$$\begin{aligned} \mu(y_2 \Delta y_1, q) &= \mu(f(x_2) \Delta f(x_1), q) \\ &= \mu(f(x_2 * x_1), q) \\ &= \mu_f(x_2 * x_1, q) \\ &\leq \max \{ \mu_f(x_3 * x_2, q), \mu_f(x_3 * x_1, q) \} \\ &= \max \{ \mu[f(x_3 * x_2, q)], \mu[f(x_3 * x_1, q)] \} \\ &= \max \{ \mu[f(x_3, q) \Delta f(x_2, q)], \mu[f(x_3, q) \Delta f(x_1, q)] \} \\ &= \max \{ \mu[(y_3, q) \Delta (y_2, q)], \mu[(y_3, q) \Delta (y_1, q)] \} \end{aligned}$$

$\Rightarrow \mu$ is an anti Q- fuzzy PMS-ideal of Y.

Theorem 4.6 : Let $f: X \rightarrow Y$ be a homomorphism of PMS- algebra. If μ is an anti Q-fuzzy PMS-ideal of Y then μ_f is an anti Q-fuzzy PMS-ideal of X.

Proof: Let μ be an anti Q- fuzzy PMS-ideal of Y.

Let $x, y, z \in X$.

$$\begin{aligned} \mu_f(0, q) &= \mu(f(0, q)) \\ &\leq \mu(f(x, q)) = \mu_f(x, q) \end{aligned}$$

$$\Rightarrow \mu_f(0, q) \leq \mu_f(x, q).$$

$$\begin{aligned} \mu_f(y * x, q) &= \mu[f(y * x, q)] \\ &= \mu[f(y, q) \Delta f(x, q)] \\ &\leq \max \{ \mu(f(z, q) \Delta f(y, q)), \mu(f(z, q) \Delta f(x, q)) \} \\ &= \max \{ \mu(f(z * y, q)), \mu(f(z * x, q)) \} \\ &= \max \{ \mu_f(z * y, q), \mu_f(z * x, q) \} \end{aligned}$$

$$\therefore \mu_f(y * x, q) \leq \max \{ \mu_f(z * y, q), \mu_f(z * x, q) \}.$$

Hence μ_f is an anti Q-fuzzy R-closed PMS-ideal of X.

5. CARTESIAN PRODUCT OF ANTI Q-FUZZY PMS-IDEALS OF PMS-ALGEBRAS

In this section, we introduce the concept of Cartesian product of anti Q-fuzzy PMS-ideals of PMS-algebra.

Definition 5.1 : Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by

$$(\mu \times \delta)(x, y) = \min \{ \mu(x), \delta(y) \}, \text{ for all } x, y \in X.$$

Definition 5.2 : Let μ and δ be the anti fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by

$$(\mu \times \delta)(x, y) = \max \{ \mu(x), \delta(y) \}, \text{ for all } x, y \in X.$$

Definition 5.3: Let μ and δ be the anti Q-fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by

$$(\mu \times \delta)((x, y), q) = \max \{ \mu(x, q), \delta(y, q) \}, \text{ for all } x, y \in X \text{ and } q \in Q.$$

Theorem 5.4 : If μ and δ are anti Q-fuzzy PMS-ideals in a PMS- algebra X, then $\mu \times \delta$ is an anti Q-fuzzy PMS-ideal in $X \times X$.

Proof: Let $(x_1, x_2) \in X \times X$ and $q \in Q$.

$$\begin{aligned} (\mu \times \delta)((x_1 * 0, x_2 * 0), q) &= \max \{ \mu(x_1 * 0, q), \delta(x_2 * 0, q) \} \\ &\leq \max \{ \mu(x_1, q), \delta(x_2, q) \} \\ &= (\mu \times \delta)((x_1, x_2), q) \end{aligned}$$

$$\therefore (\mu \times \delta)((x_1 * 0, x_2 * 0), q) \leq (\mu \times \delta)((x_1, x_2), q)$$

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

$$\begin{aligned} (\mu \times \delta)[(y_1, y_2) * (x_1, x_2), q] &= (\mu \times \delta)[(y_1 * x_1, y_2 * x_2), q] \\ &= \max \{ \mu(y_1 * x_1, q), \delta(y_2 * x_2, q) \} \\ &\leq \max \{ \max \{ \mu(z_1 * y_1, q), \mu(z_1 * x_1, q) \}, \max \{ \delta(z_2 * y_2, q), \delta(z_2 * x_2, q) \} \} \\ &= \max \{ \max \{ \mu(z_1 * y_1, q), \delta(z_2 * y_2, q) \}, \max \{ \mu(z_1 * x_1, q), \delta(z_2 * x_2, q) \} \} \\ &= \max \{ (\mu \times \delta)((z_1 * y_1, q), (z_2 * y_2, q)), (\mu \times \delta)((z_1 * x_1, q), (z_2 * x_2, q)) \} \\ &= \max \{ (\mu \times \delta)[((z_1, z_2), q) * ((y_1, y_2), q)], (\mu \times \delta)[((z_1, z_2), q) * ((x_1, x_2), q)] \} \end{aligned}$$

Hence, $\mu \times \delta$ is an anti Q-fuzzy PMS- ideal in $X \times X$.

Theorem 5.5: Let μ & δ be fuzzy sets in a PMS-algebra X such that $\mu \times \delta$ is an anti Q-fuzzy PMS-ideal of $X \times X$.

Then (i) Either $\mu(0, q) \leq \mu(x, q)$ (or) $\delta(0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.

(ii) If $\mu(0, q) \leq \mu(x, q)$ for all $x \in X$ and $q \in Q$, then either $\delta(0, q) \leq \mu(x, q)$ (or) $\delta(0, q) \leq \delta(x, q)$

(iii) If $\delta(0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$, then either $\mu(0, q) \leq \mu(x, q)$ (or) $\mu(0, q) \leq \delta(x, q)$.

Proof: Straightforward.

Theorem 5.6: Let μ & δ be fuzzy sets in a PMS-algebra X such that $\mu \times \delta$ is an anti Q-fuzzy PMS-ideal of $X \times X$. Then either μ or δ is an anti Q-fuzzy PMS-ideal of X .

Proof: First we prove that δ is an anti Q- fuzzy PMS-ideal of X .

Since by 5.5(i) either $\mu(0,q) \leq \mu(x, q)$ or $\delta(0,q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.

Assume that $\delta(0,q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$. It follows from 6.2(iii) that either

$$\mu(0,q) \leq \mu(x,q) \text{ (or) } \mu(0,q) \leq \delta(x,q).$$

If $\mu(0,q) \leq \delta(x, q)$, for any $x \in X$ and $q \in Q$, then

$$\begin{aligned} \delta(x, q) &= \max \{ \mu(0,q), \delta(x, q) \} = (\mu \times \delta) ((0, x), q) \\ \delta(y * x, q) &= (\mu \times \delta) [(0, y * x), q] \\ &\leq \max \{ (\mu \times \delta) [((0,z), q) * ((0,y), q)], (\mu \times \delta) [((0,z), q) * ((0, x), q)] \} \\ &= \max \{ (\mu \times \delta) [(0 * 0, z * y), q], (\mu \times \delta) [(0 * 0, z * x), q] \} \\ &= \max \{ (\mu \times \delta) [(0, z * y), q], (\mu \times \delta) [(0, z * x), q] \} \\ &= \max \{ \delta(z * y, q), \delta(z * x, q) \} \end{aligned}$$

$$\Rightarrow \delta(y * x, q) \leq \max \{ \delta(z * y, q), \delta(z * x, q) \}$$

Hence δ is an anti fuzzy PMS-ideal of X .

Similarly we will prove that μ is an anti Q- fuzzy PMS-ideal of X .

6. CONCLUSION

In this article we have discussed anti Q-fuzzy PMS- ideal of PMS-algebras and its lower level cuts in detail. We hope that this work would lay other foundations for further study of the theory of PMS-algebras. In our future study of fuzzy structure of PMS-algebra, can be extended to the topics, intuitionistic fuzzy set, interval valued fuzzy sets, for more interesting results.

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