Some Graph Operations on Signed Product Cordial Labeling Graphs

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ABSTRACT

A vertex labeling $f: V(G) \rightarrow \{-1,1\}$ of a graph G with induced edge labeling$f^{*}: E(G) \rightarrow \{-1,1\}$ defined by $f^{*}(uv) = f(u)f(v)$ is called a signed product cordial labeling if the number of vertices with labels -1 and +1 differs at most 1 as well as the number of edges with labels -1 and +1 differs at most 1. In this paper, we prove some results on signed product cordial labeling of graphs in the context of path union at vertex of flower F_n, binary tree and star. Also, we prove total graph of path P_n and $K_{1,n} \odot G^{'}$ ($G^{'}$ be the null graph with two vertices) are signed and total signed product cordial labeling graphs.

1 Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Gross and Yellen[8]. We will provide brief summary of definitions and other information which are prerequisites for the present study.

Definition 1.1.A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called label of vertex v of Gunder$f$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G labeled with 0 and 1 respectively under f. A mapping $f^{*} : E(G) \rightarrow \{0,1\}$ is called binary edge labeling of G. Let, $e_{f^*}(0)$, $e_{f^*}(1)$ the number of edges of G having labels 0 and 1 respectively underf^{*}.

Definition 1.2.A binary vertex labeling of graph G with induced edge labeling $f^{*}: E(G) \rightarrow \{0,1\}$ defined by $f^{*}(e = uv) = |f(u) - f(v)|$is called cordial labeling if$|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. A graph G is cordial if it admits a cordial labeling.

The concept of cordial labeling was introduced by Cahit[5] as a weaker version of graceful and harmonious labeling. This concept is explored by many researchers like Andar et.al.,[1,2], Vaidya and Dani [14,15]. Vaidya and Vihol [14] have proved that middle graphs of path, crown, $K_{1,n}$, Tadpole T(n,l+1) are cordial graph. Vaidya and Shah[17,18] have discussed cordial labeling of some bistar related graphs and cordial labeling of snake related graphs. Motivated through the concept of cordial labeling Babujee and Shobana [3] introduced the concepts of cordial languages and cordial numbers. Lawrence and Koilraj [10] have discussed cordial labeling for the spliting graph of some standard graphs.

Definition 1.3.(Babujee and Shobana [4]) A vertex labeling $f : V(G) \to \{-1,1\}$ of a graph G with induced edge labeling $f^*: E(G) \to \{-1,1\}$, defined by $f^*(uv) = f(u)f(v)$, is called signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$. A graph G is signed product cordial if it admits a signed product cordial labeling.

Santhi and James Albert [12] have introduced the concept of total signed product cordial labeling.

Definition 1.4.Let $f : V(G) \to \{-1,1\}$ with induced edge labeling $f^* : E(G) \to \{-1,1\}$ defined by $f^*(uv) = f(u)f(v)$. Then f is called a total signed product cordial labeling if $|(v_f(-1) + e_{f^*}(-1)) - (v_f(1) + e_{f^*}(1))| \leq 1$. A graph with a total signed product cordial labeling is called a total signed product cordial graph.

Definition 1.5. The total graph $T(G)$ of a graph G is the graph whose vertex set is V ∪ E, with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident.

Definition 1.6. (shee and Ho [13]) Let $G_1, G_2, \cdots, G_n, n \geq 2$ be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for $i = 1,2,\cdots, n-1$ is called path union of G and is defined by $G(n)$.

Definition 1.7.Let $G_1, G_2, \cdots, G_n, n \geq 2$ be n copies of a graph G. The graph $G_u(n)$obtained by adding edge between the vertex u in G_i and the copy of u in G_{i+1} in$G_i$for $(i = 1,2,\cdots, n-1)$, we call $G_u(n)$is called the path union of n copies of G at u.

Defintion 1.8.**(Frucht and Harary [7])**Let G_1 and G_2 be two simple, connected graphs. The corona $G_1 \odot G_2$of two graphs G_1 and G_2 is the graph obtained by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2, and then joining the i^{th} copy of G_2 with G_1.

2 MAIN RESULTS

Theorem 2.1. Let $G = F_n$and w be the centre vertex of F_n . Then $G_w(2)$is signed product corial labeling graph.

Proof

Let u and v be the centre vertices of the flower graphs G_1 and G_2.

Let us denote the vertices in the cycle of G_1 and G_2 as u_i and $v_i (1 \leq i \leq n)$ respectively.

Let the end vertices of $G_1$and G_2 as u_i' and $v_i' (1 \leq i \leq n)$ respectively.

Let $w_1, w_2, \cdots, w_k$ be the vertices of path P_k with $u = w_1$ and $v = w_k k \geq 2$

Then $|V(G_w(2))| = 4n + k$ and $|E(G_w(2))| = 8n + k - 1$

Define $f: V(G_w(2)) \to \{-1,1\}$ as given below.

$$f(u) = -1$$

$$f(v) = 1$$

$$f(u_i) = f(v_i') = 1$$

$$f(v_i) = f(u_i') = -1$$

$$f(w_i) = \begin{cases} -1, & (1,2)(mod\ 4), \\ 1 & (0,3)(mod\ 4). \end{cases}$$

Case(i) Suppose $k = 2$.

$$v_f(-1) = v_f(1) = 2n + 1$$

$$e_{f^*}(-1) = e_{f^*}(1) = 4n + 1$$

Case(ii) Suppose $k \equiv 2\ (mod\ 4\ )$ and $k \neq 2$.

$$v_f(-1) = v_f(1) = 2n + 3$$

$$e_{f^*}(-1) = 4n + 3$$

$$e_{f^*}(1) = 4n + 2$$

Case(iii) Suppose$k \equiv 1\ (mod\ 4\ )$.

$$v_f(-1) = 2n + 3$$

$$v_f(1) = 2n + 2$$

$$e_{f^*}(-1) = e_{f^*}(1) = 4n + 2$$

Case(iv) Suppose $k \equiv 3\ (mod\ 4\ )$.

$$v_f(-1) = 2n + 2$$

$$v_f(1) = 2n + 1$$

$$e_{f^*}(-1) = e_{f^*}(1) = 4n + 1$$

Case(v) Suppose$k \equiv 0\ (mod\ 4\ )$.

$$v_f(-1) = v_f(1) = 2n + 2$$

$$e_{f^*}(-1) = 4n + 1$$

$$e_{f^*}(1) = 4n + 2$$

By all the above cases, we have

$|v_f(-1) - v_f(1)| \leq 1$ and $|e_f{}^*(-1) - e_f{}^*(1)| \leq 1$.

Hence, G is a signed product cordial graph.

Example2.2.Suppose $G = F_3$and w be the centre vertex of G.Then $G_w(2)$ with its signed product cordial labeling.

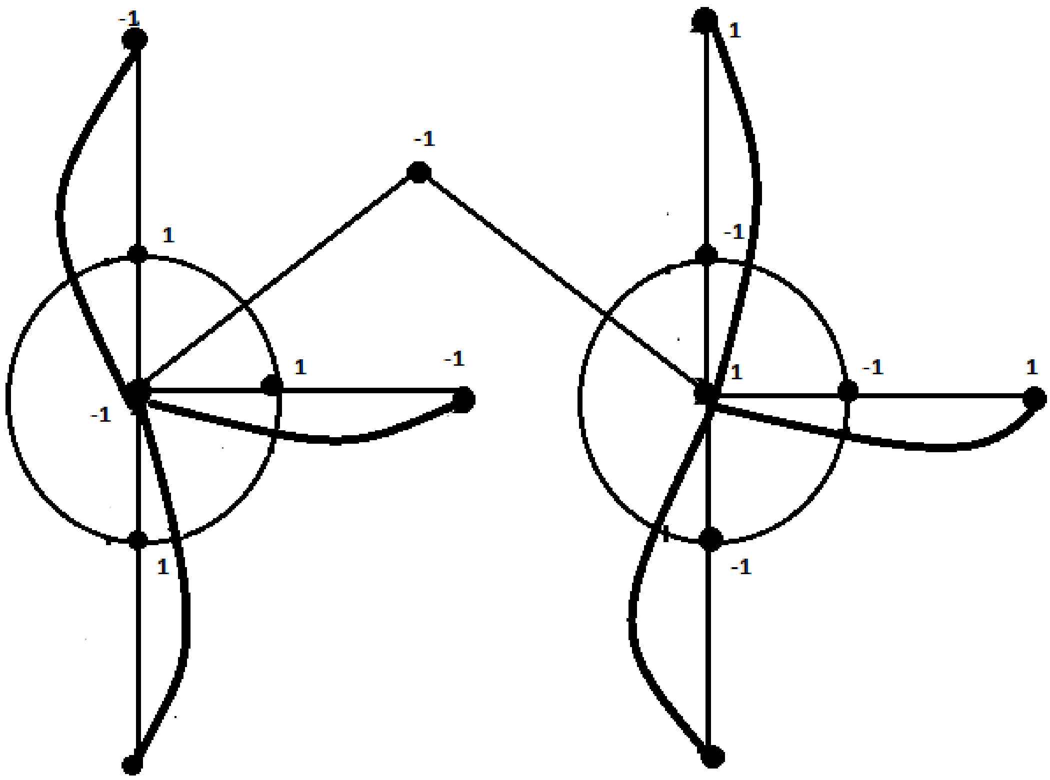

Fig.1.

Theorem 2.3.Path union of two copies of binary tree at the root vertex is signed product cordial graph.

Proof

Let G be the path union of two copies binary tree at the root vertex with m $(m \geq 1)$ levels.

Let G_1 and G_2 be two copies of binary tree with m levels.

Let u and v be the root of G_1 and G_2 respectively.

Clearly i^{th} level of G_1 and G_2 has 2^i vertices.

Then $|V(G_i)| = 2^{m+1} - 1$ and $|E(G_i)| = 2^{m+1} - 2$ (i=1,2)

Let u_i and $v_i$be the vertices in the i^{th} level of G_1 and G_2respectively. $(i = 1,2,\cdots,2^{i+1} - 1)$.

Let w_i be the vertices of path $P_k(k \geq 2)$ then $w_1 = u$ and $w_n = v$.

Define $f: V(G) \rightarrow \{-1,1\}$ as given below.

$$f(u_i) = \begin{cases} 1, & i \text{ is odd}, \\ -1, & i \text{ is even}. \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i \text{ is odd}, \\ -1, & i \text{ is even}. \end{cases}$$

$$f(w_i) = \begin{cases} -1, & i \equiv (1,2) \ (mod\ 4), \\ 1, & i \equiv (0,3)(mod\ 4). \end{cases}$$

$f(w_n) = 1$ if $k \equiv 2\ (mod\ 4)$

By the above labeling pattern, we have,

Case(i) Suppose $k = 2$.

$$v_f(-1) = v_f(1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(-1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(1) = \frac{|V(G)|}{2} - 1$$

Case(ii) Suppose $k \equiv 0\ (mod\ 4)$.

$$v_f(-1) = v_f(1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(-1) = \frac{|V(G)|}{2} - 1$$

Case(ii) Suppose$k \equiv (1,3)\ (mod\ 4)$.

$$v_f(-1) = \frac{|V(G)| + 1}{2}$$

$$v_f(1) = \frac{|V(G)| - 1}{2}$$

$$e_{f^*}(-1) = e_{f^*}(1) = \frac{|V(G)|}{2} - 1$$

Case(ii) Suppose$k \equiv 2\ (mod\ 4)$ and $k \neq 2$.

$$v_f(-1) = v_f(1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(-1) = \frac{|V(G)|}{2}$$

$$e_{f^*}(1) = \frac{|V(G)|}{2} - 1$$

By all the above cases, we have

$$|v_f(-1)-v_f(1)|\leq 1 \text{ and } |e_{f^*}(-1)-e_{f^*}(1)|\leq 1.$$

Hence, G is signed product cordial labeling graph.

Example 2.4.

The graph obtained by joining two copies of binary tree (7 vertices) with its signed product cordial labeling.

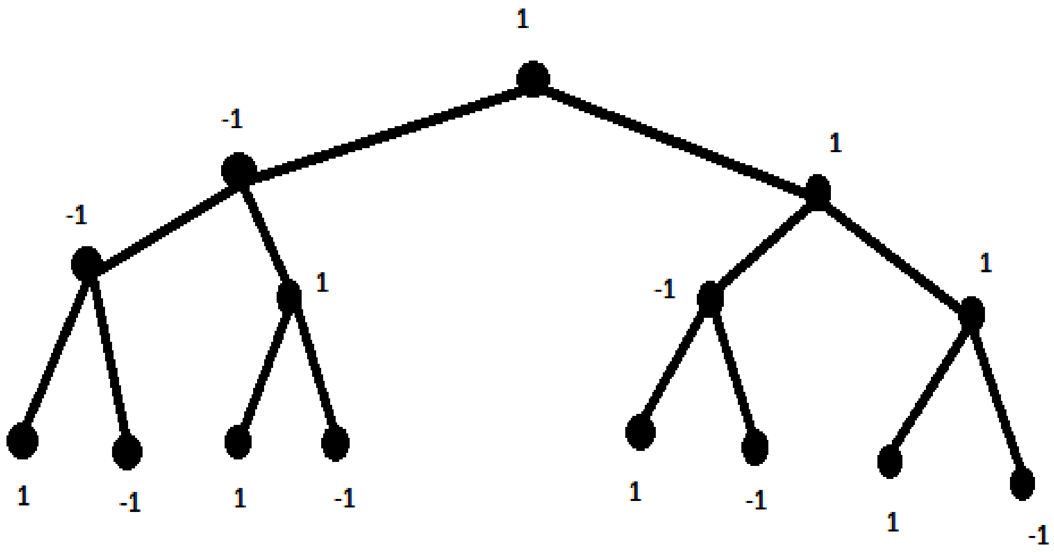

Fig. 2.

Theorem 2.5. Path union of two copies of binary tree at the root vertex is a total signed product cordial graph.

Proof

Let G be the graph obtained by joining two copies of binary tree by path P_k, labeling the vertices as in the above theorem 2.3., we have

$$\left|(v_f(-1)+e_{f^*}(-1))-(v_f(1)+e_{f^*}(1))\right|\leq 1.$$

Hence, G is a total signed product cordial graph.

Theorem 2.6. Path union of two copies of $K_{1,n}$ by P_2 is a signed product cordial graph.

Proof

Let G be the path union of two copies of $K_{1,n}$ by P_2.

Let u and v be the centre vertices of G_1 and G_2 respectively.

Let u_i and v_i be the remaining vertices of G_1 and G_2 respctively. $(i=1,2,\dots,n)$

Then $|V(G_i)|=n+1$ and $|E(G_i)|=n$ (i= 1,2).

Let w_i be the vertices of path P_2 with $w_1 = u$ and $w_2 = v$.

Define $f: V(G) \to \{-1,1\}$ as given below.

$$f(u) = -1$$

$$f(u) = -1$$

$$f(u_i) = \begin{cases} -1, & i \text{ is odd,} \\ 1, & i \text{ is even} \end{cases}$$

$$f(w_1) = -1$$

$$f(w_2) = 1$$

Case (i) Suppose $n \equiv (1,3)\ (mod\ 4)$.

$$v_f(-1) = v_f(1) = n + 1$$

$$e_{f^*}(-1) = n$$

$$e_{f^*}(1) = n + 1$$

Case (ii) Suppose $n \equiv (0,2)\ (mod\ 4)$.

$$v_f(-1) = v_f(1) = n + 1$$

$$e_{f^*}(-1) = n + 1$$

$$e_{f^*}(1) = n$$

Therefore, $|v_f(-1) - v_f(1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$.

Hence, G is a signed product cordial graph.

Example 2.7. Path union of two copies of star by P_2 with its signed product cordial labeling

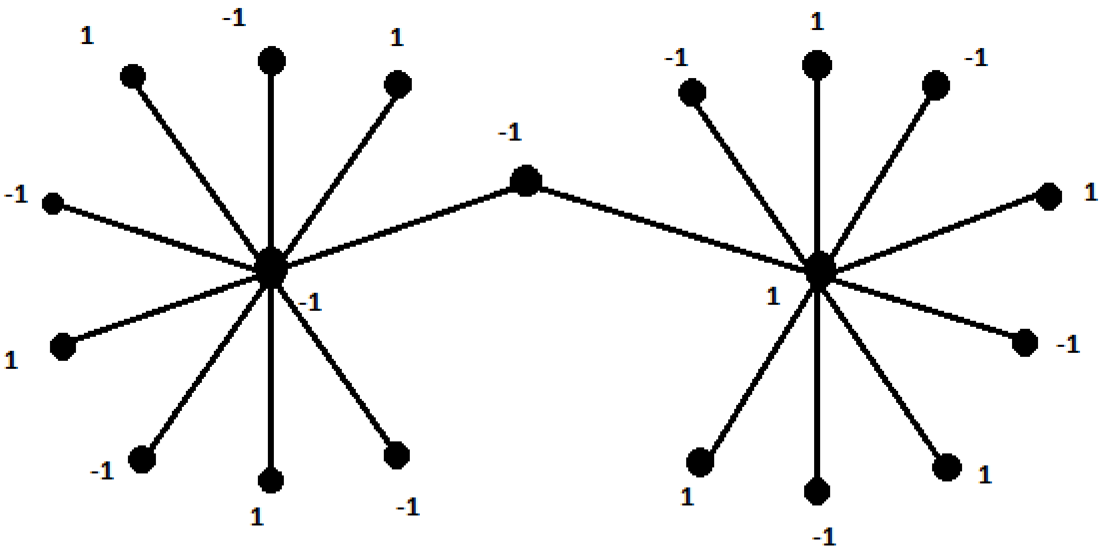

Fig.3.

Theorem 2.8. Path union of two copies of star by P_2 is a total signed product cordial graph.

Proof

Let G be the path union of two copies of $K_{1,n}$by P_2. Labeling the vertices as in the above theorem 2.6., we have

Case (i) Suppose $n \equiv (1,3)(mod\ 4)$.

$$v_f(-1) + e_{f^*}(-1) = 2n + 1$$

$$v_f(1) + e_{f^*}(1) = 2n + 2$$

Case (ii) Suppose $n \equiv (0,2)(mod\ 4)$.

$$v_f(-1) + e_{f^*}(-1) = 2n + 2$$

$$v_f(1) + e_{f^*}(1) = 2n + 1$$

By the above cases, we have

$$|(v_f(-1) + e_{f^*}(-1)) - (v_f(1) + e_{f^*}(1))| = 1$$

Therefore, $|(v_f(-1) + e_{f^*}(-1)) - (v_f(1) + e_{f^*}(1))| \leq 1$.

Hence, G is a total signed product cordial grapih.

Theorem 2.9. $T(P_n)$ is a total signed product cordial graph.

Proof

Let $G = T(P_n)$ be the total graph of P_n.

Let v_i and e_j be vertices and edges of P_n (i = 1,2,…,n) and (j = 1,2,…,n-1)

Then $|V(G)| = 2n-1$ and $|E(G)| = 2n+1$.

Define $f : V(G) \to \{-1,1\}$ as given below.

$$f(v_i) = \begin{cases} -1, & 1 \le i \le n, \\ 1, & otherwise. \end{cases}$$

$$f(e_i) = \begin{cases} 1, & 1 \le i \le n-1, \\ -1, & otherwise. \end{cases}$$

By the above labeling pattern, we have

$$v_f(-1) = n+1$$

$$v_f(1) = n$$

$$e_{f^*}(-1) = 2n$$

$$e_{f^*}(1) = n+1.$$

Thus, we have

$$|(v_f(-1) + e_{f^*}(-1)) - (v_f(1) + e_{f^*}(1))| \le 1.$$

Hence, $T(P_n)$ is a total sigmed product cordial graph.

Example 2.10

$T(P_6)$ with its total signed product cordial labeling.

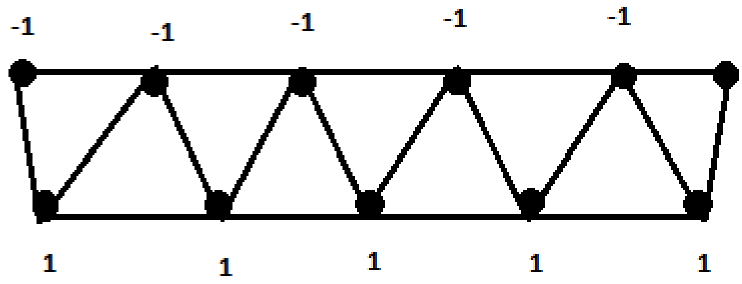

Fig.4.

Theorem 2.11. $G = (K_{1,n} \odot G')$ is a signed product cordial graph.

Proof

Let $G = (K_{1,n} \odot G')$ be a graph with $V(G) = \{u, u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n, w_1, w_2, \cdots, w_n\}$ t and $E(G) = \{uu_i, 1 \le i \le n\} \cup \{u_i v_i, 1 \le i \le n\} \cup \{u_i w_i, 1 \le i \le n\}$.

Define $f : V(G) \to \{-1, 1\}$ as given below.

$$f(u) = -1.$$

$$f(u_i) = \begin{cases} 1, & i \text{ is odd}, \\ -1, & i \text{ is even}. \end{cases}$$

$$f(v_i) = 1 \qquad 1 \le i \le n$$

$$f(w_i) = -1 \qquad 1 \le i \le n$$

Case (i) Suppose n is odd.

$$v_f(-1) = v_f(1) = \frac{3n+3}{2}$$

$$e_{f^*}(-1) = \frac{3n+3}{2}$$

$$e_{f^*}(1) = \frac{3n+1}{2}$$

Case (ii) Suppose n is even.

$$v_f(-1) = \frac{3n+2}{2}$$

$$v_f(1) = \frac{3n+4}{2}$$

$$e_{f^*}(-1) = e_{f^*}(1) = \frac{3n+2}{2}$$

By the above cases, we have

$$|v_f(-1) - v_f(1)| \le 1 \text{ and } |e_{f^*}(-1) - e_{f^*}(1)| \le 1.$$

Hence, $G = (K_{1,n} \odot G')$ is a signed product cordial graph.

Example

$G = (K_{1,7} \odot G')$ with its signed product cordial graph.

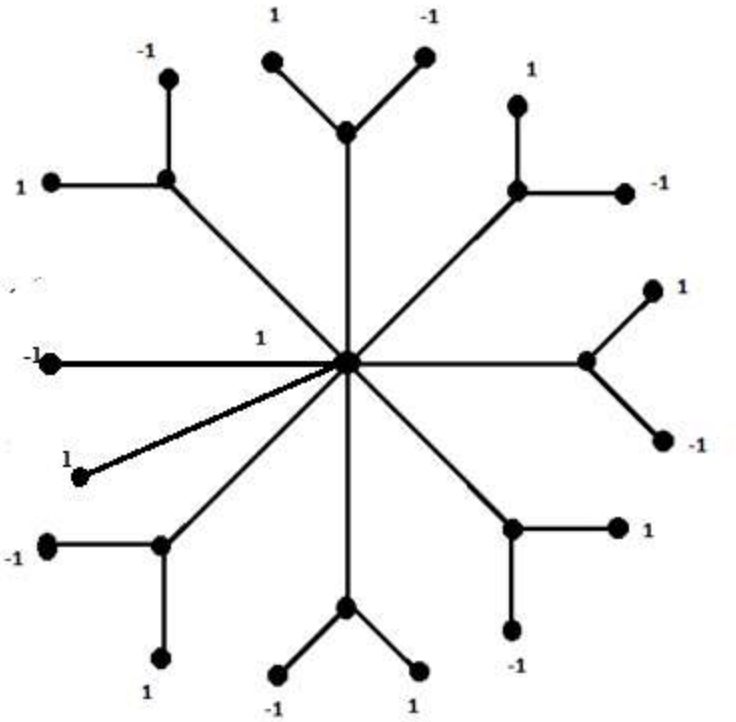

Fig.5.

Theorem 2.12. $G = (K_{1,n} \odot G')$ is a total signed product cordial graph.

Proof

Let $G = (K_{1,n} \odot G')$be a graph with labeling pattern as in the above theorem 2.11., we have

Case (i) Suppose n is odd.

$$v_f(-1) + e_{f^*}(-1) = 3n + 3$$

$$v_f(1) + e_{f^*}(1) = 3n + 2$$

Case (ii) Suppose n is even.

$$v_f(-1) + e_{f^*}(-1) = 3n + 2$$

$$v_f(1) + e_{f^*}(1) = 3n + 3$$

Therefore, $|(v_f(-1) + e_{f^*}(-1)) - (v_f(-1) + e_{f^*}(-1))| \le 1$.

Hence, $G = (K_{1,n} \odot G')$is a total signed product cordial graph.

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