# An Application Of Generalized Trapezoidal Fuzzy Soft Sets For Finding The Best Tennis Palyer 

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#### Abstract

: This paper deals the concept of generalized trapezoidal fuzzy soft sets for solving multi attribute decision making problem. Generalized intuitionistic fuzzy soft set is the extension of intuitionistic fuzzy soft sets. In this paper, we are analysing the performance of five tennis players and also finding the best player among the five by using generalized trapezoidal fuzzy soft set theory.


## Keywords:

Fuzzy soft sets, Generalized fuzzy soft sets, Generalized trapezoidal Fuzzy soft sets and multi criteria decision making problem.

## Introduction:

The applications of soft set theory are many including the smoothness of functions, operation research, game etc. Maji et. Al introduced the study on hybrid structures involving fuzzy sets and soft sets. The trapezoidal fuzzy number is an important concept of fuzzy set and also it can be applied in many fields. The membership function of a trapezoidal fuzzy number is piecewise linear and trapezoidal. Xiao presented the concept of the trapezoidal fuzzy soft sets by combining both trapezoidal fuzzy number and soft set models.

## Definition :

Let $\mathrm{U}=\left\{x_{1}, x_{2}, \ldots \ldots . x_{n}\right\}$ be the universal set of elements and $\mathrm{E}=\left\{e_{1}, e_{2}, e_{3}, \ldots \ldots . e_{m}\right\}$ be the universal set of parameters. Let $F$ : $E \rightarrow I^{U}$ and $\mu$ be a fuzzy subset of $E$, ie., $\mu: E \rightarrow I=[0,1]$, where $I^{U}$ be the collection of all fuzzy subsets of U. A pair $(F, \mu)$ is called the fuzzy soft set, FSS over $U$. It is denoted by $F_{\mu}(e)=(F(e), \mu(e))$.

## Trapezoidal fuzzy number:

A fuzzy number $\tilde{A}$ is trapezoidal fuzzy number denoted by $\tilde{A}=\left(a_{1}, a_{2}, a_{3}, a_{4}\right)$ where $a_{1}, a_{2}, a_{4}$ and $a_{3}$ are real numbers and its membership function $\mu_{A}(\mathrm{x})$ is given by

$$
\mu_{A}(\mathrm{x})=\left\{\begin{array}{cc}
0, & \mathrm{x}<a_{1} \\
\frac{x-a_{1}}{a_{2}-a_{1}}, & a_{1} \leq x<a_{2} \\
1, & a_{2} \leq x \leq a_{3} \\
\frac{x-a_{4}}{a_{4}-a_{3},} & a_{3} \leq x<a_{4} \\
0, & \text { otherwise }
\end{array}\right.
$$

## Linguistic assessment for ratings



The membership function of a trapezoidal fuzzy number is piecewise linear and trapezoidal which can capture the vagueness of linguistic assessments.

## Example:

The linguistic variable "medium good" can be represented as $(0.5,0.6,0.7,0.8)$ and the membership function is

$$
\mu_{\text {medium good }}(\mathrm{x})=\left\{\begin{array}{cl}
0, & \mathrm{x}<0.2 \\
\frac{x-0.5}{0.6-0.5}, & 0.5 \leq x<0.6 \\
1, & 0.6 \leq x \leq 0.7 \\
\frac{x-0.8}{0.7-0.8}, & 0.5 \leq x<0.6 \\
0, & x>0.8
\end{array}\right.
$$

## Trapezoidal fuzzy set:

A set which is consisted by a trapezoidal fuzzy number or several trapezoidal fuzzy numbers is called trapezoidal fuzzy set and it is denoted by $\tilde{I}$.

## Trapezoidal fuzzy soft set:

Let $U$ be the universe set, $\operatorname{TF}(\mathrm{U})$ be the set of all trapezoidal fuzzy subsets of U . The trapezoidal fuzzy soft set over U is defined by a pair $(\tilde{F}, \mathrm{~A})$ where $\tilde{F}$ is a mapping given by

$$
\tilde{F}: \mathrm{A} \rightarrow \mathrm{TF}(\mathrm{U}) .
$$

## Generalized trapezoidal fuzzy soft set (GTFSS):

Let $U$ be the universe set and $E$ be the set of parameters. The pair ( $U, E$ ) is called soft universe. Suppose that $\mathrm{F}: \mathrm{E} \rightarrow \mathrm{TF}(\mathrm{U})$ and $\tilde{f}: \mathrm{E} \rightarrow \tilde{I}$ where $\tilde{f}$ is a trapezoidal fuzzy subset of E . We say that $\tilde{F}_{\tilde{f}}$ generalized trapezoidal fuzzy soft set over (U, E) if

$$
\tilde{F}_{\tilde{f}}: E \rightarrow T F(U) \times \tilde{I},
$$

Where $\tilde{F}_{\tilde{f}}(e)=(\tilde{F}(e), \tilde{f}(\mathrm{e}))$, such that $\forall e \in E, \tilde{F}(e) \in T F(U)$ and $\tilde{f}(\mathrm{e}) \in \tilde{I}$.
For each parameter $e_{i}, \tilde{F}_{\tilde{f}}\left(e_{i}\right)=\left(\tilde{F}\left(e_{i}\right), \tilde{f}\left(e_{i}\right)\right)$ indicates not only the trapezoidal fuzzy membership degree of belongingness of the elements of U in $\mathrm{F}\left(e_{i}\right)$ but also the trapezoidal fuzzy membership degree of possibility of such belongingness of the parameters of E which is represented by $\tilde{f}\left(e_{i}\right)$.

We can note that a generalized trapezoidal fuzzy soft set $\tilde{F}_{\tilde{f}}$ is actually a soft set because it is still a mapping from parameters to $T F(U) \times \tilde{I}$, and it can be written as

$$
\tilde{F}_{\tilde{f}}(e)=(\tilde{F}(e), \tilde{f}(e))
$$

Where,

$$
\tilde{F}(e)=\left\{\frac{u}{\mu_{\tilde{F}(e)}^{1}(u), \mu_{\tilde{F}(e)}^{2}(u), \mu_{\tilde{F}(e)}^{3}(u), \mu_{\tilde{F}(e)}^{4}(u)}, u \in U\right\}
$$

$\tilde{f}(\mathrm{e})=\left(\mu_{\tilde{f}(\mathrm{e})}^{1}, \mu_{\tilde{f}(\mathrm{e})}^{2}, \mu_{\tilde{f}(\mathrm{e})}^{3}, \mu_{\tilde{f}(\mathrm{e})}^{4}\right)$.

## Union of two generalized trapezoidal fuzzy soft sets:

Let $\tilde{F}_{\tilde{f}}$ and $\tilde{G}_{\tilde{g}}$ be the two generalized trapezoidal fuzzy soft sets over (U, E). Then the union of these two generalized trapezoidal fuzzy soft sets is denoted by
$\tilde{F}_{\tilde{f}} \widetilde{U} \tilde{G}_{\tilde{g}}$ and it is defined by the mapping given by

$$
\widetilde{H}_{\widetilde{h}}: E \rightarrow T F(U) \times \tilde{I} \forall u \in U
$$

Such that $\widetilde{H}_{\widetilde{h}}(e)=(\widetilde{H}(e), \tilde{h}(\mathrm{e}))$
Where,

$$
\begin{aligned}
\widetilde{H}(e) & =\tilde{F}(e) \widetilde{\cup} \tilde{G}(e) \\
& =\left\{\frac{u}{\mu_{\tilde{F}(e)}^{1}(u) \widetilde{\cup} \mu_{\tilde{G}(e)}^{1}(u)}: u \in U\right\} \\
& =\left\{\frac{u}{\left.\mu_{\tilde{F}(e)}^{1}(u) \vee \mu_{\tilde{G}(e)^{1}}^{1}(u), \mu_{\tilde{F}(e)^{2}}(u) \vee \mu_{\widetilde{G}(e)^{2}}^{(u), \mu_{\tilde{F}(e)^{3}}^{3}(u) \vee \mu_{\widetilde{G}(e)}^{3}(u), \mu_{\tilde{F}(e)^{4}}^{(u) \mu_{\tilde{G}(e)}^{4}}(u)}, u \in U\right\}}\right.
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{h}(\mathrm{e}) & =\tilde{f}(\mathrm{e}) \widetilde{\cup} \tilde{g}(\mathrm{e}) \\
& =\left(\mu_{\tilde{f}(\mathrm{e})}^{1} \vee \mu_{\tilde{g}(\mathrm{e})}^{1}, \mu_{\tilde{f}(\mathrm{e})}^{2} \vee \mu_{\tilde{g}(\mathrm{e})}^{2}, \mu_{\tilde{f}(\mathrm{e})}^{3} \vee \mu_{\tilde{g}(\mathrm{e})}^{3}, \mu_{\tilde{f}(\mathrm{e})}^{4} \vee \mu_{\tilde{g}(\mathrm{e})}^{4}\right) .
\end{aligned}
$$

## Example:

Consider the generalized trapezoidal fuzzy soft set $\tilde{F}_{\tilde{f}}$ over (U, E) as

$$
\begin{aligned}
& \tilde{F}_{\tilde{f}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.1,0.2,0.2,0.3)}, \frac{t_{2}}{(0.1,0.2,0.2,0.3)}, \frac{t_{3}}{(0.4,0.5,0.5,0.6)},\right. \\
& \left.\left.\frac{t_{4}}{(0.5,0.6,0.7,0.8)}\right\},(0.4,0.5,0.5,0.6)\right) \\
& \tilde{F}_{\tilde{f}}\left(e_{2}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.4,0.5,0.5,0.6)},\right. \\
& \left.\left.\frac{t_{4}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right) \\
& \tilde{F}_{\tilde{f}}\left(e_{3}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{3}}{(0.5,0.6,0.7,0.8)},\right. \\
& \left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right) \\
& \tilde{F}_{\tilde{f}}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.5,0.6,0.7,0.8)}, \frac{t_{2}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.7,0.8,0.8,0.9)},\right. \\
& \left.\left.\frac{t_{4}}{(0.7,0.8,0.8,0.9)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{aligned}
$$

Let us consider the another generalized trapezoidal fuzzy soft set $\tilde{G}_{\tilde{g}}$ over (U,E) is defined as follows:

$$
\begin{gathered}
\tilde{G}_{\tilde{g}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.1,0.2,0.2,0.3)}, \frac{t_{2}}{(0.1,0.2,0.2,0.3)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)},\right. \\
\\
\begin{array}{c}
\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
\tilde{G}_{\tilde{g}}\left(e_{2}\right)=\left\{\frac{t_{1}}{(0.8,0.9,1.0,1.0)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.1,0.2,0.2,0.3)},\right. \\
\\
\\
\left.\tilde{G}_{\tilde{g}}\left(e_{3}\right)=\left\{\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
\\
\left.t_{1}, 0.5,0.5,0.6\right)
\end{array} \frac{t_{1}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.7,0.8,0.8,0.9)},
\end{gathered}
$$

$$
\begin{gathered}
\left.\left.\frac{t_{4}}{(0.5,0.6,0.7,0.8)}\right\},(0.2,0.3,0.4,0.5)\right) \\
\tilde{G}_{\tilde{g}}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.4,0.5,0.5,0.6)}, \frac{t_{2}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)},\right. \\
\left.\left.\frac{t_{4}}{(0.4,0.5,0.5,0.6)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{gathered}
$$

Then the union of $\tilde{F}_{\tilde{f}} \widetilde{\cup} \tilde{G}_{\tilde{g}}$ is

$$
\begin{aligned}
& \widetilde{H}_{\tilde{f h}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.4,0.5,0.5,0.6)},\right. \\
& \left.\left.\frac{t_{4}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right) \\
& \widetilde{H}_{\tilde{f h}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.8,0.9,01.0,1.0)}, \frac{t_{2}}{(0.7,0.8,0.8,0.9)}, \frac{t_{3}}{(0.5,0.6,0.7,0.8)},\right. \\
& \left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
& \tilde{F}_{f}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.5,0.6,0.7,0.8)}, \frac{t_{2}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.7,0.8,0.8,0.9)},\right. \\
& \left.\left.\frac{t_{4}}{(0.7,0.8,0.8,0.9)}\right\},(0.2,0.3,0.4,0.5)\right)
\end{aligned}
$$

## Intersection of two generalized trapezoidal fuzzy soft sets:

Let $\tilde{F}_{\tilde{f}}$ and $\tilde{G}_{\tilde{g}}$ be the two generalized trapezoidal fuzzy soft sets over (U, E). Then the intersection of these two generalized trapezoidal fuzzy soft sets is denoted by
$\tilde{F}_{\tilde{f}} \widetilde{\cap} \tilde{G}_{\tilde{g}}$ and it is defined by the mapping given by

$$
\widetilde{H}_{\widetilde{h}}: E \rightarrow T F(U) \times \tilde{I} \forall u \in U
$$

Such that $\widetilde{H}_{\widetilde{h}}(e)=(\widetilde{H}(e), \widetilde{h}(\mathrm{e}))$

Where, $\widetilde{H}(e)=\tilde{F}(e) \widetilde{\cap} \tilde{G}(e)$

$$
\begin{aligned}
& =\left\{\frac{u}{\mu_{\tilde{F}(e)}^{1}(u) \tilde{\cap} \mu_{\tilde{G}(e)}^{1}(u)}: u \in U\right\} \\
& =\left\{\frac{u}{\mu_{\tilde{F}(e)}^{1}(u) \wedge \mu_{\tilde{G}(e)}^{1}(u), \mu_{\tilde{F}(e)}^{2}(u) \wedge \mu_{\tilde{G}(e)}^{2}(u), \mu_{\tilde{F}(e)}^{3}(u) \wedge \mu_{\tilde{G}(e)}^{3}(u), \mu_{\tilde{F}(e)}^{4}(u) \wedge \mu_{\tilde{G}(e)}^{4}(u)}, u \in U\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{h}(\mathrm{e}) & =\tilde{f}(\mathrm{e}) \widetilde{n} \tilde{g}(\mathrm{e}) \\
& =\left(\mu_{\tilde{f}(\mathrm{e})}^{1} \wedge \mu_{\tilde{g}(\mathrm{e})}^{1}, \mu_{\tilde{f}(\mathrm{e})}^{2} \wedge \mu_{\tilde{g}(\mathrm{e})}^{2}, \mu_{\tilde{f}(\mathrm{e})}^{3} \wedge \mu_{\tilde{g}(\mathrm{e})}^{3}, \mu_{\tilde{f}(\mathrm{e})}^{4} \wedge \mu_{\tilde{g}(\mathrm{e})}^{4}\right) .
\end{aligned}
$$

## Example:

Consider the generalized trapezoidal fuzzy soft set $\tilde{F}_{\tilde{f}}$ over (U, E) as

$$
\begin{gathered}
\tilde{F}_{\tilde{f}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.1,0.2,0.2,0.3)}, \frac{t_{2}}{(0.1,0.2,0.2,0.3)}, \frac{t_{3}}{(0.4,0.5,0.5,0.6)},\right. \\
\left.\frac{t_{4}}{(0.5,0.6,0.7,0.8)}\right\},(0.4,0.5,0.5,0.6), \\
\tilde{F}_{\tilde{f}}\left(e_{2}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.4,0.5,0.5,0.6)},\right. \\
\\
\left.\tilde{F}_{\tilde{f}}\left(e_{3}\right)=\left\{\frac{t_{4}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right) \\
\tilde{F}_{\tilde{f}}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{3}}{(0.5,0.6,0.7,0.8)},\right. \\
\\
\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right) \\
\left.\left.\frac{t_{4}}{(0.7,0.8,0.8,0.9)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{gathered}
$$

Let us consider the another generalized trapezoidal fuzzy soft set $\tilde{G}_{\tilde{g}}$ over (U,E) is defined as follows:

$$
\begin{aligned}
& \tilde{G}_{\tilde{g}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.1,0.2,0.2,0.3)}, \frac{t_{2}}{(0.1,0.2,0.2,0.3)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)},\right. \\
&\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
& \tilde{G}_{\tilde{g}}\left(e_{2}\right)=\left\{\frac{t_{1}}{(0.8,0.9,1.0,1.0)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.1,0.2,0.2,0.3)},\right. \\
&\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
& \tilde{G}_{\tilde{g}}\left(e_{3}\right)=\left\{\frac{t_{1}}{(0.4,0.5,0.5,0.6)}, \frac{t_{1}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.7,0.8,0.8,0.9)},\right.
\end{aligned}
$$

$$
\left.\left.\frac{t_{4}}{(0.5,0.6,0.7,0.8)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\begin{gathered}
\tilde{G}_{\tilde{g}}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.4,0.5,0.5,0.6)}, \frac{t_{2}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)}\right. \\
\left.\left.\frac{t_{4}}{(0.4,0.5,0.5,0.6)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{gathered}
$$

Then the intersection, $\tilde{F}_{\tilde{f}} \widetilde{\cap} \tilde{G}_{\tilde{g}}$ is

$$
\begin{gathered}
\widetilde{H}_{\widetilde{h}}\left(e_{1}\right)=\left\{\frac{t_{1}}{(0.1,0.2,0.2,0.3)}, \frac{t_{2}}{(0.1,0.2,0.2,0.3)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)}\right. \\
\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right)
\end{gathered}
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{2}\right)=\left\{\frac{t_{1}}{(0.7,0.8,0.8,0.9)}, \frac{t_{2}}{(0.2,0.3,0.4,0.5)}, \frac{t_{3}}{(0.1,0.2,0.2,0.3)},\right.
$$

$$
\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{3}\right)=\left\{\frac{t_{1}}{(0.4,0.5,0.5,0.6)}, \frac{t_{1}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.5,0.6,0.7,0.8)},\right.
$$

$$
\left.\left.\frac{t_{4}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{4}\right)=\left\{\frac{t_{1}}{(0.4,0.5,0.5,0.6)}, \frac{t_{2}}{(0.5,0.6,0.7,0.8)}, \frac{t_{3}}{(0.2,0.3,0.4,0.5)}\right.
$$

$$
\left.\left.\frac{t_{4}}{(0.4,0.5,0.5,0.6)}\right\},(0.1,0.2,0.2,0.3)\right)
$$

## The AND operator:

Let $\left(\tilde{F}_{\tilde{f}}, A\right)$ and $\left(\tilde{G}_{\tilde{g}}, B\right)$ be the two generalized trapezoidal fuzzy soft sets over
(U, E). Then the AND operator between these two generalized trapezoidal fuzzy soft sets is denoted by
" $\left(\tilde{F}_{\tilde{f}}, A\right) \operatorname{AND}\left(\tilde{G}_{\tilde{g}}, B\right)$ " or $\tilde{F}_{\tilde{f}} \wedge \tilde{G}_{\tilde{g}}=\left(\widetilde{H}_{\tilde{h}}, A \times B\right)$
Where, $\widetilde{H}_{\widetilde{h}}(\alpha, \beta)=(\widetilde{H}(\alpha, \beta), \tilde{h}(\alpha, \beta)), \forall(\alpha, \beta) \in A \times B$
Such that $\widetilde{H}(\alpha, \beta)=\tilde{F}(\alpha) \widetilde{\cap} \tilde{G}(\beta)$

$$
=\left\{\frac{u}{\mu_{\tilde{F}(\alpha)}(u) \widetilde{n} \mu_{\widetilde{G}(\alpha)}(u)}: u \in U\right\} \text { and }
$$

$$
\tilde{h}(\alpha, \beta)=\mu_{\tilde{F}(\alpha)} \widetilde{\cap} \mu_{\tilde{G}(\alpha)}
$$

## The OR operator:

Let $\left(\tilde{F}_{\tilde{f}}, A\right)$ and $\left(\tilde{G}_{\tilde{g}}, B\right)$ be the two generalized trapezoidal fuzzy soft sets over ( $\mathrm{U}, \mathrm{E}$ ). Then the OR operator between these two generalized trapezoidal fuzzy soft sets is denoted by

$$
\text { " }\left(\tilde{F}_{\tilde{f}}, A\right) \text { OR }\left(\tilde{G}_{\tilde{g}}, B\right) " \text { or } \tilde{F}_{\tilde{f}} \vee \tilde{G}_{\tilde{g}}=\left(\widetilde{H}_{\tilde{h}}, A \times B\right)
$$

Where, $\widetilde{H}_{\widetilde{h}}(\alpha, \beta)=(\widetilde{H}(\alpha, \beta), \tilde{h}(\alpha, \beta)), \forall(\alpha, \beta) \in A \times B$

Such that $\widetilde{H}(\alpha, \beta)=\tilde{F}(\alpha) \widetilde{U} \tilde{G}(\beta)$

$$
=\left\{\frac{u}{\mu_{\widetilde{F}(\alpha)}(u) \widetilde{\widetilde{U}} \mu_{\widetilde{G}(\alpha)}(u)}: u \in U\right\} \text { and }
$$

$\tilde{h}(\alpha, \beta)=\mu_{\tilde{F}(\alpha)} \widetilde{\cup} \mu_{\tilde{G}(\alpha)}$.

## Defuzzification method of a trapezoidal fuzzy number:

Y. Celik and S. Yamakintroduced the method of defuzzification of a trapezoidal fuzzy number. Trapezoidal number is parametrized by $\left(a_{1}, a_{2}, a_{3}, a_{4}\right)$, then the defuzzification value $t$ of the trapezoidal fuzzy number is calculated by

$$
\begin{aligned}
& \left(t-a_{2}\right)+\frac{1}{2}\left(a_{2}-a_{1}\right)=\left(a_{3}-t\right)+\frac{1}{2}\left(a_{4}-a_{3}\right) \\
& \quad \Rightarrow 2 t=\frac{1}{2}\left[a_{4}-a_{3}-a_{2}+a_{1}+2 a_{2}+2 a_{3}\right] \\
& \mathrm{t}=\frac{a_{1}+a_{2}+a_{3}+a_{4}}{4}
\end{aligned}
$$

## Application of generalized trapezoidal fuzzy soft sets

In group decision making problems, everyone has different opinions. The attribute of the parameters are vague and also imprecise. Comparing to trapezoidal fuzzy soft set, generalized trapezoidal fuzzy soft set is more realistic and it gives accurate value although each person has various opinions on the vague attributes of the same parameter.

## Algorithm

Write the ratings of the five players under various skills for Mr. Y.

1. Write the corresponding generalized trapezoidal fuzzy soft sets $\tilde{F}_{\tilde{f}}$ by using step 1 .
2. Write the ratings of the five players under various skills for Mr. Z .
3. Write the corresponding generalized trapezoidal fuzzy soft sets $\tilde{G}_{\tilde{g}}$ by using step 3 .
4. By using the "AND" operator on $\tilde{F}_{\tilde{f}}$ and $\tilde{G}_{\tilde{g}}$, we can obtain $\widetilde{H}_{\widetilde{h}}$.
5. Find the defuzzification value of $\widetilde{H}_{\widetilde{h}}$ and write it in the tabular form.
6. Find the highest grade and possibility grade then write it in the grade table.
7. Find the scores for each players who are in the grade table.
8. Select the maximum score.
9. The corresponding player with the maximum score is the best one.

## Finding the best tennis player among the five players:

Let $\mathrm{U}=\left\{p_{1}, p_{2}, p_{3}, p_{4}, p_{5}\right\}$ be the set of five players. Suppose that two senior tennis players Mr. Y and Mr. Z are invited to select the best tennis player among five players. The judges asked to select the best one by observing their individual talents. Here the talents are nothing but the skills which is more important to be a best tennis player. There are such more skills to be a tennis player such as general athletic ability, explosiveness, balance, hand-eye co ordination, ball judgement and so on.

But here we are considering only five skills such as general atheletic ability, explosiveness, balance, hand-eye co ordination and ball judgement.
i.e.,
$e_{2}=$ general athletic ability, $e_{5}=$ explosiveness, $e_{6}=$ balace, $e_{8}=$ hand - eye co ordination, $e_{9}=$ ball judgement $\}$

By observing these skills from the players, they have to select the best one. Mr. Y and Mr. Z has different opinions about the players.

Step 1: Write the ratings of the five players under various skills for Mr. Y.

Mr. Y describes the five players under various skills with linguistic variables intuitively as below:

| U | $e_{2}$ | $e_{5}$ | $e_{6}$ | $e_{8}$ | $e_{9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{1}$ | Medium good | Good | Very good | Fair | Poor |
| $p_{2}$ | Poor | Good | Fair | Poor | Medium poor |
| $p_{3}$ | Good | Fair | Medium poor | Fair | Good |
| $p_{4}$ | Very good | Very good | Good | Fair | Poor |
| $p_{5}$ | Poor | Good | Very good | Medium good | Medium poor |
| $\tilde{f}$ | Poor | Very good | Good | Very god | Medium good |

Step 2: Write the corresponding generalized trapezoidal fuzzy soft sets $\tilde{F}_{\tilde{f}}$ by using step 1

From the tabular column, we can obtain a corresponding generalized trapezoidal fuzzy soft set $\tilde{F}_{\tilde{f}}$ as follows:

$$
\begin{aligned}
\tilde{F}_{\tilde{f}}\left(e_{2}\right)= & \left(\left\{\frac{p_{1}}{(0.5,0.6,0.7,0.8)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)^{\prime}}\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.8,0.9,0.9,1.0)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.4,0.5,0.5,0.6)\right) \\
\tilde{F}_{\tilde{f}}\left(e_{5}\right)= & \left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.7,0.8,0.8,0.9)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)^{\prime}}\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.8,0.9,0.9,1.0)}, \frac{p_{5}}{(0.7,0.8,0.8,0.9)}\right\},(0.8,0.9,0.9,1.0)\right)
\end{aligned}
$$

Step 3: Write the ratings of the five players under various skills to Mr. Z.

$$
\begin{aligned}
\tilde{F}_{\tilde{f}}\left(e_{6}\right)= & \left(\left\{\frac{p_{1}}{(0.8,0.9,0.9,1.0)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.7,0.8,0.8,0.9)}, \frac{p_{5}}{(0.8,0.9,0.9,1.0)}\right\},(0.7,0.8,0.8,0.9)\right) \\
\tilde{F}_{\tilde{f}}\left(e_{8}\right)= & \left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.5,0.6,0.7,0.8)}\right\},(0.5,0.6,0.7,0.8)\right) \\
\tilde{F}_{\tilde{f}}\left(e_{9}\right)= & \left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
\end{aligned}
$$

## Step 4:

Write the corresponding generalized trapezoidal fuzzy soft sets $\tilde{G}_{\tilde{g}}$ by using step 3 .

| U | $e_{2}$ | $e_{5}$ | $e_{6}$ | $e_{8}$ | $e_{9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{1}$ | Good | Fair | good | Fair | Medium Poor |
| $p_{2}$ | Good | Medium Good | Fair | Medium Poor | Poor |
| $p_{3}$ | Very Good | Fair | Medium good | Medium good | Good |
| $p_{4}$ | Fair | Very good | Fair | Poor | Medium good |
| $p_{5}$ | Poor | Very Good | good | fair | Medium poor |
| $\tilde{g}$ | Medium poor | Good | Very good | Good | Very good |

$$
\begin{aligned}
\tilde{G}_{\tilde{g}}\left(e_{2}\right)= & \left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.7,0.8,0.8,0.9)}, \frac{p_{3}}{(0.8,0.9,0.9,1.0)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right) \\
\tilde{G}_{\tilde{g}}\left(e_{5}\right)= & \left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.5,0.6,0.7,0.8)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.8,0.9,0.9,1.0)}, \frac{p_{5}}{(0.8,0.9,0.9,1.0)}\right\},(0.7,0.8,0.8,0.9)\right) \\
\tilde{G}_{\tilde{g}}\left(e_{6}\right)= & \left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.7,0.8,0.8,0.9)}\right\},(0.8,0.9,0.9,1.0)\right) \\
\tilde{G}_{\tilde{g}}\left(e_{8}\right)= & \left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.5,0.6,0.7,0.8)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.4,0.5,0.5,0.6)}\right\},(0.7,0.8,0.8,0.9)\right)
\end{aligned}
$$

$$
\begin{array}{r}
\tilde{G}_{\tilde{g}}\left(e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.2,0.3,0.4,0.5)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right. \\
\left.\frac{p_{4}}{(0.5,0.6,0.7,0.8)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\}
\end{array}
$$

Step 5: By using "AND" operator on $\tilde{F}_{\tilde{f}}$ and $\tilde{G}_{\tilde{g}}$, we can obtain $\widetilde{H}_{\tilde{h}}$

Here we are using the AND operator since the different opinions of the judges has to be considered and it is denoted by the generalized trapezoidal fuzzy soft set $\widetilde{H}_{\widetilde{h}}$.

$$
\begin{array}{r}
\widetilde{H}_{\tilde{h}}\left(e_{2}, e_{2}\right)=\left(\left\{\frac{p_{1}}{(0.5,0.6,0.7,0.8)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right. \\
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.1,0.2,0.2,0.3)\right) \\
\widetilde{H}_{\widetilde{h}}\left(e_{2}, e_{5}\right)= \\
\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right. \\
\\
\left.\left.\quad \frac{p_{4}}{(0.8,0.9,0.9,1.0)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{array}
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{2}, e_{6}\right)=\left(\left\{\frac{p_{1}}{(0.5,0.6,0.7,0.8)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right.
$$

$$
\begin{array}{r}
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.1,0.2,0.2,0.3)\right) \\
\tilde{H}_{\widetilde{n}}\left(e_{2}, e_{8}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.5,0.6,0.7,0.8)},\right.\right. \\
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.1,0.2,0.2,0.3)\right)
\end{array}
$$

$$
\widetilde{H}_{\tilde{h}}\left(e_{2}, e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.2,0.3,0.4,0.5)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)}\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.5,0.6,0.7,0.8)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.1,0.2,0.2,0.3)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{5}, e_{2}\right)=\left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.7,0.8,0.8,0.9)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\begin{aligned}
\widetilde{H}_{\overparen{h}}\left(e_{5}, e_{5}\right)= & \left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.5,0.6,0.7,0.8)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right. \\
& \left.\left.\frac{p_{4}}{(0.8,0.9,0.9,1.0)}, \frac{p_{5}}{(0.7,0.8,0.8,0.9)}\right\},(0.7,0.8,0.8,0.9)\right) \\
\widetilde{H}_{\overparen{h}}\left(e_{5}, e_{6}\right)= & \left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
\end{aligned}
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.7,0.8,0.8,0.9)}\right\},(0.8,0.9,0.9,1.0)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{5}, e_{8}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.4,0.5,0.5,0.6)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{5}, e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.2,0.3,0.4,0.5)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.4,0.5,0.5,0.6)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\tilde{h}}\left(e_{6}, e_{2}\right)=\left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\widetilde{H}_{\overparen{h}}\left(e_{6}, e_{5}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.7,0.8,0.8,0.9)}, \frac{p_{5}}{(0.8,0.9,0.9,1.0)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\widetilde{n}}\left(e_{6}, e_{6}\right)=\left(\left\{\frac{p_{1}}{(0.7,0.8,0.8,0.9)}, \frac{p_{2}}{(0.4,0.5,0.5,0.6)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.7,0.8,0.8,0.9)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{6}, e_{8}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.4,0.5,0.5,0.6)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{6}, e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.2,0.3,0.4,0.5)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.2,0.3,0.4,0.5)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.5,0.6,0.7,0.8)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.7,0.8,0.8,0.9)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{8}, e_{2}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{8}, e_{5}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.5,0.6,0.7,0.8)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{8}, e_{6}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.5,0.6,0.7,0.8)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{8}, e_{8}\right)=\left(\left\{\frac{p_{1}}{(0.4,0.5,0.5,0.6)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.4,0.5,0.5,0.6)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{8}, e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.2,0.3,0.4,0.5)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.4,0.5,0.5,0.6)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{9}, e_{2}\right)=\left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.1,0.2,0.2,0.3)}\right\},(0.2,0.3,0.4,0.5)\right)
$$

$$
\widetilde{H}_{\widetilde{h}}\left(e_{9}, e_{5}\right)=\left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.4,0.5,0.5,0.6)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\tilde{h}}\left(e_{9}, e_{6}\right)=\left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\widetilde{H}_{\tilde{h}}\left(e_{9}, e_{8}\right)=\left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.2,0.3,0.4,0.5)}, \frac{p_{3}}{(0.5,0.6,0.7,0.8)},\right.\right.
$$

$$
\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
$$

$$
\begin{aligned}
& \widetilde{H}_{\widetilde{h}}\left(e_{9}, e_{9}\right)=\left(\left\{\frac{p_{1}}{(0.1,0.2,0.2,0.3)}, \frac{p_{2}}{(0.1,0.2,0.2,0.3)}, \frac{p_{3}}{(0.7,0.8,0.8,0.9)}\right.\right. \\
&\left.\left.\frac{p_{4}}{(0.1,0.2,0.2,0.3)}, \frac{p_{5}}{(0.2,0.3,0.4,0.5)}\right\},(0.5,0.6,0.7,0.8)\right)
\end{aligned}
$$

Step 6: Find the defuzzification value of $\widetilde{H}_{\widetilde{h}}$ and write it in the tabular form
From the generalized trapezoidal fuzzy soft set $\widetilde{H}_{\widetilde{h}}$ we can immediately write the following defuzzification table.

Table 1: Defuzzification table

|  | $p_{1}$ | $p_{2}$ | $p_{3}$ | $p_{4}$ | $p_{5}$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(e_{2}, e_{2}\right)$ | 0.65 | 0.2 | 0.8 | 0.5 | 0.2 | 0.2 |
| $\left(e_{2}, e_{5}\right)$ | 0.5 | 0.2 | 0.5 | 0.9 | 0.2 | 0.2 |
| $\left(e_{2}, e_{6}\right)$ | 0.65 | 0.2 | 0.8 | 0.9 | 0.2 | 0.2 |
| $\left(e_{2}, e_{8}\right)$ | 0.5 | 0.2 | 0.65 | 0.2 | 0.2 | 0.2 |
| $\left(e_{2}, e_{9}\right)$ | 0.35 | 0.2 | 0.8 | 0.65 | 0.2 | 0.2 |
| $\left(e_{5}, e_{2}\right)$ | 0.8 | 0.8 | 0.5 | 0.5 | 0.2 | 0.35 |
| $\left(e_{5}, e_{5}\right)$ | 0.5 | 0.65 | 0.5 | 0.9 | 0.8 | 0.8 |
| $\left(e_{5}, e_{6}\right)$ | 0.8 | 0.5 | 0.5 | 0.5 | 0.8 | 0.9 |
| $\left(e_{5}, e_{8}\right)$ | 0.5 | 0.35 | 0.5 | 0.2 | 0.5 | 0.8 |
| $\left(e_{5}, e_{9}\right)$ | 0.35 | 0.2 | 0.5 | 0.2 | 0.5 | 0.8 |
| $\left(e_{6}, e_{2}\right)$ | 0.8 | 0.5 | 0.35 | 0.5 | 0.2 | 0.35 |
| $\left(e_{6}, e_{5}\right)$ | 0.5 | 0.5 | 0.35 | 0.8 | 0.9 | 0.8 |
| $\left(e_{6}, e_{6}\right)$ | 0.8 | 0.5 | 0.35 | 0.5 | 0.8 | 0.8 |
| $\left(e_{6}, e_{8}\right)$ | 0.5 | 0.35 | 0.35 | 0.2 | 0.5 | 0.8 |
| $\left(e_{6}, e_{9}\right)$ | 0.35 | 0.2 | 0.35 | 0.65 | 0.35 | 0.8 |
| $\left(e_{8}, e_{2}\right)$ | 0.5 | 0.2 | 0.5 | 0.5 | 0.2 | 0.35 |
| $\left(e_{8}, e_{5}\right)$ | 0.5 | 0.2 | 0.5 | 0.5 | 0.65 | 0.65 |
| $\left(e_{8}, e_{6}\right)$ | 0.5 | 0.2 | 0.5 | 0.5 | 0.65 | 0.65 |


| $\left(e_{8}, e_{8}\right)$ | 0.5 | 0.2 | 0.5 | 0.2 | 0.5 | 0.65 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(e_{8}, e_{9}\right)$ | 0.35 | 0.2 | 0.5 | 0.5 | 0.35 | 0.65 |
| $\left(e_{9}, e_{2}\right)$ | 0.2 | 0.35 | 0.8 | 0.2 | 0.2 | 0.35 |
| $\left(e_{9}, e_{5}\right)$ | 0.2 | 0.35 | 0.5 | 0.2 | 0.35 | 0.65 |
| $\left(e_{9}, e_{6}\right)$ | 0.2 | 0.35 | 0.8 | 0.2 | 0.35 | 0.65 |
| $\left(e_{9}, e_{8}\right)$ | 0.2 | 0.35 | 0.65 | 0.2 | 0.35 | 0.65 |
| $\left(e_{9}, e_{9}\right)$ | 0.2 | 0.2 | 0.8 | 0.2 | 0.35 | 0.65 |

We can find highest score and possibility score by using the defuzzification table.

Table 2: Grade table

|  | Players | Highest grade | Possibility grade |
| :---: | :---: | :--- | :--- |
| $\left(e_{2}, e_{2}\right)$ | $p_{3}$ | - | - |
| $\left(e_{2}, e_{5}\right)$ | $p_{4}$ | 0.9 | 0.2 |
| $\left(e_{2}, e_{6}\right)$ | $p_{4}$ | 0.9 | 0.2 |
| $\left(e_{2}, e_{8}\right)$ | $p_{3}$ | 0.65 | 0.2 |
| $\left(e_{2}, e_{9}\right)$ | $p_{3}$ | 0.8 | 0.2 |
| $\left(e_{5}, e_{2}\right)$ | $p_{1}, p_{2}$ | 0.8 | 0.35 |
| $\left(e_{5}, e_{5}\right)$ | $p_{4}$ | - | - |
| $\left(e_{5}, e_{6}\right)$ | $p_{1}, p_{5}$ | 0.8 | 0.9 |
| $\left(e_{5}, e_{8}\right)$ | $p_{1}, p_{3}, p_{5}$ | 0.5 | 0.8 |
| $\left(e_{5}, e_{9}\right)$ | $p_{3}, p_{5}$ | 0.5 | 0.8 |
| $\left(e_{6}, e_{2}\right)$ | $p_{1}$ | 0.8 | 0.8 |
| $\left(e_{6}, e_{5}\right)$ | $p_{5}$ | 0.9 |  |
| $\left(e_{6}, e_{6}\right)$ | $p_{1}, p_{5}$ | - | 0.8 |
|  |  |  |  |


| $\left(e_{6}, e_{8}\right)$ | $p_{1}, p_{5}$ | 0.5 | 0.8 |
| :---: | :---: | :--- | :--- |
| $\left(e_{6}, e_{9}\right)$ | $p_{4}$ | 0.65 | 0.8 |
| $\left(e_{8}, e_{2}\right)$ | $p_{1}, p_{3}, p_{4}$ | 0.5 | 0.35 |
| $\left(e_{8}, e_{5}\right)$ | $p_{1}, p_{3}, p_{4}$ | 0.5 | 0.65 |
| $\left(e_{8}, e_{6}\right)$ | $p_{1}, p_{3}, p_{4}$ | 0.5 | 0.65 |
| $\left(e_{8}, e_{8}\right)$ | $p_{1}, p_{3}, p_{5}$ | - | - |
| $\left(e_{8}, e_{9}\right)$ | $p_{3}, p_{4}$ | 0.5 | 0.65 |
| $\left(e_{9}, e_{2}\right)$ | $p_{3}$ | 0.8 | 0.35 |
| $\left(e_{9}, e_{5}\right)$ | $p_{3}$ | 0.5 | 0.65 |
| $\left(e_{9}, e_{6}\right)$ | $p_{3}$ | 0.8 | 0.65 |
| $\left(e_{9}, e_{8}\right)$ | $p_{3}$ | 0.65 | 0.65 |
| $\left(e_{9}, e_{9}\right)$ | $p_{3}$ | - | - |
|  |  |  |  |

Table 3: Score table

| Players | score |
| :---: | :---: |
| $p_{1}$ | 1.68 |
| $p_{2}$ | 0.28 |
| $p_{3}$ | $\mathbf{3 . 3 6 5}$ |
| $p_{4}$ | 2.03 |
| $p_{5}$ | 2.64 |

The highest score in the table is 3.365 corresponding to the player $p_{3}$. Thus the player $p_{3}$ is the best badminton player among all the other players.

## Conclusion:

Mr. Y and Mr. Z has different opinions on the attributes of the same player. Suppose Mr. Y may think that player $p_{4}$ used the skill "smash" is good but Mr. Z may thinks that the same player used that skill is "fair". It may be impossible to obtain more accurate values for the subjective assessments. Trapezoidal membership function are enough to capture the vagueness of the attribute of the parameter. Therefore generalized trapezoidal fuzzy soft set is more effective to express the decision making problems when the attribute of the parameters is imprecise and vague.

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