Single Valued Neutrosophic Ideals of BCK-Algebras

Martina Jency .J[#1], Arockiarani. I[*2]

[#1]Research Scholar, Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India.
[*2]Associate Professor, Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India.

Abstract

The focus of this paper is to present Single valued neutrosophic ideals of a BCK-algebras, give some results on the single valued neutrosophic implicative ideals, single valued neutrosophic positive implicative ideals, single valued neutrosophic commutative ideals.

Keywords—*Single valued neutrosophic sub-algebra, Single valued neutrosophic ideal, Single valued neutrosophic positive implicative ideal, single valued neutrosophic commutative ideal.*

I. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [12] several researches were conducted on the generalizations of the notion of fuzzy sets. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1], as a generalization of the notion of fuzzy set. In 1995, Florentin Smarandache [2,3] initiated the concept of neutrosophic set. Recently H.Wang et.al [4] introduced an instance of neutrosophic set known as single valued neutrosophic set which was motivated from the practical point of view and that can be used in real scientific and engineering applications.

BCK/BCI-algebras are algebraic structures, introduced by K. Iseki [5,6] in 1966, that describe ´ fragments of the propositional calculus involving implication known as BCK/BCI-logics. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

Y. B. Jun (together with Hong, Kim, Meng, Roh, and Song) considered the fuzzification of ideals and sub-algebras in BCK-algebras ([7, 8, 9, 10, 11]). In this paper, we establish the single valued neutrosophication of the concept of sub-algebras and ideals in BCK-algebras, and investigate some of their properties.

II. Preliminaries

Definition 2.1

An algebra $(X,*,0)$ of type (2, 0) is called a BCK-algebra if it satisfies the following axioms:

(a1) $((x*y)*(x*z))*(z*y)=0$,

(a2) $(x*(x*y))*y=0$,

(a3) $x*x=0$,

(a4) $x*y=0$ and $y*x=0$ imply that $x=y$

(a5) $0*x=0$ for all $x,y,z\in X$.

Definition 2.2

A partial ordering "$\le$" on X can be defined by $x\le y$ if and only if $x*y=0$.

Definition 2.3

In any BCK-algebra X the following holds:

(P1) $x*0=x$

(P2) $x*y\le x$

(P3) $(x*y)*z=(x*z)*y$

(P4) $(x*z)*(y*z)\le x*y$

(P5) $x*(x*(x*y))=x*y$

(P6) $x\le y \Rightarrow x*z\le y*z$ and $z*y\le z*x$, for all $x,y,z\in X$.

Definition 2.4

A subset S of a BCK-algebra X is called a subalgebra of X if $x*y\in S$ whenever $x,y\in S$.

Definition 2.5

A non-empty subset I of a BCK-algebra X is called an ideal of X if it satisfies:

(C1) $0 \in I$,

(C2) $x * y \in I$ and $y \in I$ imply $x \in I$

Proposition 2.6

In a BCK-algebra X, the following holds, for all $x, y, z \in X$.

(i) $((x * z) * z) * (y * z) \leq (x * y) * z.$

(ii) $(x * z) * (x * (x * z)) = (x * z) * z$

(iii) $(x * (y * (y * x))) * (y * (x * (y * (y * x)))) \leq x * y$.

III .SINGLE VALUED NEUTROSOPHIC IDEALS

In what follows, let *X* denote a BCK-algebra unless otherwise specified.

Definition 3.1

An SVNS $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ in *X* is called a single *valued neutrosophic sub-algebra* of *X* if it satisfies:

1. $\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$
2. $\beta_A(x * y) \geq \min\{\beta_A(x), \beta_A(y)\}$
3. $\gamma_A(x * y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$ for all $x, y \in X$.

Example 3.2

Consider a BCK-algebra $X=\{0, a, b, c\}$ with the following Cayley table:

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Let $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ be a SVNS in *X* defined by

$$\alpha_A(0) = \alpha_A(a) = \alpha_A(c) = 0.7 > 0.3 = \alpha_A(b),$$
$$\beta_A(0) = \beta_A(a) = \beta_A(c) = 0.6 > 0.2 = \beta_A(b),$$
$$\gamma_A(0) = \gamma_A(a) = \gamma_A(c) = 0.2 < 0.5 = \gamma_A(b).$$

Then $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic sub-algebra of *X*.

Proposition 3.3

Every single valued neutrosophic sub-algebra $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ *of X satisfies the inequalities* $\alpha_A(0) \geq \alpha_A(x)$, $\beta_A(0) \geq \beta_A(x)$ *and* $\gamma_A(0) \leq \gamma_A(x)$ *for all* $x \in X$.

Proof:

For any $x \in X$, we have

$$\alpha_A(0) = \alpha_A(x * x) \geq \min\{\alpha_A(x), \alpha_A(x)\} = \alpha_A(x),$$

$$\beta_A(0) = \beta_A(x * x) \geq \min\{\beta_A(x), \beta_A(x)\} = \beta_A(x),$$

$$\gamma_A(0) = \gamma_A(x * x) \leq \max\{\gamma_A(x), \gamma_A(x)\} = \gamma_A(x).$$

This completes the proof.

Definition 3.4

An SVNS $A=\langle \alpha_A,\beta_A,\gamma_A\rangle$ in X is called a single *valued neutrosophic ideal* of X if it satisfies the following inequalities:

1. $\alpha_A(0) \geq \alpha_A(x)$, $\beta_A(0) \geq \beta_A(x)$ and $\gamma_A(0) \leq \gamma_A(x)$,
2. $\alpha_A(x) \geq \min\{\alpha_A(x*y),\alpha_A(y)\}$,
3. $\beta_A(x) \geq \min\{\beta_A(x*y),\beta_A(y)\}$,
4. $\gamma_A(x) \leq \max\{\gamma_A(x*y),\gamma_A(y)\}$, for all $x,y \in X$.

Example 3.5

Let $X=\{0,1,2,3,4\}$ be a BCK-algebra with the following Cayley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Define a SVNS $A=\langle \alpha_A,\beta_A,\gamma_A\rangle$ in X as follows:

$$\alpha_A(0)=\alpha_A(2)=1,\ \alpha_A(1)=\alpha_A(3)=\alpha_A(4)=r,$$
$$\beta_A(0)=\beta_A(2)=1,\ \beta_A(1)=\beta_A(3)=\beta_A(4)=s,$$
$$\gamma_A(0)=\gamma_A(2)=0,\ \ \gamma_A(1)=\gamma_A(3)=\gamma_A(4)=t$$

where $r \in [0,1]$, $s \in [0,1]$, $t \in [0,1]$ and $r+s+t \leq 3$. By routine calculation we know that $A=\langle \alpha_A,\beta_A,\gamma_A\rangle$ is a single *valued neutrosophic ideal* of X.

Lemma 3.6

Let a SVNS $A=\langle \alpha_A,\beta_A,\gamma_A\rangle$ in X be a single valued neutrosophic ideal of X. If the inequality $x*y \leq z$ holds in X, then

$\alpha_A(x) \geq \min\{\alpha_A(y),\alpha_A(z)\}$, $\beta_A(x) \geq \min\{\beta_A(y),\beta_A(z)\}$ and $\gamma_A(x) \leq \max\{\gamma_A(y),\gamma_A(z)\}$

Proof:

Let $x,y,z \in X$ be such that $x*y \leq z$. Then $(x*y)*z = 0$, and thus

$$\alpha_A(x) \geq \min\{\alpha_A(x*y),\alpha_A(y)\}$$
$$\geq \min\{\min\{\alpha_A((x*y)*z),\alpha_A(z)\},\alpha_A(y)\}$$
$$=\min\{\min\{\alpha_A(0),\alpha_A(z)\},\alpha_A(y)\}$$
$$=\min\{\alpha_A(y),\alpha_A(z)\}$$

$$\beta_A(x) \geq \min\{\beta_A(x*y),\beta_A(y)\}$$
$$\geq \min\{\min\{\beta_A((x*y)*z),\beta_A(z)\},\beta_A(y)\}$$
$$=\min\{\min\{\beta_A(0),\beta_A(z)\},\beta_A(y)\}$$
$$=\min\{\beta_A(y),\beta_A(z)\}.$$

$$\gamma_A(x) \le \max\{\gamma_A(x*y), \gamma_A(y)\}$$

$$\le \max\{\max\{\gamma_A((x*y)*z), \gamma_A(z)\}, \gamma_A(y)\}$$

$$= \max\{\max\{\gamma_A(0), \gamma_A(z)\}, \gamma_A(y)\}$$

$$= \max\{\gamma_A(y), \gamma_A(z)\}.$$

this completes the proof.

Lemma 3.7

Let $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ be a single valued neutrosophic ideal of X. If $x \le y$ in X, then

$\alpha_A(x) \ge \alpha_A(y)$, $\beta_A(x) \ge \beta_A(y)$, $\gamma_A(x) \le \gamma_A(y)$ that is, α_A and β_A are order-reserving and γ_A is order-preserving.

Proof:

Let $x,y \in X$ be such that $x \le y$. Then $x*y = 0$ and so

$$\alpha_A(x) \ge \min\{\alpha_A(x*y), \alpha_A(y)\} = \min\{\alpha_A(0), \alpha_A(y)\} = \alpha_A(y)$$

$$\beta_A(x) \ge \min\{\beta_A(x*y), \beta_A(y)\} = \min\{\beta_A(0), \beta_A(y)\} = \beta_A(y)$$

$$\gamma_A(x) \le \max\{\gamma_A(x*y), \gamma_A(y)\} = \max\{\gamma_A(0), \gamma_A(y)\} = \gamma_A(y)$$

This completes the proof.

Theorem 3.8

If $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic ideal of X, then for any $x, a_1, a_2, \ldots, a_n \in X$, $(\cdots((x*a_1)*a_2)*\cdots)*a_n = 0$ implies

$$\alpha_A(x) \ge \min\{\alpha_A(a_1), \alpha_A(a_2), \ldots, \alpha_A(a_n)\} \quad \beta_A(x) \ge \min\{\beta_A(a_1), \beta_A(a_2), \ldots, \beta_A(a_n)\}$$

$$\gamma_A(x) \le \max\{\gamma_A(a_1), \gamma_A(a_2), \ldots, \gamma_A(a_n)\}$$

Proof:

Using induction on n and Lemmas 3.6 and 3.7, the proof is straightforward.

Theorem 3.9

Every single valued neutrosophic ideal of X is a single valued neutrosophic sub-algebra of X.

Proof:

Let $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ be a single valued neutrosophic ideal of X. Since $x*y \le x$ for all $x,y \in X$, it follows from Lemma 3.7 that $\alpha_A(x*y) \ge \alpha_A(x)$, $\beta_A(x*y) \ge \beta_A(x)$, $\gamma_A(x*y) \le \gamma_A(x)$ so by Definition 3.4,

$$\alpha_A(x*y) \ge \alpha_A(x) \ge \min\{\alpha_A(x*y), \alpha_A(y)\} \ge \min\{\alpha_A(x), \alpha_A(y)\}$$

$$\beta_A(x*y) \ge \beta_A(x) \ge \min\{\beta_A(x*y), \beta_A(y)\} \ge \min\{\beta_A(x), \beta_A(y)\}$$

$$\gamma_A(x*y) \le \gamma_A(x) \le \max\{\gamma_A(x*y), \gamma_A(y)\} \le \max\{\gamma_A(x), \gamma_A(y)\}$$

This shows that $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic sub-algebra of X.

The converse of Theorem 3.9 may not be true. For example, the single valued neutrosophic sub-algebra $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ in Example 3.2 is not a single valued neutrosophic ideal of X since $\gamma_A(b) = 0.5 > 0.2 = \max\{\gamma_A(b*a), \gamma(a)\}$.

We now give a condition for a single valued neutrosophic sub-algebra to be a single valued neutrosophic ideal.

Theorem 3.10

Let $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ be a single valued neutrosophic sub-algebra of X such that $\alpha_A(x)\geq\min\{\alpha_A(y),\alpha_A(z)\}$, $\beta_A(x)\geq\min\{\beta_A(y),\beta_A(z)\}$, $\gamma_A(x)\leq\max\{\gamma_A(y),\gamma_A(z)\}$ for all x,y,z ∈ X satisfying the inequality $x*y\leq z$. Then $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic ideal of X.

Proof:

Let $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ be a single valued neutrosophic sub-algebra of *X*. Recall that $\alpha_A(0)\geq\alpha_A(x),\beta_A(0)\geq\beta_A(x)$ and $\gamma_A(0)\leq\gamma_A(x)$ for all *X*. Since $x*(x*y)\leq y$, it follows from the hypothesis that $\alpha_A(x)\geq\min\{\alpha_A(x*y),\alpha_A(y)\}$, $\beta_A(x)\geq\min\{\beta_A(x*y),\beta_A(y)\}$ and $\gamma_A(x)\leq\max\{\gamma_A(x*y),\gamma_A(y)\}$.Hence $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic ideal of *X*.

Lemma 3.11

A SVNS $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic ideal of X if and only if the fuzzy sets α_A, β_A and $\bar{\gamma}_A$ are fuzzy ideals of X.

Proof:

Let $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ be a single valued neutrosophic ideal of *X*. Clearly, α_A. β_A is a fuzzy ideal of *X*. For every *x, y* ∈ *X*, we have

$\bar{\gamma}_A(0)=1-\gamma_A(0)\geq1-\gamma_A(x)=\bar{\gamma}_A(x)$, $\bar{\gamma}_A(x)=1-\gamma_A(x)\geq1-\max\{\gamma_A(x*y),\gamma_A(y)\}$
$=\min\{1-\gamma_A(x*y),1-\gamma_A(y)\}$ $=\min\{\bar{\gamma}_A(x*y),\bar{\gamma}_A(y)\}$.Hence $\bar{\gamma}_A$ is a fuzzy ideal of *X*.

Conversely, assume that α_A,β_A and $\bar{\gamma}_A$ are fuzzy ideals of *X*. For every *x,y* ∈ *X*, we get
$\alpha_A(0)\geq\alpha_A(x)$, $\beta_A(0)\geq\beta_A(x)$, $1-\gamma_A(0)=\bar{\gamma}_A(0)\geq\bar{\gamma}_A(x)=1-\gamma_A(x)$ that is,
$\gamma_A(0)\leq\gamma_A(x)$; $\alpha_A(x)\geq\min\{\alpha_A(x*y),\alpha_A(y)\}$ and
$1-\gamma_A(x)=\bar{\gamma}_A(x)\geq\min\{\bar{\gamma}_A(x*y),\bar{\gamma}_A(y)\}=\min\{1-\gamma_A(x*y),1-\gamma_A(y)\}$
$=1-\max\{\gamma_A(x*y),\gamma_A(y)\}$ that is, $\gamma_A(x)\leq\max\{\gamma_A(x*y),\gamma_A(y)\}$. Hence $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic ideal of *X*.

Definition 3.12

For any $r,s,t\in[0,1]$ and a single valued neutrosophic set $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ in a non-empty set X ,the set $U(A;\langle r,s,t\rangle)=\{x\in X/\alpha_A(x)\geq r,\beta_A(x)\geq s,\gamma_A(x)\leq t\}$ is called an upper $\langle r,s,t\rangle-$level cut of A and the set $L(A;\langle r,s,t\rangle)=\{x\in X/\alpha_A(x)\leq r,\beta_A(x)\leq s,\gamma_A(x)\geq t\}$ is called a lower $\langle r,s,t\rangle-$level cut of A.

Definition 3.13

Let $SVNS(X)$ be the family of all single valued neutrosophic ideals of X and let $r,s,t\in[0,1]$.Define binary relations $U^{\langle r,s,t\rangle}$ and $L^{\langle r,s,t\rangle}$ on $SVNS(X)$ as follows:

$(A,B)\in U^{\langle r,s,t\rangle}\Leftrightarrow U(A;\langle r,s,t\rangle)=U(B;\langle r,s,t\rangle)$

$(A,B)\in L^{\langle r,s,t\rangle}\Leftrightarrow L(A;\langle r,s,t\rangle)=L(B;\langle r,s,t\rangle)$ respectively, for $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ and $B=\langle\alpha_B,\beta_B,\gamma_B\rangle$ in $SVN(X)$.Then clearly $U^{\langle r,s,t\rangle}$ and $L^{\langle r,s,t\rangle}$ are equivalence relations on $SVN(X)$.

Definition 3.14

A SVNS $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ in a BCK-algebra *X* is a single valued neutrosophic implicative ideal (SVNI-ideal) of *X* if it satisfies

(1) $\alpha_A(0)\geq\alpha_A(x),\beta_A(0)\geq\beta_A(x)$ and $\gamma_A(0)\leq\gamma_A(x)$

(2) $\alpha_A(x)\geq\min\{\alpha_A((x*(y*x))*z),\alpha_A(z)\}$

(3) $\beta_A(x) \geq \min\{\beta_A((x*(y*x))*z), \beta_A(z)\}$

(4) $\gamma_A(x) \leq \max\{\gamma_A((x*(y*x))*z), \gamma_A(z)\}$, for all $x, y, z \in X$.

Definition 3.15

An SVNS $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ in X is a single valued neutrosophic commutative ideal (SVNCI-ideal) of X if it satisfies

(1) $\alpha_A(0) \geq \alpha_A(x), \beta_A(0) \geq \beta_A(x)$ and $\gamma_A(0) \leq \gamma_A(x)$

(2) $\alpha_A(x*(y*(y*x))) \geq \min\{\alpha_A((x*y)*z), \alpha_A(z)\}$

(3) $\beta_A(x*(y*(y*x))) \geq \min\{\beta_A((x*y)*z), \beta_A(z)\}$

(4) $\gamma_A(x*(y*(y*x))) \leq \max\{\gamma_A((x*y)*z), \gamma_A(z)\}$

for all $x,y,z \in X$.

Definition 3.16

A SVNS $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ in a BCK-algebra X is a single valued neutrosophic positive implicative ideal (SVNPI-ideal) of X if it satisfies

(1) $\alpha_A(0) \geq \alpha_A(x), \beta_A(0) \geq \beta_A(x)$ and $\gamma_A(0) \leq \gamma_A(x)$

(2) $\alpha_A(x*z) \geq \min\{\alpha_A((x*y)*z), \alpha_A(y*z)\}$

(3) $\beta_A(x*z) \geq \min\{\beta_A((x*y)*z), \beta_A(y*z)\}$

(4) $\gamma_A(x*z) \leq \max\{\gamma_A((x*y)*z), \gamma_A(y*z)\}$

for all $x,y,z \in X$.

Theorem 3.17

A single valued neutrosophic ideal $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ of X is a single valued neutrosophic implicative if and only if A is both single valued neutrosophic commutative and single valued neutrosophic positive implicative.

Proof:

Assume that $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic implicative ideal of X.

By Definition 2.6(i) and. Lemma 3.6, we have

$\min\{\alpha_A((x*y)*z), \alpha_A(y*z)\} \leq \alpha_A((x*z)*z)$

$= \alpha_A((x*z)*(x*(x*z)))$ by (2.6(ii))

$= \alpha_A(x*z)$

$\min\{\beta_A((x*y)*z), \beta_A(y*z)\} \leq \beta_A((x*z)*z)$

$= \beta_A((x*z)*(x*(x*z)))$ by (2.6(ii))

$= \beta_A(x*z)$

and $\max\{\gamma_A((x*y)*z), \gamma_A(y*z)\} \geq \gamma_A((x*z)*z)$

$= \gamma_A((x*z)*(x*(x*z)))$ by (2.6(ii))

$= \gamma_A(x*z)$

for all $x,y,z \in X$.

Then $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X.

$\alpha_A(x*y) \leq \alpha_A((x*(y*(y*x)))*(y*(x*(y*(y*x))))) = \alpha_A(x*(y*(y*x)))$

$\beta_A(x*y) \leq \beta_A((x*(y*(y*x)))*(y*(x*(y*(y*x))))) = \beta_A(x*(y*(y*x)))$

and

$\gamma_A(x*y) \geq \gamma_A((x*(y*(y*x)))*(y*(x*(y*(y*x))))) = \gamma_A(x*(y*(y*x)))$

Therefore, $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic commutative.

Conversely, suppose that $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is both single valued neutrosophic positive implicative and single valued neutrosophic commutative.
Since, $(y*(y*x))*(y*x)\le x*(y*x)$, it follows from Lemma 3.7.

$\alpha_A((y*(y*x))*(y*x))\ge\alpha_A(x*(y*x))$,
$\beta_A((y*(y*x))*(y*x))\ge\beta_A(x*(y*x))$ and
$\gamma_A((y*(y*x))*(y*x))\le\gamma_A(x*(y*x))$

Then, we have $\alpha_A(y*(y*x)*(y*x))=\alpha_A(y*(y*x))$
$\beta_A(y*(y*x)*(y*x))=\beta_A(y*(y*x))$
and $\gamma_A(y*(y*x)*(y*x))=\gamma_A(y*(y*x))$.

Therefore
$\alpha_A(x*(y*x))\le\alpha_A(y*(y*x))$, $\beta_A(x*(y*x))\le\beta_A(y*(y*x))$
and . $\gamma_A(x*(y*x))\ge\gamma_A(y*(y*x))$ ………… (1)

On the other hand since $x*y\le x*(y*x)$, we have, by Lemma 3.7
$\alpha_A(x*y)\ge\alpha_A(x*(y*x))$, $\beta_A(x*y)\ge\beta_A(x*(y*x))$ and $\gamma_A(x*y)\le\gamma_A(x*(y*x))$.

Since $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic commutative ideal of X, then we have
$\alpha_A(x*y)=\alpha_A(x*(y*(y*x)))$, $\beta_A(x*y)=\beta_A(x*(y*(y*x)))$ and
$\gamma_A(x*y)=\gamma_A(x*(y*(y*x)))$.

Hence $\alpha_A(x*(y*x))\le\alpha_A(x*(y*(y*x)))$, $\beta_A(x*(y*x))\le\beta_A(x*(y*(y*x)))$ and
$\gamma_A(x*(y*x))\ge\gamma_A(x*(y*(y*x)))$ ……….(2)

Combining (1) and (2), we obtain
$\alpha_A(x*(y*x))\le\min\{\alpha_A(x*(y*(y*x))),\alpha_A(y*(y*x))\}\le\alpha_A(x)$
$\beta_A(x*(y*x))\le\min\{\beta_A(x*(y*(y*x))),\beta_A(y*(y*x))\}\le\beta_A(x)$
and $\gamma_A(x*(y*x))\ge\max\{\gamma_A(x*(y*(y*x))),\gamma_A(y*(y*x))\}\ge\gamma_A(x)$

So $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic implicative ideal of X .
The proof is complete.

Theorem 3.18

If $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is a single valued neutrosophic ideal of X with the following conditions holds

(i) $\alpha_A(x*y)\ge\min\{\alpha_A(((x*y)*y)*z),\alpha_A(z)\}$
(ii) $\beta_A(x*y)\ge\min\{\beta_A(((x*y)*y)*z),\beta_A(z)\}$
(iii) $\gamma_A(x*y)\le\max\{\gamma_A(((x*y)*y)*z),\gamma_A(z)\}$ for all $x,y,z\in X$ .Then A is single valued neutrosophic positive implicative ideal of *X.*

Proof:

Suppose $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ is single valued neutrosophic ideal of X.
with condition (i) , (ii) and (iii). Using (P3) and (P4), we have
$((x*z)*z)*(y*z))\le(x*z)*y=(x*y)*z$, for all x, y, z ∈X, therefore by Lemma 3.7
$\alpha_A(((x*z)*z)*(y*z))\ge\alpha_A((x*y)*z)$
$\beta_A(((x*z)*z)*(y*z))\ge\beta_A((x*y)*z)$ and $\gamma_A(((x*z)*z)*(y*z))\le\gamma_A((x*y)*z)$

Now $\alpha_A(x*z)\ge\min\{\alpha_A(((x*z)*z)*(y*z)),\alpha_A(y*z)\}$
$\ge\min\{\alpha_A((x*y)*z),\alpha_A(y*z)\}$, for all $x,y,z\in X$
$\beta_A(x*z)\ge\min\{\beta_A(((x*z)*z)*(y*z)),\beta_A(y*z)\}$

$\geq \min\{\beta_A((x*y)*z), \beta_A(y*z)\}$, for all $x, y, z \in X$
and
$\gamma_A(x*z) \leq \max\{\gamma_A(((x*z)*z)*(y*z)), \gamma_A(y*z)\}$
$\leq \max\{\gamma_A((x*y)*z), \gamma_A(y*z)\}$, for all $x, y, z \in X$.

Hence $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X.

Lemma 3.19

Let $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ be a fuzzy ideal of X, then A is a single valued neutrosophic positive implicative ideal of X if and only if $\alpha_A((x*z)*(y*z)) \geq \alpha_A((x*y)*z)$, $\beta_A((x*z)*(y*z)) \geq \beta_A((x*y)*z)$ and $\gamma_A((x*z)*(y*z)) \leq \gamma_A((x*y)*z)$ for all $x, y, z \in X$.

Proof:

Suppose that $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a fuzzy ideal of X and $\alpha_A((x*z)*(y*z)) \geq \alpha_A((x*y)*z)$, $\beta_A((x*z)*(y*z)) \geq \beta_A((x*y)*z)$ and $\gamma_A((x*z)*(y*z)) \leq \gamma_A((x*y)*z)$ for all $x, y, z \in X$.

Therefore $\alpha_A(x*z) \geq \min\{\alpha_A((x*z)*(y*z)), \alpha_A(y*z)\} \geq \min\{\alpha_A((x*y)*z), \alpha_A(y*z)\}$
$\beta_A(x*z) \geq \min\{\beta_A((x*z)*(y*z)), \beta_A(y*z)\} \geq \min\{\beta_A((x*y)*z), \beta_A(y*z)\}$
$\gamma_A(x*z) \leq \max\{\gamma_A((x*z)*(y*z)), \gamma_A(y*z)\} \leq \max\{\gamma_A((x*y)*z), \gamma_A(y*z)\}$ for all $x, y, z \in X$. Thus A is a single valued neutrosophic positive implicative ideal of X. Conversely, assume that $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X implies that $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic ideal of *X*.

Let $a = x*(y*z)$ and $b = x*y$,
Since $((x*(y*z))*(x*y)) \leq y*(y*z)$, we have that
$\alpha_A((a*b)*z) = \alpha_A((x*(y*z))*((x*y)*z)) \geq \alpha_A((y*(y*z))*z) = \alpha_A(0)$
and so, $\alpha_A((x*z)*(y*z)) = \alpha_A((x*(y*z))*z) = \alpha_A(a*z) \geq \min\{\alpha_A((a*b)*z), \alpha_A(b*z)\}$
$\geq \min\{\alpha_A((a*b)*z), \alpha_A(b*z)\} \geq \min\{\alpha_A(0), \alpha_A(b*z)\} = \alpha_A(b*z)$
$= \alpha_A((x*y)*z)$.
Therefore $\alpha_A((x*z)*(y*z)) \geq \alpha_A((x*y)*z)$, for all $x, y, z \in X$.
Similarly $\beta_A((x*z)*(y*z)) \geq \beta_A((x*y)*z)$, for all $x, y, z \in X$
And $\gamma_A((a*b)*z) = \gamma_A((x*(y*z))*((x*y)*z)) \leq \gamma_A((y*(y*z))*z) = \gamma_A(0)$
and so, $\gamma_A((x*z)*(y*z)) = \gamma_A((x*(y*z))*z) = \gamma_A(a*z) \leq \max\{\gamma_A((a*b)*z), \gamma_A(b*z)\}$
$\leq \max\{\gamma_A((a*b)*z), \gamma_A(b*z)\} \leq \max\{\gamma_A(0), \gamma_A(b*z)\} = \gamma_A(b*z)$
$= \gamma_A((x*y)*z)$.
Therefore $\gamma_A((x*z)*(y*z)) \leq \gamma_A((x*y)*z)$, for all $x, y, z \in X$.
Thus $\alpha_A((x*z)*(y*z)) \geq \alpha_A((x*y)*z)$, $\beta_A((x*z)*(y*z)) \geq \beta_A((x*y)*z)$ and $\gamma_A((x*z)*(y*z)) \leq \gamma_A((x*y)*z)$, for all $x, y, z \in X$.

Theorem 3.20

If $A = \langle \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X, then

(i) For any $x, y, a, b \in X, ((x*y)*y)*a \leq b \Rightarrow \alpha_A(x*y) \geq \min\{\alpha_A(a), \alpha_A(b)\}$, $\beta_A(x*y) \geq \min\{\beta_A(a), \beta_A(b)\}$, $\gamma_A(x*y) \leq \max\{\gamma_A(a), \gamma_A(b)\}$.

(ii) For any $x,y,z,a,b \in X, ((x*y)*z)*a \le b \Rightarrow \alpha_A((x*z)*(y*z)) \ge \min\{\alpha_A(a),\alpha_A(b)\}$ $,\beta_A((x*z)*(y*z)) \ge \min\{\beta_A(a),\beta_A(b)\}$, $\gamma_A((x*z)*(y*z)) \le \max\{\gamma_A(a),\gamma_A(b)\}$.

Proof:

Suppose $A=\langle \alpha_A,\beta_A,\gamma_A \rangle$ is single valued neutrosophic positive implicative ideal of X.

Let $x,y,z \in X$ be such that $((x*y)*y)*a \le b$.Using Lemma 3.6, we have

$\alpha_A((x*y)*y) \ge \min\{\alpha_A(a),\alpha_A(b)\}, \beta_A((x*y)*y) \ge \min\{\beta_A(a),\beta_A(b)\}$ and

$\gamma_A((x*y)*y) \le \max\{\gamma_A(a),\gamma_A(b)\}$.

It follows that

$\alpha_A(x*y) \ge \min\{\alpha_A((x*y)*y),\alpha_A(y*y)\} = \min\{\alpha_A((x*y)*y),\alpha_A(0)\}$

$=\alpha_A((x*y)*y) \ge \min\{\alpha_A(a),\alpha_A(b)\}$.

$\beta_A(x*y) \ge \min\{\beta_A((x*y)*y),\beta_A(y*y)\} = \min\{\beta_A((x*y)*y),\beta_A(0)\}$

$=\beta_A((x*y)*y) \ge \min\{\beta_A(a),\beta_A(b)\}$.

And $\gamma_A(x*y) \le \max\{\gamma_A((x*y)*y),\gamma_A(y*y)\} = \max\{\gamma_A((x*y)*y),\gamma_A(0)\}$

$=\gamma_A((x*y)*y) \le \max\{\gamma_A(a),\gamma_A(b)\}$.

(ii)Now let $x,y,z \in X$ be such that $((x*y)*y)*a \le b$ .Since $A=\langle X,\alpha_A,\beta_A,\gamma_A \rangle$ is single valued neutrosophic positive implicative ideal of X , it follows from the Lemma 3.19,

$\alpha_A((x*z)*(y*z)) \ge \alpha_A((x*y)*z) \ge \min\{\alpha_A(a),\alpha_A(b)\}$,

$\beta_A((x*z)*(y*z)) \ge \beta_A((x*y)*z) \ge \min\{\beta_A(a),\beta_A(b)\}$

and $\gamma_A((x*z)*(y*z)) \le \gamma_A((x*y)*z) \le \max\{\gamma_A(a),\gamma_A(b)\}$.

This completes the proof.

Theorem 3.21

Let $A=\langle X,\alpha_A,\beta_A,\gamma_A \rangle$ be a single valued neutrosophic set in X satisfying the condition $((x*y)*y)*a \le b \Rightarrow \alpha_A(x*y) \ge \min\{\alpha_A(a),\alpha_A(b)\}, \beta_A(x*y) \ge \min\{\beta_A(a),\beta_A(b)\}$

And $\gamma_A(x*y) \le \max\{\gamma_A(a),\gamma_A(b)\}$ for any $x,y,a,b \in X$, then $A=\langle X,\alpha_A,\beta_A,\gamma_A \rangle$ is single valued neutrosophic positive implicative ideal of X.

Proof:

First we prove that $A=\langle X,\alpha_A,\beta_A,\gamma_A \rangle$ is a single valued neutrosophic ideal of X.

Let $x,y,z \in X$ be such that $x*y \le z$ .Then $(((x*0)*0)*y)*z = (x*y)*z = 0$, that is $(((x*0)*0)*y)*z \le z$.

Since, for

$x,y,a,b \in X$,

$((x*y)*y)*a \le b \Rightarrow \alpha_A(x*y) \ge \min\{\alpha_A(a),\alpha_A(b)\}, \beta_A(x*y) \ge \min\{\beta_A(a),\beta_A(b)\}$

And $\gamma_A(x*y) \le \max\{\gamma_A(a),\gamma_A(b)\}$.

Put $y=0, a=y, b=z$,

we get $\alpha_A(x) = \alpha_A(x*0) \ge \min\{\alpha_A(y),\alpha_A(z)\}, \beta_A(x) = \beta_A(x*0) \ge \min\{\beta_A(y),\beta_A(z)\}$

$\gamma_A(x) = \gamma_A(x*0) \le \max\{\gamma_A(y),\gamma_A(z)\}$. It follows that $A=\langle X,\alpha_A,\beta_A,\gamma_A \rangle$ is a single valued neutrosophic ideal of X.

Note that $(((x*y)*y)*((x*y)*y))*0 = 0$ implies $(((x*y)*y)*((x*y)*y)) \le 0, x,y \in X$ .From hypothesis we have $\alpha_A(x*y) \ge \min\{\alpha_A((x*y)*y),\alpha_A(0)\} = \alpha_A((x*y)*y)$

$\alpha_A(x*y) \ge \min\{\alpha_A((x*y)*y),\alpha_A(0)\} = \alpha_A((x*y)*y)$

$\beta_A(x*y) \ge \min\{\beta_A((x*y)*y),\beta_A(0)\} = \beta_A((x*y)*y)$ and

$\gamma_A(x*y) \le \max\{\gamma_A((x*y)*y), \gamma_A(0)\} = \gamma_A((x*y)*y)$.

And so $A = \langle X, \alpha_A, \beta_A, \gamma_A \rangle$ is single valued neutrosophic positive implicative ideal of X.

Theorem 3.22

Let $A = \langle X, \alpha_A, \beta_A, \gamma_A \rangle$ be a single valued neutrosophic set in X satisfying $((x*y)*z)*a \le b$ imply $\alpha_A((x*y)*(y*z)) \ge \min\{\alpha_A(a), \alpha_A(b)\}$, $\beta_A((x*y)*(y*z)) \ge \min\{\beta_A(a), \beta_A(b)\}$ and $\gamma_A((x*y)*(y*z)) \le \max\{\gamma_A(a), \gamma_A(b)\}$ for any $x, y, z, a, b \in X$ .Then $A = \langle X, \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X.

Proof:

Let $x, y, a, b \in X$ be such that $((x*y)*z)*a \le b$, that is $(((x*y)*y)*a)*b = 0$ therefore

$\alpha_A(x*y) = \alpha_A((x*y)*0) = \alpha_A((x*y)*(y*y)) \ge \min\{\alpha_A(a), \alpha_A(b)\}$

$\beta_A(x*y) = \beta_A((x*y)*0) = \beta_A((x*y)*(y*y)) \ge \min\{\beta_A(a), \beta_A(b)\}$

$\gamma_A(x*y) = \gamma_A((x*y)*0) = \gamma_A((x*y)*(y*y)) \le \max\{\gamma_A(a), \gamma_A(b)\}$. It follows from Theorem 3.21, $A = \langle X, \alpha_A, \beta_A, \gamma_A \rangle$ is a single valued neutrosophic positive implicative ideal of X.

IV. REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1), 1986,87-96.

[2] FlorentinSmarandache,Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy ,Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002) .

[3] FlorentinSmarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, Inter.J.PureAppl.Math., 24,2005,287-297 .

[4] Haibin Wang, Florentin Smarandache, Yanqing Zhang, Rajshekhar Sunderraman, Single valued neutrosophic sets. Technical sciences and applied Mathematics, 2012, 10-14.

[5] Iséki, K. "An algebra related with a propositional calculus", Proc. Japan Acad. Ser. A, Math. Sci., 42: (1966), 26–29,

[6] K. Iseki, S. Tanaka, An introduction to the theory of BCK-algebras, Mathematica Japonicae 23 (1978), pp. 1–26.

[7] Jun Y. B, A note on fuzzy ideals in BCK-algebras, Fuzzy Math., 5(1) (1995), 333-335.

[8] Y. B. Jun, S. M. Hong, S. J. Kim, and S. Z. Song, Fuzzy ideals and fuzzy sub-algebras of BCKalgebras, J. Fuzzy Math. 7 (1999), no. 2, 411–418.

[9] Y. B. Jun and E. H. Roh, Fuzzy commutative ideals of BCK-algebras, Fuzzy Sets and Systems 64 (1994), no. 3, 401–405.

[10] Y.B. Jun and K.H. Kim, Intuitionistic fuzzy ideals of BCK-algebras, Internat. J. Math. and Math. Sci., 24(12) (2000), 839-849.

[11] J. Meng, Y. B. Jun, and H. S. Kim, Fuzzy implicative ideals of BCK-algebras, Fuzzy Sets and Systems 89 (1997), no. 2, 243–248.

[12] Zadeh L. A, Fuzzy sets, Information and Control, 8(1965), 338-353.