Estimation of Mean Time to Recruitment in a Two Graded Manpower System with Depletion and Inter-Decision Times are Independent and Non - Identically Distributed Random Variables

S.Jenita[#1] & S.Sendhamizh Selvi[#2]

[#1] *Research Scholar, PG & Research Department of Mathematics, Government Arts College, Trichy-22,TN, India.*

[#2] *Assistant Professor, PG & Research Department of Mathematics, Government Arts College, Trichy-22,TN, India.*

Abstract

In this paper, the problem of time to recruitment prevailing in an organization with two grades when it is subjected to loss of manpower due to the policy decisions taken by the organization is studied. Two mathematical models are constructed by considering two types of thresholds in each grade namely optional and mandatory thresholds for the loss of man hours in these grades. Based on shock model approach in reliability theory, two cases are constructed using an appropriate univariate policy of recruitment is suggested in both the grades. Performance measures namely mean and variance of the time to recruitment are obtained for model I when (i)The loss of manpower form a sequence of independent and non-identically distributed exponential random variables (ii)The inter-decision times are independent and non-identically distributed exponential random variables and for model II when (i)The loss of manpower form a sequence of independent and non-identically distributed exponential random variables (ii)The inter-decision times are independent and identically distributed exponential random variables. The mean time to recruitment is obtained for these models when the optional and the mandatory thresholds follows extended exponential distribution.

Keywords:

Manpower planning, Shock models, Univariate recruitment policy, Extended Exponential distribution, Hypo-exponential distribution.

I.INTRODUCTION

Exodus of personnel is a common phenomenon in any marketing organization whenever the organization announces revised policies regarding sales target, revision of wages, incentives and perquisites. This in turn produces loss in man hours, which adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to cost of recruitment. As the loss of man hours is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. In [1] authors have stated a replacement policy for a device, which is exposed to shocks. One univariate recruitment policy based on this replacement policy under shock model approach in reliability theory suggests that recruitment is done whenever the cumulative loss of man hours crosses a particular level known as threshold or breakdown point, otherwise, the organization reaches an uneconomic status. A detailed account of the application of stochastic processes in manpower planning models can be seen from [2], [3]. The problem of finding the time to recruitment is studied for a single grade and multigrade system by several authors under different conditions.

For a single grade system, in [4] authors have suggested a new univaiate recruitment policy involving two thresholds for the loss of manpower in which one threshold is optional and the other is mandatory. According to this recruitment policy, if the total loss of man hours crosses optional threshold level the organization may or may not go for recruitment, but if the total loss of manhours crosses the mandatory threshold recruitment is necessary. In this work they have obtained the mean and variance of time to recruitment when (i) the loss of manpower form a sequence of independent and identically distributed exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables (iii) the thresholds are exponential random variables. In [5] and [7] the authors have studied this work when the distribution of the thresholds have SCBZ property and the thresholds are geometric random variables respectively, In [6] authors have also studied their work derived in [4] when (i) the distribution of optional threshold is exponential and the distribution of mandatory threshold has SCBZ property and vice versa and (ii) the inter-decision times are correlated. In [8] authors using a bivariate recruitment policy, have studied the problem of time to recruitment in a single grade organization when (i) the loss of manpower form a sequence of independent and identically distributed exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables (iii) the thresholds are exponential random variables. In [9] authors have extended their work obtained in [8] for correlated inter-decision times. In [10],[11],[12],[13] authors have extended the results of [4] for a two grade system according as the thresholds are exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables. In [14] authors have also extended the results of [6] for a two graded system. In [15] authors have also extended the results of [6] for the two grade system according as the thresholds for the first grade have SCBZ property and those for second grade are extended exponential random variables. In [16], the author has obtained mean time to recruitment for a two graded manpower system using the univariate cumulative recruitment policy considering optional and mandatory thresholds for both the grades.

The objective of the present paper is to obtain the mean time to recruitment for a two graded system using the univariate cumulative recruitment policy considering optional and mandatory thresholds for both the grades. The present paper convert the results of V.Vasudevan and A. Srinivasan [16] for a two graded manpower system when the loss of manpower and inter decision times are independent and non-identical distributed random variables. The distribution of the optional and the mandatory thresholds are extended exponential random variables.

I. MODEL DESCRIPTION FOR MODEL - I

i. Consider an organization having two grades 1 & 2 with univariate policy of recruitment which takes decisions at random epoch in $(0,\infty)$.

ii. At every decision making epoch a random number of persons quit the organization.

iii. There is an associated loss of man hours to the organization if a person quits.

iv. The loss of man hours and inter-decision times are at any decision forms a sequence of independent and non-identically distributed random variables.

v. The loss of man hours process and the process of inter-decision times are statistically independent.

vi. The loss of man hours are linear and cumulative.

III. NOTATIONS

X_i : The loss of manpowers due to the ith decision epoch i=1,2,3……forming a sequence of independent and non-identically distributed exponential random variables with parameter α_i (α_i>0).

$G_i(.)$: The Distribution function of X_i

$g_i(.)$: The probability density function of X_i with mean $\frac{1}{\alpha_i}$ (α_i >0)

S_k : Cumulative loss of manpower in the first k-decisions (k=1,2,3….)

That is $S_k = \sum_{i=0}^{k} X_i$

$G_k(.)$: The Distribution function of sum of k independent and non-identically distributed exponential random variables.

$g_k(.)$: The probability density function of S_k

we note that $G_k(t) = \sum_{i=1}^{k} c_i (1- e^{-\alpha_i t})$,

$g_k(t) = \sum_{i=1}^{k} c_i \alpha_i e^{-\alpha_i t}$, $g_k^*(s) = \sum_{i=1}^{k} c_i \frac{\alpha_i}{\alpha_i+s}$

Where $c_i = \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{\alpha_j}{\alpha_j - \alpha_i}, i = 1,2,3..k$

U_i : The inter-decision times are independent and non-identically distributed exponential random variables between (i-1) and ith decisions with parameter β_i (β_i>0)

$F_i(.)$: The Distribution function of U_i

$f_i(.)$: The probability density function of U_i with mean $\frac{1}{\beta_i}$ (β_i >0)

R_k : The waiting time upto k decisions

$F_k(.)$: The Distribution function of R_k

$f_k(.)$: The probability density function of R_k

we note that $F_k(t) = \sum_{i=1}^{k} b_i(1- e^{-\beta_i t})$,

$f_k(t) = \sum_{i=i}^{k} b_i \beta_i e^{-\beta_i t}$, $f_k^*(s) = \sum_{i=1}^{k} b_i \frac{\beta_i}{\beta_i+s}$

Where $b_i = \prod_{\substack{j=1 \\ j\neq i}}^{k} \frac{\beta_j}{\beta_j - \beta_i}, i = 1,2,3..k$

Y_1,Y_2 : The continuous random variables denoting the optional thresholds levels for the grade 1 and grade 2 follows extended exponential distribution with parameters λ_1 & λ_2 respectively.

Z_1,Z_2 : The continuous random variables denoting the mandatory thresholds levels for the grade 1 and grade 2 follows extended exponential distribution with parameters μ_1 & μ_2 respectively.

W : The continuous random variable denoting the time to recruitment in the organization.

p : The probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold Y.

$V_k(t)$: The probability that exactly k decisions are taken in [0,t).

L(.) : Distribution function of W

l(.) : The probability density function of W

$l^*(.)$: The laplace transform of l(.)

E(W) : The expected time to recruitment

CUM policy : Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

IV.MAIN RESULTS

The survival function of W is given by

$P(W > t) = \sum_{k=0}^{\infty}$ [Probability that exactly k-decisions are taken in [0,t), k = 0,1,2…] [Probability that the total number of exits in these k-decisions does not cross the optional level Y or the total number of exits in these k- decisions crosses the optional level Y but lies below the mandatory level Z and the organization is not making recruitment]

$$P(W>t)= \sum_{k=0}^{\infty} V_k(t)\, P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t)\, P(S_k \geq Y)\, P(S_k < Z)\, p \quad (1)$$

Case (i):

For maximum case,Consider $P(S_k<Y)$, Conditioning upon S_k and using the law of total probability ,We get

$$P(S_k<Y) = \int_0^{\infty} P(S_k < Y | S_k = x) g_k(x) dx$$
$$= \int_0^{\infty} P(Y = \max(Y_1, Y_2) > x)\, g_k(x) dx$$
$$= \int_0^{\infty} [1 - P(Y \leq x)] g_k(x) dx$$
$$= \int_0^{\infty} [1 - P(Y_1 \leq x) P(Y_2 \leq x)] g_k(x) dx$$
$$= \int_0^{\infty} [1 - (1 - e^{-\lambda_1 x})^2 (1 - e^{-\lambda_2 x})^2] g_k(x) dx$$
$$= 2\int_0^{\infty} e^{-\lambda_1 x} g_k(x) dx + 2\int_0^{\infty} e^{-\lambda_2 x} g_k(x) dx$$
$$-4\int_0^{\infty} e^{-(\lambda_1+\lambda_2)x} g_k(x) dx + 2\int_0^{\infty} e^{-(2\lambda_1+\lambda_2)x}\, g_k(x) dx$$
$$+2\int_0^{\infty} e^{-(\lambda_1+2\lambda_2)x}\, g_k(x) dx$$
$$-\int_0^{\infty} e^{-2\lambda_1 x} g_k(x) dx - \int_0^{\infty} e^{-2\lambda_2 x} g_k(x) dx$$
$$-\int_0^{\infty} e^{-(2\lambda_1+2\lambda_2)x} g_k(x) dx$$

$$P(S_k<Y) = 2g_k^*(\lambda_1)+2g_k^*(\lambda_2) - 4g_k^*(\lambda_1+\lambda_2) +2g_k^*(2\lambda_1+\lambda_2)+2g_k^*(\lambda_1+2\lambda_2) - g_k^*(2\lambda_1) - g_k^*(2\lambda_2) - g_k^*(2\lambda_1+2\lambda_2) \quad (2)$$

$$P(S_k < Y)= 2D_1+2D_2 - 4D_3+2D_4+2D_5-D_6-D_7-D_8 \quad (3)$$

Where $D_1 = g_k^*(\lambda_1)$, $D_2 = g_k^*(\lambda_2)$, $D_3 = g_k^*(\lambda_1+\lambda_2)$, $D_4 = g_k^*(2\lambda_1+\lambda_2)$, $D_5 = g_k^*(\lambda_1+2\lambda_2)$
$D_6 = g_k^*(2\lambda_1)$ $D_7 = g_k^*(2\lambda_2)$,
$D_8 = g_k^*(2\lambda_1+2\lambda_2)$,

Similarly

$$(S_k<Z) = 2\int_0^{\infty} e^{-\mu_1 x} g_k(x) dx + 2\int_0^{\infty} e^{-\mu_2 x} g_k(x) dx$$
$$-4\int_0^{\infty} e^{-(\mu_1+\mu_2)x} g_k(x) dx$$
$$+2\int_0^{\infty} e^{-(2\mu_1+\mu_2)x}\, g_k(x)\, dx$$
$$+2\int_0^{\infty} e^{-(\mu_1+2\mu_2)x}\, g_k(x) dx$$
$$-\int_0^{\infty} e^{-2\mu_1 x} g_k(x) dx - \int_0^{\infty} e^{-2\mu_2 x} g_k(x) dx$$
$$-\int_0^{\infty} e^{-(2\mu_1+2\mu_2)x} g_k(x) dx$$

$$P(S_k<Z) = 2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2) \quad (4)$$

$$P(S_k < Z) = 2D_9+2D_{10} - 4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16} \quad (5)$$

Where $D_9 = g_k^*(\mu_1)$, $D_{10} = g_k^*(\mu_2)$, $D_{11} = g_k^*(\mu_1+\mu_2)$,
$D_{12} = g_k^*(2\mu_1+\mu_2)$, $D_{13} = g_k^*(\mu_1+2\mu_2)$ $D_{14} = g_k^*(2\mu_1)$
$D_{15} = g_k^*(2\mu_2)$,$D_{16} = g_k^*(2\mu_1+2\mu_2)$,

Substituting (3) & (5) in (1), we get

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t) \{(2D_1+2D_2 - 4D_3+2D_4+2D_5-D_6-D_7-D_8) + p[1- (2D_1+2D_2 - 4D_3+2D_4+2D_5-D_6-D_7-D_8)] [2D_9+2D_{10} - 4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}]\}$$

$$P(W>t)= \sum_{k=0}^{\infty} V_k(t) \{(2D_1+2D_2-4D_3+2D_4+2D_5-D_6-D_7-D_8) [1-p(2D_9+2D_{10}- 4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}) + p(2D_9+2D_{10} - 4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (6)$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t) \{B_k(1- pC_k) +pC_k\} \quad (7)$$

Where $B_k = 2D_1+2D_2-4D_3+2D_4+2D_5-D_6-D_7-D_8$,$C_k$

$C_k = 2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, A_k, \text{ where } A_k=B_k(1-pC_k)+pC_k \quad (8)$$

From renewal theory (Medhi),

$$P(W>t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]A_k \quad (9)$$

$$P(W>t) = \sum_{k=0}^{\infty} F_k(t)A_k - \sum_{k=0}^{\infty} F_{k+1}(t)A_k$$

Since $L(t) = 1 - P(W>t)$ and $l(t) = \frac{d}{dt}L(t)$, we get

$$L(t) = 1 - \sum_{k=0}^{\infty} F_k(t)A_k + \sum_{k=0}^{\infty} F_{k+1}(t)A_k$$

$$l(t) = -\sum_{k=0}^{\infty} f_k(t)A_k + \sum_{k=0}^{\infty} f_{k+1}(t)A_k$$

(10)

Taking Laplace Transform on both sides,
we get

$$l^*(s) = \sum_{k=0}^{\infty} f_{k+1}^*(s)\, A_k - \sum_{k=0}^{\infty} f_k^*(s)\, A_k \quad (11)$$

Where $f_k^*(s) = \sum_{i=1}^{k} \frac{b_i\beta_i}{s+\beta_i}$

The mean time to recruitment, is known that

$$E(W) = -\left[\frac{d}{ds} l^*(s)\right]_{s=0} \quad (12)$$

Now

$$-\frac{d}{ds} f_k^*(s)|_{s=0} = E[U_1+U_2+U_3+\ldots\ldots.+U_k] = \sum_{i=1}^{k} \frac{1}{\beta_i} \quad (13)$$

From the equations (11),(12) & (13), we get

$$E(W) = \sum_{k=0}^{\infty} \sum_{i=1}^{k+1} \frac{1}{\beta_i} A_k - \sum_{k=0}^{\infty} \sum_{i=1}^{k} \frac{1}{\beta_i} A_k$$

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k \quad (14)$$

Equation (14) gives the mean time to recruitment for maximum case in Model I.

Where $A_k = (2D_1+2D_2-4D_3+2D_4+2D_5-D_6-D_7-D_8)$
$[1-p(2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}$
$-D_{15}-D_{16})] + p(2D_9+2D_{10}-4D_{11}$
$+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})$ and

$D_1 = g_k^*(\lambda_1)$, $D_2 = g_k^*(\lambda_2)$, $D_3 = g_k^*(\lambda_1+\lambda_2)$,
$D_4 = g_k^*(2\lambda_1+\lambda_2)$, $D_5 = g_k^*(\lambda_1+2\lambda_2)$, $D_6 = g_k^*(2\lambda_1)$
$D_7 = g_k^*(2\lambda_2)$, $D_8 = g_k^*(2\lambda_1+2\lambda_2)$, $D_9 = g_k^*(\mu_1)$,
$D_{10} = g_k^*(\mu_2)$, $D_{11} = g_k^*(\mu_1+\mu_2)$, $D_{12} = g_k^*(2\mu_1+\mu_2)$,
$D_{13} = g_k^*(\mu_1+2\mu_2)$, $D_{14} = g_k^*(2\mu_1)$, $D_{15} = g_k^*(2\mu_2)$,
$D_{16} = g_k^*(2\mu_1+2\mu_2)$.

Case (ii):

For minimum case, the optional and mandatory thresholds for the organization are given by

$Y = \min(Y_1,Y_2)$ and $Z = \min(Z_1,Z_2)$, with same assumptions and notations of case(i).

As in case(i),

$$P(S_k<Y) = \int_0^{\infty} P(S_K < Y | S_K = x) g_k(x)dx$$
$$= \int_0^{\infty} P(Y = \min(Y_1,Y_2) > x) g_k(x)dx$$
$$= \int_0^{\infty} P(Y_1 > x)P(Y_2 > x) g_k(x)dx$$
$$= \int_0^{\infty} (1 - P(Y_1 \le x))(1 - P(Y_2 \le x))\, g_k(x)dx$$
$$= \int_0^{\infty} [1-(1-e^{-\lambda_1 x})^2]\, [1-(1-e^{-\lambda_2 x})^2] g_k(x)dx$$
$$= 4\int_0^{\infty} e^{-(\lambda_1+\lambda_2)x} g_k(x)\, dx$$
$$-2\int_0^{\infty} e^{-(2\lambda_1+\lambda_2)x} g_k(x)dx$$
$$-2\int_0^{\infty} e^{-(\lambda_1+2\lambda_2)x} g_k(x)dx$$
$$+\int_0^{\infty} e^{-(2\lambda_1+2\lambda_2)x} g_k(x)dx$$

$$P(S_k<Y) = 4g_k^*(\lambda_1+\lambda_2) - 2g_k^*(2\lambda_1+\lambda_2) - 2g_k^*(\lambda_1+2\lambda_2) + g_k^*(2\lambda_1+2\lambda_2) \quad (15)$$

$$P(S_k<Y) = 4D_3-2D_4-2D_5+D_8 \quad (16)$$

Where $D_3 = g_k^*(\mu_1+\mu_2)$, $D_4 = g_k^*(2\mu_1+\mu_2)$,
$D_5 = g_k^*(\mu_1+2\mu_2)$, $D_8 = g_k^*(2\mu_1+2\mu_2)$

Similarly

$$P(S_k<Z) = 4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2) - 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2) \quad (17)$$

(ie) $P(S_k<Z) = 4D_{11}-2D_{12}-2D_{13}+D_{16}$ (18)

Where $D_{11} = g_k^*(\mu_1+\mu_2)$, $D_{12} = g_k^*(2\mu_1+\mu_2)$,
$D_{13} = g_k^*(\mu_1+2\mu_2)$, $D_{16} = g_k^*(2\mu_1+2\mu_2)$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, \{(4D_3-2D_4-2D_5+D_8)\, [1-p(4D_{11}-2D_{12}-2D_{13}+D_{16})] + p\,(4D_{11}-2D_{12}-2D_{13}+D_{16})\} \quad (19)$$

From Renewal Theory,

$$P(W>t)=\sum_{k=0}^{\infty} V_k(t)\{B_k(1-pC_k)+pC_k\} \quad (20)$$

Where $B_k = 4D_3-2D_4-2D_5+D_8$,

$C_k = 4D_{11}-2D_{12}-2D_{13}+D_{16}$

$$P(W>t)=\sum_{k=0}^{\infty} V_k(t)A_k, \text{ where } A_k = B_k(1-pC_k)+pC_k \quad (21)$$

As we found in case (i)

$$E(W)=\sum_{k=0}^{\infty}\frac{1}{\beta_{k+1}}A_k \quad (22)$$

Equation (22) gives the mean time to recruitment for minimum case in Model I.

Where $A_k = (4D_3-2D_4-2D_5+D_8)[1-p(4D_{11}-2D_{12}-2D_{13}+D_{16})]+p(4D_{11}-2D_{12}-2D_{13}+D_{16})$ &

$D_3 = g_k^*(\mu_1+\mu_2)$, $D_4 = g_k^*(2\mu_1+\mu_2)$,

$D_5 = g_k^*(\mu_1+2\mu_2)$, $D_8 = g_k^*(2\mu_1+2\mu_2)$,

$D_{11} = g_k^*(\mu_1+\mu_2)$, $D_{12} = g_k^*(2\mu_1+\mu_2)$,

$D_{13} = g_k^*(\mu_1+2\mu_2)$, $D_{16} = g_k^*(2\mu_1+2\mu_2)$

Model II:

Description of model II is similar to that of model I except the condition on inter-decision times.

The analytical results for the survival function of time to recruitment when the loss of manpower is independent and non-identically distributed exponential random variables with α_i and inter-decision times are independent and identically distributed exponential random variables with parameter β. The thresholds optional and mandatory follows extended exponential distribution with parameters λ_1 & λ_1 and μ_1 & μ_2 .

Case (i):

As in model I, for maximum case,

We define $A_k = [2D_1+2D_2-4D_3+2D_4+2D_5-D_6-D_7-D_8]$

$[1-p(2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})]$

$+p[2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}]$

By definition $V_k(t) = P[\ N(t) = k\ ]$, where N(t) is the number of decisions taken in [0,t).

From (8), we get

$$P(W>t)=\sum_{k=0}^{\infty} V_k(t)\{[2D_1+2D_2-4D_3+2D_4+2D_5-D_6-D_7-D_8][1-p(2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})]+p[2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}]\}$$

$$\text{(ie)}P(W>t)=\sum_{k=0}^{\infty} V_k(t)\{[2g_k^*(\lambda_1)2g_k^*(\lambda_2)-4g_k^*(\lambda_1+\lambda_2)+2g_k^*(2\lambda_1+\lambda_2)-g_k^*(2\lambda_1)-g_k^*(2\lambda_2)-g_k^*(2\lambda_1+2\lambda_2)][1-p(2g_k^*(\mu_1)+2g_k^*(\mu_2)-4g_k^*(\mu_1+\mu_2)+2g_k^*(2\mu_1+\mu_2)-g_k^*(2\mu_1)-g_k^*(2\mu_2)-g_k^*(2\mu_1+2\mu_2))]+p[2g_k^*(\mu_1)+2g_k^*(\mu_2)-4g_k^*(\mu_1+\mu_2)+2g_k^*(2\mu_1+\mu_2)-g_k^*(2\mu_1)-g_k^*(2\mu_2)-g_k^*(2\mu_1+2\mu_2)]\}$$

$$=\sum_{k=0}^{\infty} V_k(t)A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)$$

Now,

$$P(W>t)=\sum_{k=0}^{\infty} P[N(t)=k]\{A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)\} \quad (23)$$

Since W is a non-negative continuous random variable, its known from [Medhi] that

$$E[W]=\int_0^{\infty} P(W>t)\,dt$$

Therefore from the equation (23),we get

$$E[W]=\int_0^{\infty}\sum_{k=0}^{\infty} P[N(t)=k]\{A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)\} \quad (24)$$

Since N(t) is a Poisson process with rate β by hypothesis, we can write the equation (24) as

$$E[W]=\sum_{k=0}^{\infty} A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)\int_0^{\infty}\frac{e^{-\beta t}(\beta t)^k}{k!}dt$$

$$=\sum_{k=0}^{\infty} A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)\frac{\beta^k}{k!}\int_0^{\infty} t^k e^{-\beta t}dt$$

$$=\sum_{k=0}^{\infty} A_k(\lambda_1,\lambda_2,\mu_1,\mu_2)\frac{\beta^k}{k!}\frac{k!}{\beta^{k+1}}$$

$$E[W]=\frac{1}{\beta}\sum_{k=0}^{\infty} A_k(\lambda_1,\lambda_2,\mu_1,\mu_2) \quad (25)$$

Equation (25) gives the mean time to recruitment for maximum case in model II

Case (ii):

For minimum case,

We define $A_k = [4D_3-2D_4-2D_5+D_8]$

$[1-p(4D_{11}-2D_{12}-2D_{13}+D_{16})]+p[4D_{11}-2D_{12}-2D_{13}+D_{16}]$

As in case (i),

$$P(W>t)=\sum_{k=0}^{\infty} V_k(t)\{[4D_3-2D_4-2D_5+D_8]$$

[1-p(4D$_{11}$-2D$_{12}$-2D$_{13}$+D$_{16}$)]

+ p[4D$_{11}$-2D$_{12}$-2D$_{13}$+D$_{16}$]}

$$= \sum_{k=0}^{\infty} V_k(t)\ \{[4g_k^*(\lambda_1+\lambda_2) - 2g_k^*(2\lambda_1+\lambda_2) - 2g_k^*(\lambda_1+2\lambda_2) + g_k^*(2\lambda_1+2\lambda_2)]$$
$$[1 - p(4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2) - 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2)]$$
$$+p[4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2) - 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2)]\}$$

$$= \sum_{k=0}^{\infty} V_k(t)\ A_k(\lambda_1, \lambda_2, \mu_1, \mu_2)$$

$$P(W>t) = \sum_{k=0}^{\infty} P[N(t)=k]\ \{A_k(\lambda_1, \lambda_2, \mu_1, \mu_2)\}$$

and $E[W] = \int_0^{\infty} \sum_{k=0}^{\infty} P[N(t)=k]\ \{A_k(\lambda_1, \lambda_2, \mu_1, \mu_2)\}$

(ie) $E[W] = \sum_{k=0}^{\infty} A_k(\lambda_1, \lambda_2, \mu_1, \mu_2) \int_0^{\infty} \frac{e^{-\beta t}(\beta t)^k}{k!} dt$

$$= \sum_{k=0}^{\infty} A_k(\lambda_1, \lambda_2, \mu_1, \mu_2) \frac{\beta^k}{k!} \int_0^{\infty} t^k e^{-\beta t} dt$$

$$= \sum_{k=0}^{\infty} A_k(\lambda_1, \lambda_2, \mu_1, \mu_2) \frac{\beta^k}{k!} \frac{k!}{\beta^{k+1}}$$

$$E[W] = \frac{1}{\beta} \sum_{k=0}^{\infty} A_k(\lambda_1, \lambda_2, \mu_1, \mu_2) \qquad (26)$$

Equation (26) gives the mean time to recruitment for minimum case in model II

SPECIAL CASES :

Case (i) :

The optional threshold Y_1 , Y_2 follows exponential distribution with parameters (λ_1, λ_2) and the mandatory threshold Z_1,Z_2 follows extended exponential distribution with parameters (μ_1, μ_2), the loss of manpower and the inter-decision times are independent and non-identically distributed exponential random variables with parameters α_i , β_i

In maximum case,

$$P(S_k<Y) = \int_0^{\infty} [1-(1-e^{-\lambda_1 x})(1-e^{-\lambda_2 x})]\ g_k(x)dx$$

$$= \int_0^{\infty} e^{-\lambda_1 x} g_k(x)dx + \int_0^{\infty} e^{-\lambda_2 x} g_k(x)dx - \int_0^{\infty} e^{-(\lambda_1+\lambda_2)x} g_k(x)dx$$

$$P(S_k<Y) = g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1+\lambda_2) \qquad (27)$$

$$P(S_k<Y) = D_1+D_2-D_3 \qquad (28)$$

Where $D_1 = g_k^*(\lambda_1)$, $D_2 = g_k^*(\lambda_2)$,
$D_3 = g_k^*(\lambda_1+\lambda_2)$

From model I,

$$P(S_k<Z) = 2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2) \qquad (29)$$

$$P(S_k<Z) = 2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16} \qquad (30)$$

Where $D_9 = g_k^*(\mu_1)$, $D_{10} = g_k^*(\mu_2)$, $D_{11} = g_k^*(\mu_1+\mu_2)$,
$D_{12} = g_k^*(2\mu_1+\mu_2)$, $D_{13} = g_k^*(\mu_1+2\mu_2)$
$D_{14} = g_k^*(2\mu_1)$, $D_{15} = g_k^*(2\mu_2)$,
$D_{16} = g_k^*(2\mu_1+2\mu_2)$

Using the equations (28) and (30) in (1), we get

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{(D_1+D_2-D_3)+p(1-(D_1+D_2-D_3))(2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{[D_1+D_2-D_3][1-p(2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})] + p[2D_9+2D_{10}-4D_{11}+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}]$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\ A_K,$$

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k \qquad (31)$$

Where $A_k = [g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1+\lambda_2)]$
$[1-p(2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)$
$+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2))]$
$+p[2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)$
$+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2)]$

In minimum case,

$$P(S_k<Y) = \int_0^{\infty} e^{-(\lambda_1+\lambda_2)x} g_k(x)dx$$

$P(S_k<Y) = g_k^*(\lambda_1+\lambda_2)$

$P(S_k<Y) = D_3$, Where $D_3 = g_k^*(\lambda_1+\lambda_2)$ (32)

As in model I,

$$P(S_k<Z) = 4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2) - 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2)$$

$$P(S_k<Z) = 4D_{11}-2D_{12}-2D_{13}+D_{16} \quad (33)$$

Where $D_{11} = g_k^*(\mu_1+\mu_2)$,

$D_{12} = g_k^*(2\mu_1+\mu_2)$, $D_{13} = g_k^*(\mu_1+2\mu_2)$,

$D_{16} = g_k^*(2\mu_1+2\mu_2)$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\{[D_3][1-p(4D_{11}-2D_{12}-2D_{13}+D_{16})]+p[4D_{11}-2D_{12}-2D_{13}+D_{16}]$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, A_k$$

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k \quad (34)$$

Where $A_k = [g_k^*(\lambda_1+\lambda_2)]$

$[1-p(4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2)$

$- 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2))]$

$+ p[4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2)$

$- 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2)]$

Note : 1

The optional threshold Y_1, Y_2 follows exponential distribution with parameters (λ_1, λ_2) and the mandatory threshold Z_1,Z_2 follows extended exponential distribution with parameters (μ_1, μ_2), the loss of manpower are independent and non-identically distributed exponential random variables with parameters α_i and the inter-decision times are independent and identically distributed exponential random variables with parameters β.

In maximum case,

$$E(W) = \frac{1}{\beta} \sum_{k=0}^{\infty} A_k \quad (35)$$

Where

$A_k = [g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1+\lambda_2)]$

$[1-p(2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)$

$+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2))]$

$+p[2g_k^*(\mu_1)+2g_k^*(\mu_2) - 4g_k^*(\mu_1+\mu_2) + 2g_k^*(2\mu_1+\mu_2)$

$+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2)]$

In minimum case, the same results of maximum case but

$A_k = [g_k^*(\lambda_1+\lambda_2)][1-p(4g_k^*(\mu_1+\mu_2) - 2g_k^*(2\mu_1+\mu_2)$

$- 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2))] + p[4g_k^*(\mu_1+\mu_2)$

$- 2g_k^*(2\mu_1+\mu_2) - 2g_k^*(\mu_1+2\mu_2) + g_k^*(2\mu_1+2\mu_2)]$

Case (ii) :

If we interchange the distributions ,we get the same results as in case (i) with different parameters.

In maximum case,

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k$$

(36)

Where $A_k = [2g_k^*(\lambda_1)+2g_k^*(\lambda_2) - 4g_k^*(\lambda_1+\lambda_2)$

$+2g_k^*(2\lambda_1+\lambda_2)+2g_k^*(\lambda_1+2\lambda_2)$

$- g_k^*(2\lambda_1) - g_k^*(2\lambda_2) - g_k^*(2\lambda_1+2\lambda_2)]$

$[1-p(g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1+\mu_2))]$

$+ p[g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1+\mu_2)]$

In minimum case,

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}} A_k$$

Where

$A_k = [4g_k^*(\lambda_1+\lambda_2) - 2g_k^*(2\lambda_1+\lambda_2)$

$- 2g_k^*(\lambda_1+2\lambda_2) + g_k^*(2\lambda_1+2\lambda_2)]$

$[1-p(g_k^*(\mu_1+\mu_2)]+p[g_k^*(\mu_1+\mu_2)]$

Note:2

The optional threshold Y_1, Y_2 follows extended exponential distribution with parameters (λ_1, λ_2) and the mandatory threshold Z_1,Z_2 follows exponential distribution with parameters (μ_1, μ_2), the loss of manpower are independent and non-identically distributed exponential random variables with parameters α_i and the inter-decision times are independent and identically distributed exponential random variables with parameters β

In maximum case,

$$E(W) = \frac{1}{\beta} \sum_{k=0}^{\infty} A_k \quad (38)$$

Where

$A_k = [2g_k^*(\lambda_1)+2g_k^*(\lambda_2)\ 4g_k^*(\lambda_1+\lambda_2) + 2g_k^*(2\lambda_1+\lambda_2)$

$+2g_k^*(\lambda_1+2\lambda_2) - g_k^*(2\lambda_1) - g_k^*(2\lambda_2)$

$- g_k^*(2\lambda_1+2\lambda_2)]$ [1-p($g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1 + \mu_2)$]+p[$g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1 + \mu_2)$]

In minimum case, the same results of maximum case but

$$A_k = [4g_k^*(\lambda_1+\lambda_2) - 2g_k^*(2\lambda_1+\lambda_2) - 2g_k^*(\lambda_1+2\lambda_2) + g_k^*(2\lambda_1+2\lambda_2)][1-p(g_k^*(\mu_1+\mu_2))]+p[g_k^*(\mu_1+\mu_2)]$$

V.CONCLUSION

The manpower planning model developed in this work is more general compared to earlier work in the context to non-identical nature of loss of manpower, Inter-decision times. The stochastic models developed in this work can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels in the context of attrition. There is a scope for studying the applicability of the designed models using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this work. The findings given in this work enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but in a wider domain. The present work can be studied for a two sources of depletion.

REFERENCES

[1] Esary. J.D.,Marshall.A.W. and Prochan., 1973, *Ann. Probability*, 1(4),pp.627-649.

[2] Barthlomew.D.J. and Morris.B.R..1971. *Aspects of Manpower Planning*, Elsevier Publishing Company.New York.

[3] Barthlomew.D.J. and Forbes.A.,1979, Statistical Techniques for Man Power Planning, John Wiley and Sons.

[4] Esther Clara, J.B. and Srinivasan A., 2008. "*Expected Time for Recruitment in a Single Graded Manpower System with Two Thresholds"*. Proceedings of AICTE sponsored National Conference on Recent Development and Applications of Probability Theory, Random Process and Random Variables in Computer Science : 98-102.

[5] Esther Clara, J.B. and Srinivasan A., 2009 "*A Stochastic Model for the Expected Time for Recruitment in a Single Graded Manpower System with Two Thresholds having SCBZ Property*." Proceedings of the International conference of Mathematical Methods and Computation, Narosa Publishing house Pvt. Ltd. New Delhi: 274-280.

[6] Esther Clara, J.B. and Srinivasan A., 2010a, "*A Stochastic Model for the Expected Time to Recruitment in a Single Graded Manpower System with Two Types of Thresholds and Correlated Inter-decision times*." Narosa Publishing House Pvt. ltd. New Delhi. 44 – 49.

[7] Esther Clara, J.B. and Srinivasan A.,(2010b), Antarctica J.Math,7(3): pp.261-271.

[8] Esther Clara, J.B. and Srinivasan A.,(2010c), Resent Research in Science and Technology,2(2): pp.70,-75.

[9] Esther Clara, J.B. and Srinivasan A.,(2011), International Journal of Mathematical Sciences and Engineering Applications, Vol.5, No.111,pp.281-288.

[10] Vasudevan.V and Srinivasan.S.,(2011a), *"Variance of the Time to Recruitment in an Organization with Two Grades"*, Recent Research in Science and Technology, 3(1),128-131.

[11] Vasudevan.V and Srinivasan.S.,(2011b), "*A Stochastic Model for the Expected Time to Recruitment in a Two Graded Manpower System"*, Antaretica Journal of Mathematics,Vol.8

[12] Vasudevan.V and Srinivasan.A.,(2011c), "*A Manpower for a Two Graded System with a Univariate Policy of Recruitment"*, The International review of Pure and Applied Mathematics, Vol.7 (1), pp.79-88.

[13] Vasudevan.V and Srinivasan.A.,(2011d), "*A Stochastic Model for the Expected Time to Recruitment in a Two Graded Manpower System with Two Discrete Thresholds"*, International Journal of Applied Mathematical Analysis and Applications, Vol.6,No.1, pp.141-148.

[14] Vasudevan.V and Srinivasan.A.,(2011e), "*Expected Time to Recruitment in an Organization with Two Grades using a Univariate Recruitment Policy involving Two Thresholds"*, Recent research in Science and Technology.

[15] Vasudevan.V and Srinivasan.A.,(2011f), "*Stochastic Model for Time to Recruitment in an Organization with two grades"*, Advances in Applied Mathematical Analysis.

[16] Vasudevan.V and Srinivasan.A., (2011), "*A Stochastic Model for the Expected Time to Recruitment in a Two graded Manpower system with Two Types of Thresholds"*, Acta Ciencia Indica, Vol.XXXVIIIM,No.1.11.